Quantum Field Theory
Homework set 1, Due Tu Oct 19

1. Show that the combination
$$\frac{d^3p}{2E}, \qquad \text{with } \ E=\sqrt{\vec{p}^{\,2}+m^2}$$
which occurs frequently in phase space calculation integration is invariant under Lorentz transformation.

2. Consider the combination of the form,
$$(\phi_1,\phi_2)\equiv\int d^3x\,[\phi_1^*\partial_0\phi_2-\phi_2\partial_0\phi_1^*]$$
   (a) Show that this combination is time independent

   (b) Write the plane wave solutions in the form,
$$f_p^{(\pm)}(x)=e^{\mp ip\cdot x}\frac{1}{\sqrt{(2\pi)^3\,2\omega_p}}, \qquad \text{where } p_0=\omega_p=\sqrt{\vec{p}^{\,2}+\mu^2}\geq 0$$
   Show that
$$\int d^3x\left[f_{p'}^{(\pm)*}(x)\,i\overleftrightarrow{\partial_0}f_p^{(\pm)}(x)\right]=\pm\delta^3\left(\vec{p}-\vec{p}'\right)$$
   and
$$\int d^3x\left[f_{p'}^{(\pm)*}(x)\,i\overleftrightarrow{\partial_0}f_p^{(\mp)}(x)\right]=0$$
   (c) If we write the general solution $\phi(x)$ in the form,
$$\phi(x)=\int d^3p\left[a_pf_p^{(+)}(x)+a_p^*f_p^{(-)}(x)\right]$$
   compute the coefficients, a_p and a_p^*.

3. Consider a system where 2 particles interacting with eac other through potential energy $V\left(\vec{x}_1-\vec{x}_2\right)$ so that the Lagrangian is of the form,
$$L=\frac{m_1}{2}\left(\frac{d\vec{x}_1}{dt}\right)^2+\frac{m_2}{2}\left(\frac{d\vec{x}_2}{dt}\right)^2-V\left(\vec{x}_1-\vec{x}_2\right)$$
   (a) Show that this Lagrangian is invariant under the spatial translation given by
$$\vec{x}_1\to\vec{x}_1^{\,\prime}=\vec{x}_1+\vec{a}, \qquad \vec{x}_2\to\vec{x}_2^{\,\prime}=\vec{x}_2+\vec{a},$$
   where $\vec{a}$ is an arbitrary vector.

   (b) Use Noether's theorem to construct the conserved quantity corresponding to this symmetry.

4. (Optional) Construct the Lorentz transformation for motion of coordinate axis in arbitrary ddirection by using the fact that coordinates perpendicualr to the direction of motion remain unchanged.

5. (Optional) Show that 2 consecutive Lorentz transformtion for motion in the same direction is also a Lorentz tranformation in the same direction. What is the corresponding velocity for the resulting transformation.