Quantum Field Theory

Ling-Fong Li

National Center for Theoretical Science

Path integral formalism has close relationship to classical dynamics, e.g. the transition amplitude

$$\langle f|i\rangle = \int [dx]\, e^{iS/\hbar}$$

as $\hbar \to 0$, the trajectory with smallest S dominates, the action principle. Here uses the ordinary functions not the operators. Later in non-Abelian gauge theory, to remove unphysical degrees of freedom can be accomodated in the path integral formalism by imposing constraints in the integral.

Quantum Mechanics in 1-dimension

In QM, transition from $|q,t\rangle$ to $\langle q',t'|$, can be written as,

$$\langle q't'|qt\rangle = \langle q|'^{-iH(t-t')}|q\rangle$$

where $|q\rangle's$ are eigenstates of position operator Q in the Schrodinger picture,

$$Q|q\rangle = q|q\rangle$$

and $|q,t\rangle$ denotes corresponding state in Heisenberg picture,

$$|q,t\rangle = e^{iHt}|q\rangle$$

Ipath integral formalism,this can be written as

$$\langle q't'|qt\rangle = N\int [dq] exp\{i\int_t^{t'} d\tau L(q,\dot{q})\}$$

To get this formula, divide the interval $(t\prime, t)$ into n intervals ,

$$\delta t = \frac{t' - t}{n}$$

and write ,

$$\langle q'|e^{-iH(t'-t)}|q\rangle = \int dq_1 ... dq_{n-1} \langle q'|e^{-iH\delta t}|q_{n-1}\rangle\langle q_{n-1}|e^{-iH\delta t}|q_{n-2}\rangle ... \langle q_1|e^{-iH\delta t}|q\rangle$$

For δt small enough,

$$\langle q'|e^{-iH\delta t}|q\rangle = \langle q'|(1 - iH(P, Q)\delta t)|q\rangle + O\left((\delta t)^2\right) + ...$$

Suppose,

$$H(P, Q) = \frac{p^2}{2m} + V(Q)$$

then

$$\begin{aligned} \langle q'|H|q\rangle &= \langle q'|\frac{p^2}{2m}|q\rangle + V(\frac{q + q'}{2})\delta(q - q') \\ &= \int \langle q'|\frac{p^2}{2m}|p\rangle\langle p|q\rangle(\frac{dp}{2\pi}) + V(\frac{q + q'}{2}) \int \frac{dp}{2\pi} e^{ip(q'-q)} \\ &= \int \frac{dp}{2\pi} e^{ip(q'-q)} [\frac{p^2}{2m} + V(\frac{q + q'}{2})] \end{aligned}$$

where

$$\langle p|q\rangle = e^{-ipq}$$

is the momentum eigenfunction. Exponentiation of this infinitesmal result

$$\begin{aligned}\langle q'|e^{-iH\delta t}|q\rangle &\simeq \int \frac{dp}{2\pi} e^{ip(q'-q)}\{1 - i\delta t[\frac{p^2}{2m} + V(\frac{q+q'}{2})]\} \\ &\simeq \int \frac{dp}{2\pi} \exp\left[ip(q'-q)\right] \exp\left[-i\delta t[\frac{p^2}{2m} + V(\frac{q+q'}{2})]\right]\end{aligned}$$

The whole transition matrix element can then be written as

$$\langle q'|e^{-iH(t'-t)}|q\rangle \cong \int (\frac{dp_1}{2\pi})...(\frac{dp_n}{2\pi}) \int dq_1...dq_{n-1} \exp\{i\left[\sum_{i=1}^{n} p_i(q_i - q_{i-1}) - (\delta t)H(p_i, \frac{q_i + q_{i+1}}{2})\right]\}$$

This can be written formally as

$$\langle q'|e^{-iH(t'-t)}|q\rangle = \int [\frac{dpdq}{2\pi}] exp\{i \int_t^{t'} dt[p\dot{q} - H(p,q)]\}$$

$$\equiv \lim_{n\to\infty} \int (\frac{dp_1}{2\pi})...(\frac{dp_n}{2\pi}) \int dq_1...dq_{n-1} exp\{i\sum_{i=1} \delta t[p_i(\frac{q_i - q_{i-1}}{\delta t}) - H(p_i, \frac{q_i + q_{i+1}}{2})]\}$$

If Hamiltonian depends quadractically on p, use the formula

$$\int_{-\infty}^{+\infty} \frac{dx}{2\pi} e^{-ax^2+bx} = \frac{1}{\sqrt{4\pi a}} e^{\frac{b^2}{4a}}$$

to get

$$\int \frac{dp_i}{2\pi} \exp[\frac{-i\delta t}{2m} p_i^2 + ip_i(q_i - q_{i-1})] = (\frac{m}{2\pi i\delta t})^{1/2} exp[\frac{im(q_i - q_{i-1})^2}{2\delta t}]$$

Then

$$\langle q'|e^{-iH(t'-t)}|q\rangle = \lim_{n\to\infty}\left(\frac{m}{2\pi i\delta t}\right)^{n/2}\int\prod_{i=1}^{n-1}dq_i\exp\{i\sum_{i=1}^{n}\delta t[\frac{m}{2}(\frac{q_i-q_{i-1}}{\delta t})^2-V]\}$$

or

$$\langle q't'|qt\rangle = \langle q'|e^{-iH(t'-t)}|q\rangle = N\int[dq]\exp\{i\int_t^{t'}d\tau[\frac{m}{2}\dot{q}^2-V(q)]\}$$

This is the path integral representation for amplitude from initial state $|q,t\rangle$ to final state $\langle q',t'|$. Or

$$\langle q't'|qt\rangle == N\int[dq]\exp iS$$

Green's functions

To generalize this to field theory where the basic entity is the vacuum expectation value of field operators, we consider

$$G(t_1,t_2) = \langle 0|T(Q^H(t_1)Q^H(t_2))|0\rangle$$

Inserting complete sets of states, we get

$$G(t_1,t_2) = \int dqdq' \langle 0|q',t'\rangle\langle q',t'|T(Q^H(t_1)Q^H(t_2))|q,t\rangle\langle q,t|0\rangle$$

The matrix element

$$\langle 0|q,t\rangle = \phi_0(q)e^{-iE_0 t} = \phi_0(q,t)$$

is the wavefunction for ground state. Consider the case

$$t' > t_1 > t_2 > t,$$

we can write

$$\langle q',t'|T(Q^H(t_1)Q^H(t_2))|q,t\rangle = \langle q'|e^{-iH(t'-t_1)}Q^s e^{-iH(t_1-t_2)}Q^s e^{-iH(t_2-t)}|q\rangle$$

$$= \int \langle q'|e^{-iH(t'-t_1)}|q_1\rangle q_1 \langle q_1|e^{-iH(t_1-t_2)}|q_2\rangle q_2 \langle q_2|e^{-iH(t_2-t)}|q\rangle dq_1 dq_2$$

$$= \int [\frac{dpdq}{2\pi}] q_1(t_1)q_2(t_2) exp\{i\int_t^{t'} d\tau [p\dot{q} - H(p,q)]\}$$

For the other time sequence

$$t' > t_2 > t_1 > t,$$

we get same formula, because path integral orders the time sequence automatically through the division of time interval into small pieces. The Green's function is then

$$G(t_1,t_2) = \int dqdq'\phi_0(q',t')\phi_0^*(q,t)\int[\frac{dpdq}{2\pi}]q_1(t_1)q_2(t_2)exp\{i\int_t^{t'}d\tau[p\dot{q}-H(p,q)]\} \quad (1)$$

We remove wavefunction $\phi_0(q,t)$ by the following procedure. Write

$$\langle q',t'|\theta(t_1,t_2)|q,t\rangle = \int dQdQ'\langle q',t'|Q',T'\rangle\langle Q',T'|\theta(t_1,t_2)|Q,T\rangle\langle Q,t|q,t\rangle$$

where

$$\theta(t_1,t_2) = T(Q^H(t_1)Q^H(t_2))$$

Let $|n>$ be eigenstate with energy E_n and wave function ϕ_n, i.e.,

$$H|n>= E_n|n>, \quad \langle q|n\rangle = \phi_n^*(q)$$

Then

$$\langle q',t'|Q',t'\rangle = \langle q'|e^{-iH(t'-T')}|Q'\rangle = \sum_n\langle q'|n\rangle e^{-iE_n(t'-T')}\langle n|Q'\rangle = \sum_n \phi_n^*(q')\phi_n(Q')e^{-iE_n(t'-T')}$$

To isolate the ground state wavefunction, we take an "unusual limit",

$$\lim_{t'\to -i\infty}\langle q',t'|Q',T'\rangle = \phi_0^*(q')\phi_0(Q')e^{-E_0|t'|}e^{iE_0T'}$$

Similarity,

$$\lim_{t\to i\infty}\langle Q,T|q,t\rangle = \phi_0(q)\phi_0^*(Q)e^{-E_0|t|}e^{-iE_0T}$$

With these we write

$$\lim_{\substack{t'\to -i\infty\\ t\to i\infty}}\langle q',t'|\theta(t_1,t_2)|q,t\rangle = \int dQdQ'\phi_0^*(q')\phi_0(Q')\langle Q',T'|\theta(t_1,t_2)|Q,T\rangle\phi_0^*(Q)\phi_0(q)e^{-E_0|t'|}e^{iE_0T'}e^{-iE_0T}e^{-E_0|t|}$$

$$= \phi_0^*(q')\phi_0(q)e^{-E_0|t'|}e^{-E_0|t|}G(t_1,t_2)$$

It is easy to see that

$$\lim_{\substack{t'\to -i\infty\\ t\to i\infty}}\langle q',t'|q,t\rangle = \phi_0^*(q')\phi_0(q)e^{-E_0|t'|}e^{-E_0|t|}$$

Finally, the Green function can be written as,

$$G(t_1,t_2) = \lim_{\substack{t'\to -i\infty\\ t\to i\infty}}\left[\frac{\langle q',t'|T(Q^H(t_1)Q^H(t_2))|q,t\rangle}{\langle q',t'|q,t\rangle}\right]$$

$$= \lim_{\substack{t'\to -i\infty\\ t\to i\infty}}\frac{1}{\langle q',t'|q,t\rangle}\int\left[\frac{dpdq}{2\pi}\right]q(t_1)q(t_2)\exp\{i\int_t^{t'}d\tau[p\dot{q}-H(p,q)]\}$$

This can generalized to n-point Green's function with the result,

$$G(t_1,t_2,...,t_n) = \langle 0|T(q(t_1)q(t_2)...q(t_n))|0\rangle$$

$$= \lim_{\substack{t' \to -i\infty \\ t \to i\infty}} \frac{1}{\langle q', t' | q, t \rangle} \int \left[\frac{dpdq}{2\pi} \right] q(t_1)q(t_2)...q(t_n) \exp\{i \int_t^{t'} d\tau [p\dot{q} - H(p,q)]\}$$

It is very useful to introduce generating functional for these n-point functions

$$W[J] = \lim_{\substack{t' \to -i\infty \\ t \to i\infty}} \frac{1}{\langle q', t' | q, t \rangle} \int \left[\frac{dpdq}{2\pi} \right] \exp\{i \int_t^{t'} d\tau [p\dot{q} - H(p,q) + J(\tau)q(\tau)]\}$$

Then

$$G(t_1, t_2, ..., t_n) = (-i)^n \left. \frac{\delta^n}{\delta J(t_1)...\delta J(t_n)} \right|_{J=0}$$

The unphysical limit, $t' \to -i\infty, t \to i\infty$, should be interpreted in term of Eudidean Green's functions defined by

$$S^{(n)}(\tau_1, \tau_2, ..., \tau_n) = i^n G^{(n)}(-i\tau_1, -i\tau_2, ..., -i\tau_n)$$

Generating functional for $S^{(n)}$ is then

$$W_E[J] = \lim_{\substack{\tau' \to \infty \\ \tau \to -\infty}} \int [dq] \frac{1}{\langle q', t' | q, t \rangle} \exp\{\int_\tau^{\tau'} d\tau'' [-\frac{m}{2} \left(\frac{dq}{d\tau''} \right)^2 - V(q) + J(\tau'')q(\tau'')]\}$$

Since we can adjust the zero point of $V(q)$ such that

$$\frac{m}{2} \left(\frac{dq}{d\tau} \right)^2 + V(q) > 0$$

which provides the damping to give a converging Gaussian integral. In this form, we can see that any constant in the path integral which is independent of q will be canceled out in the generation functional.

Field Theory

From quantum mechanics to field theory of a scalar field $\phi(x)$ replace,

$$\prod_{i=1}^{\infty}[dq_i dp_i] \longrightarrow [d\phi(x) d\pi(x)]$$

$$L(q,\dot{q}) \longrightarrow \int \mathcal{L}(\phi,\partial_\mu\phi) d^3x \qquad H(p,q) \longrightarrow \int \mathcal{H}(\phi,\pi) d^3x$$

Generating functional is

$$W[J] \backsim \int [d\phi] \exp\{i \int d^4x [\mathcal{L}(\phi,\partial_\mu\phi) + J(x)\phi(x)]\}$$

functional derivative is defined by

$$\frac{\delta F[\phi(x)]}{\delta\phi(y)} = \lim_{\varepsilon\to 0} \frac{F[\phi(x) + \varepsilon\delta(x-y)] - F[\phi(x)]}{\varepsilon}$$

Then

$$\frac{\delta W[J]}{\delta J(y)} = i \int [d\phi]\, \phi(y) \exp\{i \int d^4x [\mathcal{L}(\phi,\partial_\mu\phi) + J(x)\phi(x)]\} \tag{2}$$

and

$$\frac{\delta^2 W[J]}{\delta J(y_1)\,\delta J(y_2)} = (i)^2 \int [d\phi]\, \phi(y_1)\, \phi(y_2) \exp\{i \int d^4x [\mathcal{L}(\phi,\partial_\mu\phi) + J(x)\phi(x)]\}$$

Consider $\lambda\phi^4$ theory

$$\mathcal{L}(\phi) = \mathcal{L}_0(\phi) + \mathcal{L}_1(\phi)$$

$$\mathcal{L}_0(\phi) = \frac{1}{2}(\partial_\lambda \phi)^2 - \frac{\mu^2}{2}\phi^2, \qquad \mathcal{L}_1(\phi) = -\frac{\lambda}{4!}\phi^4$$

Use Euclidean time the generating functional

$$W\left[J\right] = \int [d\phi] \exp\{-\int d^4x[\frac{1}{2}(\frac{\partial\phi}{\partial\tau})^2 + \frac{1}{2}(\overrightarrow{\bigtriangledown}\phi)^2 + \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 - J\phi]\}$$

can be written as

$$W\left[J\right] = \left[\exp\int d^4x\mathcal{L}_I\left(\frac{\delta}{\delta J\left(x\right)}\right)\right] W_0\left[J\right]$$

We have used Eq(2) to write the interaction term in terms of function derivative with repect to the source $J\left(x\right)$ .Here $W_0\left[J\right]$ is the free field generating function

$$W_0\left[J\right] = \int [d\phi] \exp[-\frac{1}{2}\int d^4xd^4y\phi(x)K(x,y)\phi(y) + \int d^4zJ(z)\phi(z)]$$

and

$$K\left(x,y\right) = \delta^4\left(x-y\right)\left(-\frac{\partial^2}{\partial\tau^2} - \overrightarrow{\bigtriangledown}^2 + \mu^2\right)$$

. The Gaussian integral for many variables is

$$\int d\phi_1 d\phi_2 ... d\phi_n \exp\left[-\frac{1}{2}\sum_{i,j}\phi_i K_{ij}\phi_j + \sum_k J_k\phi_k\right] \backsim \frac{1}{\sqrt{detK}} \exp\left[\frac{1}{2}\sum_{i,j}J_i(K^{-1})_{ij}J_j\right]$$

Apply this to the case of scalar fields,

$$W_0\left[J\right] = \exp\left[\frac{1}{2}\int d^4xd^4yJ(x)\triangle(x,y)J(y)\right]$$

where

$$\int d^4yK(x,y)\triangle(y,z) = \delta^4(x-z)$$

$\triangle(x,y)$ can be calculated by Fourier transform to give,

$$\triangle(x,y) = \int \frac{d^4k_E}{(2\pi)^4}\frac{e^{ik_E(x-y)}}{k_E^2+\mu^2}$$

where $k_E = (ik_0, \overrightarrow{k})$,the Euclidean momentum
Perturbative expansion in power of λ gives

$$W\left[J\right] = W_0\left[J\right]\{1+\lambda w_1\left[J\right]+\lambda^2 w_2\left[J\right]+...\}$$

where

$$w_1 = -\frac{1}{4!}W_0^{-1}\left[J\right]\{\int d^4x\left[\frac{\delta}{\delta J(x)}\right]^4\}W_0\left[J\right]$$

$$w_2 = -\frac{1}{2\,(4!)^2}W_0^{-1}\left[J\right]\{\int d^4x\left[\frac{\delta}{\delta J(x)}\right]^4\}^2W_0\left[J\right]$$

Use explicit form for $W_0[J]$,

$$W_0[J] = 1 + \frac{1}{2}\int d^4x d^4y J(x) \triangle(x,y) J(y) + \left(\frac{1}{2}\right)^2 \frac{1}{2!}\int d^4y_1 d^4y_2 d^4y_3 d^4y_4 \left[J(y_1)\triangle(y_1,y_2)J(y_2)J(y_3)\triangle(y_3,y_4)J(y_4)\right] + ...$$

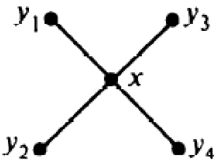
We get for w_1,

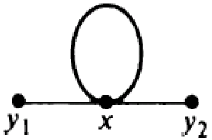
$$w_1 = -\frac{1}{4!}\left[\int \triangle(x,y_1)\triangle(x,y_2)\triangle(x,y_3)\triangle(x,y_4)J(y_1)J(y_2)J(y_3)J(y_4) + 3!\triangle(x,y_1)\triangle(x,y_2)J(y_1)J(y_2)\triangle(x,\right.$$

we dropped all J independent terms, and all (x_i, y_i) are integrated over. In this computation we have used the identity,

$$\frac{\delta}{\delta J(x)}\int d^4y_1 J(y_1) f(y_1) = \int \delta^4(x-y_1) d^4y_1 f(y_1) = f(x)$$

Graphical representation for w_1





The connected Green's function is

$$G^{(n)}(x_1, x_2, ...x_n) = \frac{\delta^n \ln W[J]}{\delta J(x_1)\delta J(x_2)...\delta J(x_n)}|_{J=0}$$

Thus replacing y_i by external x_i, we get contributions for 4-point, 2-point functions.

Grassmann algebra

For fermion fields in path integral, we need to use anti-commuting c-number functions.This can be realized as elements of Grassmann algebra.

In an n-dimensional Grassmann algebra,the n generators $\theta_1, \theta_2, \theta_3, ..., \theta_n$ satisfy the anti-commutation relations,

$$\{\theta_i, \theta_j\} = 0 \qquad i, j = 1, 2, ..., n$$

and every element can be expanded in a finite series,

$$P(\theta) = P_0 + P^{(1)}_{i_1}\theta_{i_1} + P^{(2)}_{i_1 i_2}\theta_{i_1}\theta_{i_2} + ... + P_{i_1 ... i_n}\theta_{i_1}...\theta_{i_n}$$

Simplest case:n=1

$$\{\theta, \theta\} = 0 \qquad \text{or} \qquad \theta^2 = 0 \qquad P(\theta) = P_0 + \theta P_1$$

We can define the "differentiation" and "integration" as follows,

$$\frac{d}{d\theta}\theta = \theta\frac{\overleftarrow{d}}{d\theta} = 1 \quad \Longrightarrow \frac{d}{d\theta}P(\theta) = P_1$$

Integration is defined in such a way that it is invariant under translation,

$$\int d\theta P(\theta) = \int d\theta P(\theta + \alpha)$$

α is another Grassmann variable. This implies

$$\int d\theta = 0$$

We can normalize the integral such that

$$\int d\theta\theta = 1$$

Then

$$\int d\theta P(\theta) = P_1 = \frac{d}{d\theta} P(\theta)$$

Consider a change of variable

$$\theta \longrightarrow \widetilde{\theta} = a + b\theta$$

Since

$$\int d\widetilde{\theta} P\left(\widetilde{\theta}\right) = \frac{d}{d\widetilde{\theta}} P\left(\widetilde{\theta}\right) = P_1$$

$$\int d\theta P\left(\widetilde{\theta}\right) = \int d\theta \left[P_0 + \widetilde{\theta} P_1\right] = \int d\theta \left[P_0 + (a + b\theta) P_1\right] = bP_1$$

we get

$$\int d\widetilde{\theta} P\left(\widetilde{\theta}\right) = \int d\theta \left(\frac{d\widetilde{\theta}}{d\theta}\right)^{-1} P\left(\widetilde{\theta}(\theta)\right)$$

The "Jacobian" is the inverse of that for c-number integration.
Generalize to n-dimensional Grassmann algebra,

$$\frac{d}{d\theta_i}(\theta_1, \theta_2, \theta_3, ..., \theta_n) = \delta_{i_1}\theta_2...\theta_n - \delta_{i_2}\theta_1\theta_3...\theta_n + ... + (-1)^{n-1}\delta_{in}\theta_1\theta_2...\theta_{n-1}$$

$$\{d\theta_i, d\theta_j\} = 0$$

$$\int d\theta_i = 0 \qquad \int d\theta_i \theta_j = \delta_{ij}$$

For a change of variables of the form

$$\widetilde{\theta}_i = b_{ij}\theta_j$$

we have

$$\int d\widetilde{\theta}_n d\widetilde{\theta_{n-1}} ... d\widetilde{\theta}_1 P\left(\widetilde{\theta}\right) = \int d\theta_n ... d\theta_1 \left[det \frac{d\widetilde{\theta}}{d\theta}\right]^{-1} P\left(\widetilde{\theta}\left(\theta\right)\right)$$

Proof:

$$\widetilde{\theta}_1\widetilde{\theta}_2 ... \widetilde{\theta}_n = b_{1i_1} b_{2i_2} ... b_{ni_n} \theta_{i_1} ... \theta_{i_n}$$

RHS is non-zero only if $i_1, i_2 ..., i_n$ are all different and we can write

$$\begin{aligned} \widetilde{\theta}_1\widetilde{\theta}_2 ... \widetilde{\theta}_n &= b_{1i_1} b_{2i_2} ... b_{ni_n} \epsilon_{i_1, i_2 ..., i_n} \theta_{i_1} ... \theta_{i_n} \\ &= (\det b)\, \theta_1\theta_2\theta_3 ... \theta_n \end{aligned}$$

From the normalization condition,

$$1 = \int d\widetilde{\theta}_n d\widetilde{\theta_{n-1}} ... d\widetilde{\theta}_1 \left(\widetilde{\theta}_1\widetilde{\theta}_2 ... \widetilde{\theta}_n\right) = (\det b) \int d\widetilde{\theta}_n d\widetilde{\theta_{n-1}} ... d\widetilde{\theta}_1 \left(\theta_1\theta_2\theta_3 ... \theta_n\right)$$

we see that

$$d\widetilde{\theta}_n d\widetilde{\theta_{n-1}} ... d\widetilde{\theta}_1 = (\det b)^{-1} d\theta_1 ... d\theta_n$$

In field theory, we need Gaussian integral of the form,

$$G(A) \equiv \int d\theta_n ... d\theta_1 \exp\left(\frac{1}{2}(\theta, A\theta)\right) \qquad \text{where } (\theta, A\theta) = \theta_i A_{ij} \theta_j$$

First consider n=2

$$A = \begin{pmatrix} 0 & A_{12} \\ -A_{12} & 0 \end{pmatrix}$$

Then

$$G(A) = \int d\theta_2 d\theta_1 \exp(\theta_1 \theta_2 A_{12}) \simeq \int d\theta_2 d\theta_1 (1 + \theta_1 \theta_2 A_{12}) = A_{12} = \sqrt{\det A}$$

generalization to arbitrary n

$$G(A) = \int d\theta_n ... d\theta_1 \exp\left(\frac{1}{2}(\theta, A\theta)\right) = \sqrt{\det A} \qquad \text{n even}$$

and for "complex" Grassmann variables

$$\int d\theta_n d\bar{\theta}_n d\theta_{n-1} d\bar{\theta}_{n-1} ... d\theta_1 d\bar{\theta}_1 exp\left(\bar{\theta}, A\theta\right) = \det A$$

For the Fermion fields, the generating functional is of the form,

$$W[\eta, \overline{\eta}] = \int [d\psi(x)] \left[d\overline{\psi}(x)\right] \exp\{i \int d^4x \left[\mathcal{L}\left(\psi, \overline{\psi}\right) + \overline{\psi}\eta + \overline{\eta}\psi\right]\}$$

If $\mathcal{L}$ depends on $\psi, \overline{\psi}$ quadratically

$$\mathcal{L} = \left(\overline{\psi}, A\psi\right)$$

then we have

$$W = \int \left[d\psi\left(x\right)\right]\left[d\overline{\psi}\left(x\right)\right] \exp\{\int d^4x\overline{\psi}A\psi\} = \det A$$