33-779 Nuclear and Particle Physics I

March 24, 2010

Homework set 3, Due Tue April 13

1. The Dirac equation for free particle is given by,

$$\left(i\gamma^{\mu}\partial_{\mu}-m\right)\psi\left(x\right)=0$$

Under the parity transformation the space-time coordiante transform as

$$x^{\mu}\rightarrow x'^{\mu}=\left(x_0,-x_1,-x_2,-x_3\right)$$

The Dirac equation in the new coordinate system is of the form,

$$\left(i\gamma^{\mu}\partial'_{\mu}-m\right)\psi'\left(x'\right)=0$$

Find the relation between $\psi\left(x\right)$ and $\psi'\left(x'\right)$.

2. Let ϕ be a free scalar field satisfying the field equation,

$$\left(\partial^{\mu}\partial_{\mu}+\mu^2\right)\phi\left(x\right)=0$$

(a) Show that the propagator defined by

$$\Delta_F\left(x-y\right)\equiv\left\langle 0\left|T\left(\phi\left(x\right)\phi\left(y\right)\right)\right|0\right\rangle=\theta\left(x_0-y_0\right)\phi\left(x\right)\phi\left(y\right)+\theta\left(y_0-x_0\right)\phi\left(y\right)\phi\left(x\right)$$

can be written as

$$\Delta_F\left(x-y\right)=\int\frac{d^4k}{\left(2\pi\right)^4}e^{ik\cdot(x-y)}\frac{i}{k^2-\mu^2+i\varepsilon}$$

(b) Show that the unequal time commutator for these free fields is given by

$$i\Delta\left(x-y\right)\equiv\left[\phi\left(x\right),\phi\left(y\right)\right]=\int\frac{d^3k}{\left(2\pi\right)^3 2\omega_k}\left[e^{-ik\cdot(x-y)}-e^{ik\cdot(x-y)}\right]$$

(c) Show that $\Delta\left(x-y\right)=0$ for space-like separation, i.e.

$$\Delta\left(x-y\right)=0,\qquad \text{if}\quad \left(x-y\right)^2<0$$

3. Consider the Lagrangian density given by

$$L=\frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi-\frac{1}{2}\mu^2\phi^2+J\left(x\right)\phi,\qquad J\left(x\right)\quad\text{arbitray function}$$

(a) Show that the equation of motion is of the form,

$$\left(\partial^{\mu}\partial_{\mu}+\mu^2\right)\phi\left(x\right)=J\left(x\right)$$

(b) Find the conjugate momenta and impose the quantization conditions.

(c) Find the creation and annihilation operators.

4. Dirac particle in the presence of electromagnetic field satisfies the equation,

$$\left[\gamma^{\mu}\left(i\partial_{\mu}-eA_{\mu}\right)-m\right]\psi\left(x\right)=0$$

Or

$$i\frac{\partial\psi}{\partial t}=\left[\vec{\alpha}\cdot\left(\vec{p}-e\vec{A}\right)+\beta m+e\Phi\right]\psi$$

In the non-relativistic limit, we can write

$$\psi\left(x\right)=e^{-imt}\begin{pmatrix}u\\l\end{pmatrix}$$

Show that the upper component satisfies the equation,

$$i\frac{\partial u}{\partial t}=\left[\frac{1}{2m}\left(\vec{p}-e\vec{A}\right)^{2}-\frac{e}{m}\vec{\sigma}\cdot\vec{B}+e\Phi\right]u$$

For the case of weak uniform magnetic field $\vec{B}$ we can take $\vec{A}=\frac{1}{2}\vec{B}\times\vec{r}$. Show that

$$i\frac{\partial u}{\partial t}=\left[\frac{1}{2m}\left(\vec{p}\right)^{2}-\frac{e}{2m}\left(\vec{L}+2\vec{S}\right)\cdot\vec{B}\right]u.$$