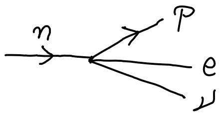
Weak Interaction before gauge theory

4-fermion interaction (Fermi 1934)

Neutron β-decay : $n \rightarrow p + e^- + \bar{\nu}$

Fermi proposed the following Lagrangian

$$\mathcal{L}_F = -\frac{G_F}{\sqrt{2}}\left[\bar{p}(x)\gamma_\mu n(x)\right]\left[\bar{e}(x)\gamma^\mu \nu(x)\right] + h.c$$



$$G_F \cong \frac{10^{-5}}{M_p^2}$$

1956 Parity violation (Lee and Yang)

V-A theory

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} J_\mu^+ J^\mu + h.c.$$

$$J_\lambda(x) = J_{e\lambda}(x) + J_{h\lambda}(x)$$

leptonic current $\quad J_e^\lambda(x) = \bar{\nu}_e \gamma^\lambda (1-\gamma_5) e + \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\mu$

hadronic current $\quad J_h^\lambda(x) = \bar{u}\gamma^\lambda(1-\gamma_5)(\cos\theta_c\, d + \sin\theta_c\, s)$

θ_c : Cabibbo angle.

Note that in V-A form the fermion fields are all left-handed.

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi \qquad J_e^\lambda = 2\bar{\nu}_L\gamma^\lambda e_L + 2\bar{\nu}_{\mu L}\gamma^\lambda \mu_L \cdots$$

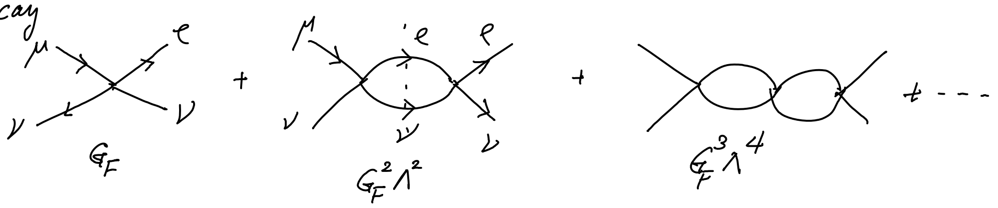
Phenomenologically, the V-A theory has been quite successful in most of the weak interactions phenomena.

Difficulties :

1) Not renormalizable

4 fermion interaction operator has dimension 6

e.g μ-decay



2) Violate unitarity

tree amplitude for $\nu_\mu e \rightarrow \mu \nu_e$ has only $J=1$. partial wave at high energies and cross section

$$\sigma(\nu_\mu e) \sim G_F^2 s \qquad s = 2m_e E$$

unitarity for $J=1$ cross section $\quad \sigma(J=1) < \frac{1}{s}$

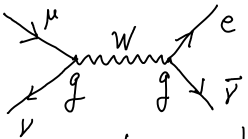
Thus $\sigma(\nu_\mu e)$ violates unitarity for $E \sim 300$ Gev

Intermediate Boson Theory (IVB)

In analogy with QED, we can introduce vector boson W to couple to the V-A current,

$\mathcal{L}_1 = g\,(J_\mu W^\mu + h.c)$

For example, the μ decay is now mediated by W-exchange



Since weak interaction is short range, we need W-boson to be massive $M_W \neq 0$

W-boson propagator
$$\frac{-g^{\mu\nu} + \frac{k^\mu k^\nu}{M_W^2}}{k^2 - M_W^2} \longrightarrow \frac{+g^{\mu\nu}}{M_W^2} \quad \text{when } |k_\mu| \ll M_W$$

This reproduces 4-fermion interaction with $\frac{g^2}{M_W^2} = \frac{G_F}{\sqrt{2}}$

In this theory, the scattering $\nu_\mu e \to \mu \nu$ no long violates unitarity.
But the violation of unitarity shows up in other processes like

$$\nu + \bar{\nu} \longrightarrow W^+ + W^-$$

and the theory is still non-renormalizable.

Construction of SU(2)xU(1) model

2008年11月23日
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1) Choice of group

In IVB theory,

$$\mathcal{L}_W = g\,(J_\mu W^\mu + h.c.)$$

For simplicity we neglect all other fermions except ν, e $\Rightarrow$

$$J_\mu = \bar{\nu}\gamma_\mu(1-\gamma_5)e$$

Recall that in the electromagnetic interaction, we have

$$\mathcal{L}_{em} = e\,J_\mu^{em}A^\mu, \qquad J_\mu^{em} = -\bar{e}\gamma_\mu e$$

Define the em and weak charges as

$$T_+ = \frac{1}{2}\int d^3x\, J_0(x) = \frac{1}{2}\int d^3x\, \nu^+(1-\gamma_5)e, \qquad T_- = (T_+)^+$$

$$Q = \int d^3x\, J_0^{em}(x) = -\int d^3x\, e^+e$$

We can compute the commutator

$$[T_+, T_-] = 2T_3$$

$$T_3 = \frac{1}{4}\int d^3x\,[\nu^+(1-\gamma_5)\nu - e^+(1-\gamma_5)e] \neq Q$$

This means that T_+, T_-, and Q do not form an SU(2) algebra.
We need to include T_3 as part of generators, i.e. T_+, T_-, T_3, Q. This leads to the gauge group

$$SU(2)\times U(1)$$

The Lagrangian for the gauge fields is then

$$\mathcal{L}_1 = -\frac{1}{4}F^{i\mu\nu}F^i_{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

where
$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\,\epsilon^{ijk}A^j_\mu A^k_\nu$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Fermions

Clearly, ν, e form a doublet under SU(2)

$$\ell_L = \begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$$

For conveience, we introduce left-handed and right-handed fields

$$\psi_L = \frac{1}{2}(1-\gamma_5)\psi, \qquad \psi_R = \frac{1}{2}(1+\gamma_5)\psi \implies \psi = \psi_L + \psi_R$$

We can write

$$T_+ = \int(\nu_L^+ e_L)\,d^3x \qquad T_- = \int(e_L^+\nu_L)\,d^3x, \qquad Q = \int(e_L^+e_L + e_R^+e_R)\,d^3x$$

Note that

$$Q - T_3 = \int\left[-\frac{1}{2}(\nu_L^+\nu_L + e_L^+e_L) - e_R^+e_R\right]d^3x$$

It is straightforward to show that

$$[Q-T_3, T_i] = 0, \qquad i=1,2,3$$

thus we can take $(Q-T_3)$ to be the $U(1)$ charge

$$Y = 2(Q-T_3)$$

The Y charges for fermions are

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad Y=-1 \quad , \qquad e_R \quad Y=-2$$

The Lagrangian for gauge coupling

$$\mathcal{L}_2 = \bar{l}_L i\gamma^\mu D_\mu l_L + \bar{l}_R i\gamma^\mu D_\mu l_R$$

Where $D_\mu \psi = (\partial_\mu - ig\frac{\vec{\tau}\cdot\vec{A}_\mu}{2} - ig'\frac{Y}{2}B_\mu)\psi$

e.g. $D_\mu l_L = (\partial_\mu - ig\frac{\vec{\tau}\cdot\vec{A}_\mu}{2} + ig'\frac{B_\mu}{2}) l_L$

2) Spontaneous Symmetry Breaking

The symmetry breaking pattern we want is

$$SU(2)\times U(1) \longrightarrow U(1)_{em}$$

choose scalar fields in $SU(2)$ doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad Y=1$$

The Lagrangian containing ϕ

$$\mathcal{L}_3 = (D_\mu\phi)^\dagger (D^\mu\phi) - V(\phi)$$

$$D_\mu\phi = (\partial_\mu - \frac{ig}{2}\vec{\tau}\cdot\vec{A}_\mu - \frac{ig'}{2}B_\mu)\phi$$

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\mathcal{L}_4 = f\,\bar{l}_L\phi e_R + h.c.$$

As we have seen before the spontaneous symmetry breaking is generated by the vaccum expectation value

$$\langle\phi\rangle_0 \equiv \langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$$

Write the scalar field in the form,

$$\phi(x) = U^{-1}(\vec{\xi})\begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix} \qquad \text{where } U(\vec{\xi}) = \exp\left[\frac{i\vec{\xi}(x)\cdot\vec{\tau}}{v}\right]$$

Gauge transformation

$$\phi' = U(\vec{\xi})\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\eta(x) \end{pmatrix} \qquad \frac{\vec{\tau}\cdot\vec{A}'_\mu}{2} = U(\vec{\xi})\left(\frac{\vec{\tau}\cdot\vec{A}_\mu}{2}\right)U^{-1}(\vec{\xi}) - \frac{i}{g}(\partial_\mu U)U^{-1}$$

$\vec{\xi}(x)$: disappear from the Lagrangian,

From $\mathcal{L}_4$ (Yukawa coupling), VEV gives

From $\mathcal{L}_4$ (Yukawa coupling), VEV gives

$$\mathcal{L}_4 = f\frac{\langle\phi\rangle}{\sqrt{2}}(\bar{\ell}_L e_R + h.c.) + \frac{f\eta(x)}{\sqrt{2}}(\bar{\ell}_L e_R + h.c.)$$ electron mass $m_e = f\langle\phi\rangle$

<u>Mass spectrum</u>

1) fermion mass

$$m_e = \frac{fv}{\sqrt{2}}$$

2) Scalar mass (Higgs)

$$V(\phi') = \mu^2\eta^2 + \lambda v\eta^3 + \frac{\lambda}{4}\eta^4 \implies m_\eta = \sqrt{2}\mu$$

3) gauge boson masses

From the covariant derivative in $\mathcal{L}_3$

$$\mathcal{L}_3 = \frac{v^2}{2}\chi^+\left(g\frac{\vec{\tau}\cdot\vec{A}'_\mu}{2} + \frac{g'}{2}B'_\mu\right)\left(g\frac{\vec{\tau}\cdot\vec{A}'^\mu}{2} + \frac{g'}{2}B'^\mu\right)\chi \qquad \chi = \begin{pmatrix}0\\1\end{pmatrix}$$

$$= \frac{v^2}{8}\left\{g^2[(A^1_\mu)^2 + (A^2_\mu)^2] + (gA^3_\mu - g'B_\mu)^2\right\}$$

$$= M_W^2 W^{+\mu}W^-_\mu + \frac{1}{2}M_Z^2 Z^\mu Z_\mu$$

where $W^+_\mu = \frac{1}{\sqrt{2}}(A^1_\mu - iA^2_\mu)$. $\qquad M_W^2 = \frac{g^2v^2}{4}$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gA^3_\mu - g'B_\mu) \qquad M_Z^2 = \frac{(g^2+g'^2)}{4}v^2$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'A^3_\mu + gB_\mu) \qquad \text{photon massless}$$

Define $\tan\theta_W = \frac{g'}{g}$ $\qquad \theta_W$: Weinberg angle

then we can write

$$Z_\mu = \cos\theta_W A^3_\mu - \sin\theta_W B_\mu \qquad M_Z^2 = \frac{g^2v^2}{4}\sec^2\theta_W$$

$$A_\mu = \sin\theta_W A^3_\mu + \cos\theta_W B_\mu$$

Note that $\rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W} = 1.$

<u>Charged current</u>

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}(J^+_\mu W^{+\mu} + h.c.) \qquad J^+_\mu = J^1_\mu + iJ^2_\mu = \frac{1}{2}\bar{\nu}\gamma_\mu(1-\gamma_5)e$$

Again to get 4-fermion interaction as low energy limit, we require

$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} \implies \frac{g^2}{8\cdot\frac{1}{16}g^2v^2} = \qquad v = \sqrt{\frac{\sqrt{2}}{G_F}} \simeq 246\ \text{GeV}$$

<u>Neutral Current</u>

$$\mathcal{L}_{NC} = gJ^3_\mu A^{3\mu} + \frac{g'}{2}J^Y_\mu B^\mu = eJ^{em}_\mu A^\mu + \frac{g}{\cos\theta_W}J^Z_\mu Z^\mu$$

Where $\quad e = g\sin\theta_W$

$$J^Z_\mu = J^3_\mu - \sin^2\theta_W J^{em}_\mu$$

Neutral Current

$$\mathcal{L}_{NC} = g J^3_\mu A^{3\mu} + \frac{g'}{2} J^Y_\mu B^\mu = e J^{em}_\mu A^\mu + \frac{g}{\cos\theta_W} J^Z_\mu Z^\mu$$

where $e = g\sin\theta_W$

$$J^Z_\mu = J^3_\mu - \sin^2\theta_W J^{em}_\mu$$

weak neutral current

Standard Model

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Neutral current

Neutral current interaction

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W} J^z_\mu Z^\mu$$

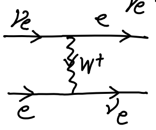
where $J^z_\mu = J^3_\mu - \sin^2\theta_W J^{em}_\mu$

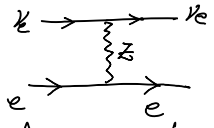
weak neutral charge $\quad Q^z = \int J^z_0 \, d^3x = (T_3 - \sin^2\theta_W Q)$

This means that the coupling strength of fermions to Z-boson is proportional to the quantum number $(T_3 - \sin^2\theta_W Q)$

In particular, Z boson can contribute to the scattering

$$\nu_e + e \longrightarrow \nu_e + e$$





the measurement of this cross section in the 1970's gives

$$\sin^2\theta_W \simeq 0.22$$

this yields

$$M_W \approx 80\ \text{GeV} \qquad M_Z \approx 90\ \text{GeV}$$

Generalization to more than one family

From 4-fermion and IVB theory, the weak current of leptons and hadrons give the following multiplets,

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L, \quad \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L. \qquad e_R, \mu_R.$$

$$\begin{pmatrix}u \\ d_\theta\end{pmatrix}_L, \qquad u_R, d_R, s_R.$$

where $\quad d_\theta = \cos\theta_c\, d + \sin\theta_c\, s$

the neutral current in the down quark sector is of the form

$$\mathcal{L}_{NC} = \Big[\bar{d}_{\theta L}\gamma_\mu\big(-\tfrac{1}{2} + \sin^2\theta_W \tfrac{1}{3}\big) d_{\theta L} - \sin^2\theta_W \tfrac{1}{3}\,(\bar{d}_R\gamma_\mu d_R + \bar{s}_R\gamma_\mu s_R)\Big] Z^\mu$$

$$\big(-\tfrac{1}{2} + \sin^2\theta_W \tfrac{1}{3}\big)\big[(\bar{d}_L\gamma_\mu d_L + \bar{s}_L\gamma_\mu s_L) + \sin\theta_W\cos\theta_W\,(\bar{d}_L\gamma_\mu s_L + \bar{s}_L\gamma_\mu d_L) + \cdots\cdots$$

gives rise to $\Delta S = 1$ neutral current processes e.g. $K_L \to \mu^+\mu^-$ with same order of magnitude as charged current

Exp.

$$R = \frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu\nu)} \leq 10^{-8}$$

Thus we can not have $\Delta S = 1$ neutral current process at the same order of magnitude as the charged current process.

GIM

Glashow, Iliopoulos and Maiani (1970) suggested the existence of 4-th quark, the charm quark c, which couples to the orthogonal combination $s_\theta = -\sin\theta_c\, d + \cos\theta_c\, s$

$$\begin{pmatrix}u \\ d_\theta\end{pmatrix}_L, \quad \begin{pmatrix}c \\ s_\theta\end{pmatrix}_L,$$

As a result, the $\Delta S = 1$, neutral current is canceled out,

$$\bar{d}_\theta\big(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W\big)\gamma_\mu d_\theta + \bar{s}_\theta\big(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W\big)\gamma_\mu s_\theta$$

$$= \big(\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W\big)\,(\bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s)$$

Quark mixing

Before the spontaneous symmetry breaking, fermions are all massless because ψ_L and ψ_R have different quantum numbers. So the mass term $(\bar{\psi}_L \psi_R + h.c)$ is not invariant under $SU(2)\times U(1)$ groups. When we have more that one doublets ψ_{iL}, ψ_{iR} all have definite quantum numbers with respect to $SU(2)\times U(1)$ group. They are called "weak eigenstates". When spontaneous symmetry breaking takes place fermions obtain their masses through Yukawa coupling,

$$\mathcal{L}_Y = (f_{ij}\, \bar{q}_{iL} u_{Rj} + f'_{ij}\, \bar{q}_{iL} d_{Rj})\phi + h.c.$$

Since the Yukawa coupling constants are arbitrary, the fermion mass matrices are in general not diagonal. When mass matrices are diagonalized we obtain the mass eigenstates which are not the same as the weak eigenstates. The mass matrices in the up and down sectors are given by

$$m^{(u)}_{ij} = f_{ij}\frac{v}{\sqrt{2}}, \qquad m^{(d)}_{ij} = f'_{ij}\frac{v}{\sqrt{2}}$$

These matrices which are sandwiched between left and right handed fields can be diagonalized by bi-unitary transformations, i.e. given m_{ij}, $\exists$ unitary matrices S and T such that

$$S^{+} m T = m_d$$

is diagonal. Basically, S is the unitary matrix which digonalizes the hermitian matrix mm^{+}, i.e.

$$S^{+}(mm^{+})S = m_d^2$$

If we write the left-handed doublets, (weak eigenstates) as

$$q_{1L} = \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \quad q_{2L} = \begin{pmatrix} c' \\ s' \end{pmatrix}_L$$

These weak eigenstates are related to mass eigenstates by unitary transformations,

$$\begin{pmatrix} u' \\ c' \end{pmatrix} = S_u \begin{pmatrix} u \\ c \end{pmatrix} \qquad \begin{pmatrix} d' \\ s' \end{pmatrix} = S_d \begin{pmatrix} d \\ s \end{pmatrix}$$

Note that in the coupling to charged gauge boson $W^{\pm}$, we have

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} W_\mu \left[\bar{q}_{1L}\gamma^\mu \tau^{+} q_{1L} + \bar{q}_{2L}\gamma^\mu \tau^{+} q_{2L}\right] + h.c.$$

and is invariant under unitary transformation in q_{1L}, q_{2L} space, i.e.

$$\begin{pmatrix} q'_{1L} \\ q'_{2L} \end{pmatrix} = V \begin{pmatrix} q_{1L} \\ q_{2L} \end{pmatrix} \qquad VV^{+} = V^{+}V = \underline{1}$$

We can use this feature to put all the mixing in the down quark sector

$$q'_{iL} : \begin{pmatrix} u \\ d'' \end{pmatrix}_L, \begin{pmatrix} c \\ s'' \end{pmatrix}_L \qquad \begin{pmatrix} d'' \\ s'' \end{pmatrix} = U \begin{pmatrix} d \\ s \end{pmatrix}$$

Clearly, we can extend this to 3 generations with result

$$q_{iL} : \begin{pmatrix} u \\ d'' \end{pmatrix}_L, \begin{pmatrix} c \\ s'' \end{pmatrix}_L, \begin{pmatrix} t \\ b'' \end{pmatrix}_L \qquad \begin{pmatrix} d'' \\ s'' \\ b'' \end{pmatrix} = U \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CP violation Phase

CP violation can come from complex coupling to gauge bosons. The gauge coupling of $W^{\pm}$ to quarks is governed by the 3×3 unitary matrix U. discussed above. This unitary matrix U can have many complex entries. However, in diagonalizing the mass mass,

$$S^{\dagger}(mm^{\dagger})S = m_d^2$$

There is ambiguity in the matrix S in the form of diagonal phases. In other words, if S diagonalizes the mass matrix, so does S'

$$S' = S \begin{pmatrix} e^{i\alpha_1} & & \\ & \ddots & \\ & & e^{i\alpha_n} \end{pmatrix}$$

We can use this property to redefine the quark fields to reduce the phases in U. It turns out that for $n \times n$ unitary matrix, number of independent physical phases left over is

$$\frac{(n-1)(n-2)}{2}$$

Thus to get CP violation we need to go to 3 generations or more (Kobayashi Maskawa).

Standard Model phenomenology

2008年12月8日
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1) W, Z gauge bosons

$$M_W = \frac{1}{2}\left(\frac{e^2}{\sqrt{2}G_F}\right)^{1/2}\frac{1}{\sin\theta_W} = \frac{37.3\,GeV}{\sin\theta_W} \simeq 80\,GeV \quad \text{for } \sin^2\theta_W = 0.23$$
80.4 GeV (Exp)

$$M_Z = \left(\frac{e^2}{\sqrt{2}G_F}\right)^{1/2}\frac{1}{\sin 2\theta_W} = \frac{74.6\,GeV}{\sin 2\theta_W} \simeq 90\,GeV \quad \text{"}$$
91.2 GeV (Exp)

W decays

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} W_\mu^+ \left[(\bar\nu_e, \bar\nu_\mu, \bar\nu_\tau)\gamma^\mu(1-\gamma_5)\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + (\bar u\, \bar c\, \bar t)\gamma^\mu(1-\gamma_5)U\begin{pmatrix} d \\ s \\ b \end{pmatrix}\right] + h.c.$$

$$\Gamma_e = \Gamma(W^- \to e + \nu) = \frac{G_F}{\sqrt{2}}\left(\frac{M_W^3}{6\pi}\right) \cong 0.25\,GeV$$

$W^+ \to e^+\nu_e$, $\mu^+\nu_\mu$, $\tau^+\nu_\tau$

$\to ud,\ us,\ ub$

$\to cd,\ cs,\ cb$

use $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$.

$\Gamma(W^+ \to ud, us, ub) = 3\Gamma_e$ 3 for colors

Similarly $\Gamma(W^+ \to cd, us, cb) = 3\Gamma_e$

$\Gamma(\text{total}) = 9\Gamma_e$

$B(W^+ \to e^+\nu) \cong \frac{1}{9} = 11.1\%$ exp $(10.8 \pm 0.09)\%$

$B(W^+ \to \text{hadrons}) \simeq \frac{6}{9} \simeq 66.67\%$ exp (67.6%)

Z decays

$Z \to \nu_i \bar\nu_i$ Theory $B(Z \to \nu\bar\nu) \simeq 20.5\%$

Exp $B(Z \to \nu\bar\nu) \simeq (20.00 \pm 0.06)\%$

The significance of this measurement is that it limits the number of light neutrinos to 3. Any possible 4-th generation neutrinos have to be above $\frac{1}{2}M_Z$.

2) Higgs particle

a) Higgs couplings to fermions $\mathcal{L}_Y = f_{ij}\bar\psi'_{iL}\phi\psi'_{jR} + h.c.$

spontaneous symmetry breaking $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\eta \end{pmatrix}$

$$\mathcal{L}_Y = m_{ij}\bar\psi'_{iL}\psi'_{jR} + \frac{f_{ij}}{\sqrt{2}}\eta(x)\bar\psi'_{iL}\psi'_{jR} + h.c. \qquad m_{ij} = \frac{v}{\sqrt{2}}f_{ij}$$

Diagonalization of mass matrix

$$m = U^+ m_d V$$

mass term $\bar\psi'_{iL} m_{ij}\psi'_{jR} = \bar\psi'_L U^+ m_d V\psi'_R = \bar\psi_L m_d \psi_R = m_i \bar\psi_{Li}\psi_{Ri}$ sum over i

Similarly UFV^+ is also diagonal $(F)_{ij} = f_{ij}$

$$\mathcal{L}_Y = m_i\bar\psi_{iL}\psi_{Ri} + \frac{m_i}{v}\eta(x)\bar\psi_{iL}\psi_{Ri} + h.c.$$

Thus Higgs couplings to fermions has the properties

(i) ... proportional to fermion mass

(i) Coupling to ψ_i is proportional to fermion mass

(ii) couplings conserve flavors and parity

b) Coupling to gauge bosons

$$\mathcal{L}_{\phi VV} = g\,\eta(x)\left[M_W W_\mu^+ W^{-\mu} + \frac{1}{2\cos\theta_W} M_Z Z^\mu Z_\mu\right]$$

again the couplings are proportional to masses

(c) mass of Higgs particle

$$m_\eta = \sqrt{2\mu^2} = \sqrt{2\lambda}\,v \qquad v \simeq 250\ \text{GeV}$$

Experimentally, there is no information on the parameter $\lambda \Rightarrow m_\eta$ is not constrained.

3) Neutrino oscillations

If ν's are all massless, there is no mixing in the lepton sector

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L, \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L, \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$$

Suppose ν's are all massive, then in analogy to the quark sector, they are linear combinations of mass eigenstates ν_1, ν_2, ν_3

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

If at time $t=0$, a beam of pure ν_e is produced, we can write

$$|\nu_e(0)\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

The time evolution is controlled by energy eigenvalues,

$$|\nu_e(t)\rangle = U_{e1}\,e^{-iE_1 t}|\nu_1\rangle + U_{e2}\,e^{-iE_2 t}|\nu_2\rangle + U_{e3}\,e^{-iE_3 t}|\nu_3\rangle$$

$$E_i^2 = p^2 + m_i^2$$

Assume $p \gg m_i$, we get $\quad E_i \approx p + m_i^2/p$

$$E_i - E_j = (m_i^2 - m_j^2)/2p$$

Define the oscillation length l_{ij} as

$$l_{ij} = \frac{2\pi}{E_i - E_j} \simeq \frac{4\pi p}{|m_i^2 - m_j^2|} = 2.5\,\text{m}\left(\frac{p(\text{MeV})}{\Delta m^2(\text{eV})}\right)$$

Consider the simple case of 2 neutrino species.

$$|\nu_e(0)\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

$$|\nu_\mu(0)\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

$$|\nu_e(t)\rangle = \cos\theta\,e^{-iE_1 t}|\nu_1\rangle + \sin\theta\,e^{-iE_2 t}|\nu_2\rangle = e^{-iE_1 t}\left(\cos\theta|\nu_1\rangle + \sin\theta\,e^{-i\frac{2\pi}{l_{12}}t}|\nu_2\rangle\right)$$

$$\langle\nu_e|\nu_e(t)\rangle = e^{-iE_1 t}\left(\cos^2\theta + \sin^2\theta\,e^{-i\frac{2\pi}{l_{12}}x}\right)$$

$P \ldots |^2 = 1 - 2\sin^2\theta\cos^2\theta(1-\cos\frac{2\pi}{\ldots}x)$

$$P_{\nu_e\to\nu_e} = |\langle \nu_e|\nu_e(t)\rangle|^2 = 1-2\sin^2\theta\cos^2\theta\left(1-\cos\frac{2\pi}{\ell_{12}}x\right)$$

$$P_{\nu_e\to\nu_\mu} = 1-\left(P_{\nu_e\to\nu_e}\right) = 2\sin\theta\cos\theta\left(1-\cos\frac{2\pi}{\ell_{12}}x\right)$$