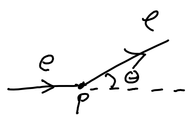
Introduction

Structure of proton

$M_p = 938.3 \ \mathrm{MeV}/c^2$.

electron proton scattering



1) Rutherford Scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}}$$

E: incident energy. θ: scattering angle.

- neglect the recoil
- proton treated as point particle

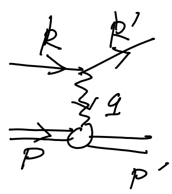
2) Mott scattering

Take into account of the spin of electron, we get Mott cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \left(1-\beta^2 \sin^2\frac{\theta}{2}\right)$$

3) Form factor

For a pointed particle $\langle p' | J_\mu^{em} | p\rangle = \bar{u}(p')\gamma_\mu u(p)$



For proton, we can parametrize

$$\langle p' | J_\mu^{em} | p\rangle = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2)\right] u(p)$$

$q = p - p'$, $F_1(q^2)$, $F_2(q^2)$ are form factors

with $F_1(0) = 1$

cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}\right] \qquad \tau = \frac{Q^2}{4M^2}$$

Rosenbluth formula

$G_E(q^2) = F_1 + \tau F_2$ electric form factor

$G_M(q^2) = F_1 + F_2$ magnetic form factor

$G_E(0) = F_1(0) = 1$ total charge

$G_M(0) = F_1(0) + F_2(0) = 1 + F_2(0)$ magnetic moment

Experimental measurements

$G_M^p(0) = 2.79\,\mu_N$, $G_M^n(0) = -1.91\,\mu_N$ $\mu_N = \frac{e}{2M_p}$ nuclear magneton

$$G_E^p(Q^2) = \frac{G_M^p(q^2)}{2.79} = \frac{G_M^n(Q^2)}{-1.91} \approx \frac{1}{(1-q^2/0.7\,\mathrm{GeV}^2)^2}$$ dipole form factor

Form factor can be related to Fourier transform of charge distribution.

$$F(q^2) = \int e^{i\vec{q}\cdot\vec{x}} \rho(x)\, d^3x \quad \Rightarrow \quad \rho(x) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} F(q^2)$$

measurement of form factors $\Longrightarrow$ charge distribution measurement of form factors $\Rightarrow$ charge distribution

For spherical charge distribution,

$$F(q^2) = 1 - \frac{1}{6}\vec{q}^{\,2}\langle r^2\rangle + \cdots$$

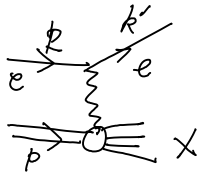
$$\langle r^2\rangle = 4\pi\int_0^\infty r^2 f(r)\, r^2 dr\,,$$ charge radius.

For proton $\quad \langle r^2\rangle_{proton} \simeq (0.86\ fm)^2$

Note that for $-q^2$ large, form factors decrease very fast $\sim \frac{1}{q^4}$

Deep Inelastic ep scattering

$e + p \rightarrow e + X$



Differential cross section can be written as

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left(2W_1\sin^2\frac{\theta}{2} + W_2\cos^2\frac{\theta}{2}\right)$$

$W_1(q^2,\nu)$, $W_2(q^2,\nu)$ are structure functions

$q = k - k' \qquad \nu = \frac{P\cdot q}{M}$

In the laboratory frame, $\nu = E - E'$ is the energy loss of lepton

Bjorken scattering

$$x = \frac{-q^2}{2M\nu} = \frac{Q^2}{2M\nu} \qquad Q^2 = -q^2$$

For large Q^2, it turns out that all structure functions have the limiting behaviors

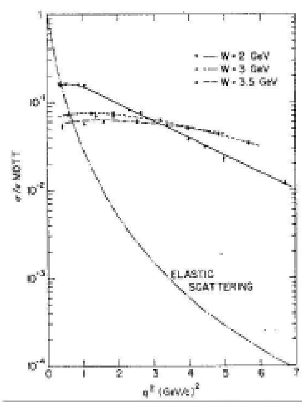
$$\left.\begin{array}{l} MW_1(Q^2,\nu) \rightarrow F_1(x) \\ \nu W_2(Q^2,\nu) \rightarrow F_2(x) \end{array}\right\}$$ independent of Q^2 for $Q^2 \gtrsim 1\ GeV^2$

If the inelastic channels have similar q^2 behavior as the elastic one, we expect the structure functions to decrease very fast as $-q^2$ increases.

The exp result is that W_1, νW_2 do not decrease much as the elastic one

$\Rightarrow$ no form factor effect

elementary particle inside proton ?



M.Breidenbach et al.,
Phys. Rev. Lett. 23 (1969) 935

Parton Model

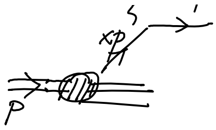
Feynman (1969) : deep inelastic scattering is viewed as due to incoherent elastic scattering from point-like constituents of the nucleon : parton.

on-shell condition :

$$p'^2 = (xp+q)^2 = (xp)^2 + 2x\,p\cdot q + q^2$$

from $(xp)^2 = p'^2 = 0 \Rightarrow x = -\dfrac{q^2}{2pq}$

scaling variable



x: fraction of proton momentum carried by the parton.

$2xF_1(x) = F_2(x)$ Callan-Gross relation

If we assume parton has spin $= 1/2$

electromagnetic current

$$J_\mu^{em} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$$

$$F_1^{ep}(x) = \frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s})$$

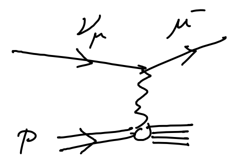
$q_i(x)$: probability of finding a parton with longitudinal momentum fraction x carrying the quantum number of quark q in the proton. (Parton distribution function)

From isospin symmetry, we get en structure functions by $u \leftrightarrow d$

$$F_1^{en}(x) = \frac{4}{9}(d+\bar{d}) + \frac{1}{9}(u+\bar{u}) + \frac{1}{9}(s+\bar{s})$$

<u>Neutrino deep inelastic scattering</u>

$$\nu_\mu + N \longrightarrow \mu + X$$
$$\nu_e + N \longrightarrow e + X$$



$$J_\mu^W \simeq \cos\theta_c\, \bar{u}\gamma^\mu(1-\gamma_5)d + \sin\theta_c\, \bar{u}\gamma^\mu(1-\gamma_5)s + \cdots$$

Here structure functions can also be expressed in terms of parton distribution functions. $q_i(x)$ ..

All data are consistent with partons carrying quark quantum numbers

Quark Model

2008年12月15日
下午 05:48

1) Isospin symmetry

In early studies of nuclear reactions, it was found that, to a good approximation, nuclear forces are independent of the electromagnetic charge carried by the nucleons — charge independence
In other words, strong interaction has an SU(2) symmetry which transforms n, into p and vice versa.

SU(2) generators, T_1, T_2, T_3 with $[T_i, T_j] = i\epsilon_{ijk} T_k$

$T_3|p\rangle = \frac{1}{2}|p\rangle$. $T_3|n\rangle = -\frac{1}{2}|n\rangle$ $T_+|n\rangle = |p\rangle$, $T_-|p\rangle = |n\rangle$ ....

H_s: strong interaction Hamiltonian $\Rightarrow [T_i, H_s] = 0$

Extension to other hadrons:

(π^+, π^0, π^-) $I=1$, $(K^+, K^0), (\bar{K}^0, K^-)$ $I=\frac{1}{2}$. η $I=0$

$(\Sigma^+, \Sigma^0, \Sigma^-)$ $I=1$. (Ξ^0, Ξ^-) $I=\frac{1}{2}$ Λ $I=0$

(ρ^+, ρ^0, ρ^-) $I=1$, (K^{+*}, K^{0*}) $(\bar{K}^{0*}, \bar{K}^{*-})$ $I=\frac{1}{2}$ ....

If isospin symmetry is exact, then all particles in the same multiplets have same masses, which is not the case in nature.

$$\frac{m_n - m_p}{m_n + m_p} \sim 0.7 \times 10^{-3} \qquad \frac{m_{\pi^+} - m_{\pi^0}}{m_{\pi^+} + m_{\pi^0}} \sim 1.7 \times 10^{-2} \quad ..$$

2) SU(3) symmetry and Quark Model

When Λ and K particles were discovered, they were produced in pair (associated production) with long life time. It was postulated that these new particles possessed a new additive quantum number, strangeness S, conserved by strong interaction but violated in decays.

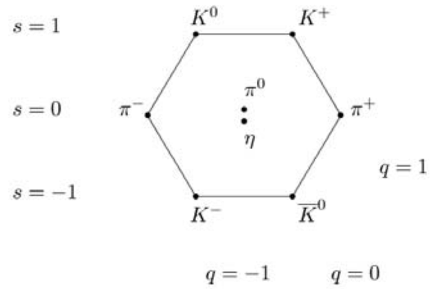
$S(\Lambda^0) = -1$. $S(K^0) = 1$ ...

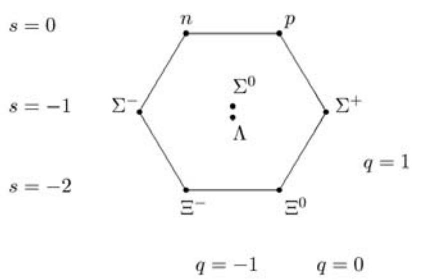
Extension to other hadrons, we can write

$$Q = T_3 + \frac{Y}{2} \qquad Y = B + S \text{ hypercharge}, \quad B: \text{baryon number}$$

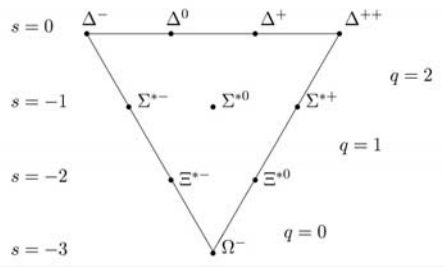
Gell-Mann-Nishijima relation

Gell-Mann, Neeman: eight-fold way
group mesons or baryons with same spin and parity





octets



decuplet

These are the same as irreducible representations of SU(3) group.

Quark Model

Since octet and decuplet are not the fundamental representation of SU(3) group, quark model was proposed, in which all hadrons are built out of spin ½ quarks which transform as members of the fundamental representation of SU(3),

$$q_i = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

quantum numbers

	Q	T	T_3	Y	S	B
u	$2/3$	$1/2$	$+1/2$	$1/3$	0	$1/3$
d	$-1/3$	$1/2$	$-1/2$	$1/3$	0	$1/3$
s	$-1/3$	0	0	$-2/3$	-1	$1/3$

mesons are $q\bar{q}$ bound states

$\pi^+ \sim \bar{d}u$ $\quad \pi^0 \sim \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$, $\quad \pi^- \sim \bar{u}d$

$K^+ \sim \bar{s}u$ $\quad K^0 \sim \bar{s}d$, $\quad \bar{K}^0 \sim \bar{d}s$, $\quad K^- \sim \bar{u}s$. $\quad \eta^0 \sim \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$

baryon are qqq bound states

$p \sim uud$, $\quad n \sim ddu$,

$\Sigma^+ \sim suu$, $\quad \Sigma^0 \sim s\frac{(ud+du)}{\sqrt{2}}$, $\quad \Sigma^- \sim sdd$

$\Xi^0 \sim ssu$, $\quad \Xi^- \sim ssd$ $\quad$, $\Lambda^0 \sim s\frac{1}{\sqrt{2}}(ud-du)$

Paradoxes of simple quark model.

1). quarks have fractional charges while all observed particles have integer charges.
At least one of the quarks is stable. none are found

2) Hadrons are exclusively made out $q\bar{q}$, qqq bound states.
qq, $qqqq$ states absent

3) $N^{*++} \sim uuu$ $\quad$ spin $\alpha_1\alpha_2\alpha_3$
ground state $l=0$, spatial wave function symmetric
total wavefunction is symmetric $\longrightarrow$ violates Pauli exclusion principle.

Color degree of freedom

To get out of these problem, one introduces color degrees of freedom for each quark and postulates that only color singlets are physical obserables. Then colors can be used to get antisymmetric wavefunction (each quark comes with 3 colors)

$u_\alpha = (u_1, u_2, u_3)$, $d_\alpha = (d_1, d_2, d_3)$ ......

Thus we have additional $SU(3)_{color}$ symmetry

e.g. $N^{*++} \sim u_\alpha(x_1) u_\beta(x_2) u_\gamma(x_3) \varepsilon^{\alpha\beta\gamma}$

Gell-Mann Okubo mass formula

Since SU(3) is not an exact symmetry, we want to see whether we can understand the pattern of the SU(3) breaking. Experimentally, SU(2) seems to be a good symmetry, we will assume isospin symmetry to set $m_u = m_d$.

0^- meson

$$m_\pi^2 = (m_0 + 2m_u)\lambda$$
$$m_K^2 = m_0 + m_u + m_s$$
$$m_\eta^2 = m_0 + \frac{2}{3}(m_u + 2m_s)$$

$$\Longrightarrow \quad \underbrace{4m_K^2}_{0.98\,(\text{Gev})^2} = \underbrace{m_\pi^2 + 3m_\eta^2}_{0.92\,(\text{Gev})^2}$$

$\frac{1}{2}^+$ baryon

$$m_N = m_0 + 3m_u$$
$$m_\Sigma = m_0 + 2m_u + m_s$$
$$m_\Xi = m_0 + m_u + 2m_s$$
$$m_\Lambda = m_0 + 2m_u + m_s$$

$$\Longrightarrow \quad \underbrace{\frac{m_\Sigma + 3m_\Lambda}{2}}_{2.23\,\text{Gev}} = \underbrace{m_N + m_\Xi}_{2.25\,\text{Gev}}$$

$\frac{3}{2}^+$ baryon

$$m_\Omega - m_{\Xi^*} = m_{\Xi^*} - m_{\Sigma^*} = m_{\Sigma^*} - m_N \qquad \text{equal spacing rule}$$

ω-ϕ mixing

for 1^- mesons. In making analogy with 0^- mesons, we would get

$$3m_{88}^2 \leftarrow 3m_\omega^2 = 4m_{K^*}^2 - m_\rho^2$$

Using $m_{K^*} = 890$ Mev, $m_\rho = 770$ Mev, we get

$$m_\omega = 926.5\ \text{Mev}$$

But experimentally, we have $m_\omega = 783$ Mev. It turns out that ϕ meson has mass $m_\phi = 1020$ Mev and has same SU(2) quantum number. In principle, when SU(3) symmetry is broken, ω-ϕ mixing is possible.

Denote the SU(3) octet state by V_8 and singlet state by V_1. Write the mass matrix as

$$M = \begin{pmatrix} m_{88}^2 & m_{18}^2 \\ m_{18}^2 & m_{11}^2 \end{pmatrix} \qquad V_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s), \quad V_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)$$

After diagonalization of M, we get

$$R^+ M R = M_d = \begin{pmatrix} m_\omega^2 & 0 \\ 0 & m_\phi^2 \end{pmatrix} \qquad R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Thus.

$$\omega = \cos\theta\, V_8 - \sin\theta\, V_1$$
$$\phi = \sin\theta\, V_8 + \cos\theta\, V_1$$

and

$$\sin\theta = \sqrt{\frac{(m_{88}^2 - m_\omega^2)}{(m_\phi^2 - m_\omega^2)}}$$

$\phi = \sin\theta\, V_8 + \cos\theta\, V_1$ $\qquad\qquad \overline{\sqrt{(m_\phi^2 - m_\omega^2)}}$

Using $m_{88} = 926.5$ MeV from Gell-Mann Okubo mass formula, we get

$$\sin\theta = 0.76$$

This is very close to the ideal mixing $\sin\theta = \sqrt{\frac{2}{3}} = 0.81$ where mass eigenstates have a simple form,

$$\omega = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

$$\phi = \bar{s}s$$

This means that the physical ϕ meson is mostly made out of s quark.

<u>Zweig rule</u>

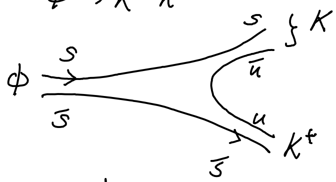
ω, ϕ have same quantum numbers, one expects they have similar decay width. Experimentally, $\omega \to 3\pi$ mostly, but $\phi \to 3\pi$ is very suppressed relative to $\phi \to KK$ channel. even though $\phi \to KK$ has very small phase space

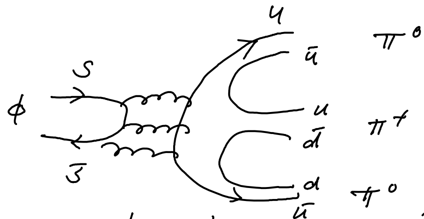
$\Gamma_\phi = 4.4$ MeV. $\qquad m_K \approx 494$ MeV

$B(\phi \to KK) \approx 85\%$ $\quad$, $B(\phi \to \pi\pi\pi) \sim 2.5\%$

<u>Quark diagrams</u>

$\phi \to K^+ K^-$





Zweig rule: processes involving quark-antiquark annihilation are highly suppressed

<u>J/ψ and charm quark</u>

In 1974 the ψ/J (3100) particle was discovered with unusally narrow width, $\Gamma \sim 70$ keV as compared to $\Gamma_\rho \sim 150$ MeV, $\Gamma_\omega \sim 10$ MeV.

Simple explanation, $\psi/J \sim \bar{c}c$ and is below the threshold of decaying into 2 mesons containing charm quark. It can only decay by $c\bar{c}$ annihilation in the initial state. By Zweig rule, these decays are highly suppressed and have very narrow width.

Asymptotic freedom

2008年12月20日
下午 02:00

1) $\lambda\phi^4$ theory

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu\phi)^2 - m^2\phi^2] - \frac{\lambda}{4!}\phi^4$$

effective coupling constant $\bar{\lambda}$ satisfies the differential equation

$$\frac{d\bar{\lambda}}{dt} = \beta(\bar{\lambda}) \quad , \quad \beta(\bar{\lambda}) \simeq \frac{3\lambda^2}{16\pi^2} + O(\lambda^3)$$

not asymptotically free.

Generalization $\lambda\phi^4 \longrightarrow \lambda_{ijkl}\phi_i\phi_j\phi_k\phi_l$, λ_{ijkl} totally symmetric

then $\beta_{ijkl} = \frac{d\lambda_{ijkl}}{dt} = \frac{1}{16\pi^2}[\lambda_{ijmn}\lambda_{mnkl} + \lambda_{ikmn}\lambda_{mnjl} + \lambda_{ilmn}\lambda_{mnjk}]$

for the special case $i=j=k=l=1$

$\beta_{1111} = \frac{3}{16\pi^2}\lambda_{11mn}\lambda_{mn11} > 0$ not asymptotically free

2) Yukawa interaction

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \frac{1}{2}[(\partial_\mu\phi)^2 - \mu^2\phi^2] - \lambda\phi^4 + f\bar{\psi}\psi\phi$$

$$\beta_\lambda = \frac{d\lambda}{dt} = A\lambda^2 + B\lambda f^2 + Cf^4 \qquad A>0$$

$$\beta_f = \frac{df}{dt} = Df^3 + E\lambda^2 f \qquad D>0$$

To get $\beta_\lambda < 0$, with $A>0$, we need $f^2 \sim \lambda$ $\Rightarrow$ we can drop E term in β_f

With $D>0$, Yukawa coupling f is not asymptotically free.

Generalization to the cases of more than one fermion fields or more scalar fields will not change the situation

3) Abelian gauge theory (QED)

$$\mathcal{L} = \bar{\psi}i\gamma^\mu(\partial_\mu - ieA_\mu)\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\frac{de}{dt} = \beta_e = \frac{e^3}{12\pi^2} + O(e^5)$$

scalar QED

$$\frac{de}{dt} = \beta_e = \frac{e^3}{48\pi^2} + O(e^5)$$

Both are not asymptotically free.

4) Non-Abelian gauge theories are asymptotically free.

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ $\qquad A_\mu = T_a A_\mu^a$

with $[T_a, T_b] = if_{abc}T_c$. $\qquad \mathrm{Tr}(T_aT_b) = \frac{1}{2}\delta_{ab}$

effective coupling constant

$\frac{dg}{dt} = \beta(g) = -\frac{g^3}{\ldots}\left(\frac{11}{\ldots}\right)t_2(V) < 0$ asymptotically free

$$\frac{dg}{dt} = \beta(g) = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}\right)t_2(V) < 0 \qquad \text{asymptotically free}$$

$$t_2(V)\,\delta^{ab} = tr[T_a(V)\,T_b(V)] \qquad t_2(V) = n \quad \text{for } SU(n)$$

If gauge fields are coupled to fermions and scalars with representation matrices, $T^a(F)$ and $T^a(S)$ respectively, then

$$\beta_g = \frac{g^3}{16\pi^2}\left[-\frac{11}{3}t_2(V) + \frac{4}{3}t_2(F) + \frac{1}{3}t_2(S)\right]$$

where $t_2(F)\,\delta^{ab} = tr(T^a(F)\,T^b(F))$

$t_2(S)\,\delta^{ab} = tr(T^a(S)\,T^b(S))$

Quark model needs colors degrees of freedom to overcome paradoxes of simple quark model. On the other hands, Bjorken scaling in deep inelastic scattering seems to require asymptotically free theory.

These suggest gauging the color degrees of freedom of quarks $\Rightarrow$ Quantum chromodynamics

$$\mathcal{L}_{QCD} = -\frac{1}{2}\,\mathrm{tr}\,(G_{\mu\nu}G^{\mu\nu}) + \sum_k \bar{q}_k\,(i\gamma^\mu D_\mu - m_k)\,q_k$$

where $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu, A_\nu]$

$D_\mu q_k = (\partial_\mu - igA_\mu)\,q_k$. $\qquad A_\mu = A_\mu^a \frac{\lambda^a}{2}$

$\beta_g = \frac{-1}{16\pi^2}(11 - \frac{2}{3}n_f) \equiv -bg^3 \qquad n_f$: # of flavors

$$\frac{d\bar{g}}{dt} = -b\bar{g}^3 \qquad t = \ln\lambda$$

$$\Rightarrow \quad \bar{g}^2(t) = \frac{g^2}{1+2bg^2t} \qquad \text{where } g = \bar{g}(g,0)$$

For large momenta, λp_i, λ large, $\bar{g}^2(t)$ decreases like $\ln\lambda$

Conveient to define
$$\alpha_s(Q^2) = \frac{\bar{g}^2(t)}{4\pi}$$
then we can write
$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+4\pi b\,\alpha_s(\mu^2)\ln(Q^2/\mu^2)}$$

Introduce Λ^2 by the relation,
$$\ln\Lambda^2 = \ln\mu^2 - \frac{1}{4\pi b\,\alpha_s(\mu^2)}$$

then
$$\alpha_s(Q^2) = \frac{4\pi}{(11-\frac{2}{3}n_f)\ln Q^2/\Lambda^2}$$

Thus the effective coupling constant $\alpha_s(Q^2)$ decreases slowly $\sim \frac{1}{\ln Q^2}$. QCD can make prediction about scaling violation (small) in the forms of integral over structure functions.

Quark confinements.

Since $\alpha_s(Q^2)$ is small for large Q^2, it is reasonable to believe that $\alpha_s(Q^2)$ is large for small Q^2. If $\alpha_s(Q^2)$ is large enough between quarks so that quarks will never get out of the hadrons. This is called quark confinement. It is most attractive way to "explain" why quarks can not be detected as free particles.

QCD and Flavor symmetry

QCD Lagrangian is of the form

QCD Lagrangian is of the form

$$\mathcal{L}_{QCD} = -\frac{1}{2} tr(G_{\mu\nu} G^{\mu\nu}) + \sum_k \bar{q}_k i\gamma^\mu D_\mu q_k + \sum_k \bar{q}_k m_k q_k$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \qquad A_\mu = A^a_\mu \frac{\lambda^a}{2}$$

$$D_\mu q_k = (\partial_\mu - ig A_\mu) q_k \qquad , q_k = (u, d, s, \cdots)$$

Consider the simple case of 3 flavors.

$$q_k = (u, d, s)$$

$$\mathcal{L}_{QCD} = -\frac{1}{2} tr(G_{\mu\nu} G^{\mu\nu}) + (\bar{u} i\gamma^\mu D_\mu u + \bar{d} i\gamma^\mu D_\mu d + \bar{s} i\gamma^\mu D_\mu s) + m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s$$

In the limit $m_u = m_d = m_s = 0$, $\mathcal{L}_{QCD}$ is invariant under $SU(3)_L \times SU(3)_R$ transformation

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}_L \to U_L \begin{pmatrix} u \\ d \\ s \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \\ s \end{pmatrix}_R \to U_R \begin{pmatrix} u \\ d \\ s \end{pmatrix}_R$$

However, hadron spectra shows only approximate $SU(3)$ symmetry, not $SU(3) \times SU(3)$ symmetry. We can reconcile this by the scheme $SU(3)_L \times SU(3)_R$ is broken spontaneously to $SU(3)$ so that particles group into $SU(3)$ multiplet. This would require 8 Goldstone bosons which are massless. However, in real world quark masses are not zero, these Goldstone bosons are not exactly massless. But if this symmetry breaking makes sense at all, these Goldstone bosons should be light. Thus we can identify them as pseudoscalar mesons. In other worlds, pseudoscalar mesons are "almost" Goldstone bosons.