Quantum Field Theory II

April 25, 2011

Homework set 3, Due Tu May 10

1. In the elastic electron proton scattering the F_1 form factor is just the Fourier transform of the charge distribution ρ,

$$F_1\left(\vec{q}^{\,2}\right) = \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{x}} \rho\left(\vec{x}\right)$$

Use the experimental measurement

$$G_E\left(q^2\right) = \frac{1}{\left(1-\frac{q^2}{a^2}\right)^2}, \qquad \text{where} \quad a^2 = 0.71 Gev^2$$

to compute the charge distribution and the charge radius., assuming that $q^2 \simeq -\vec{q}^{\,2}$.

2. In parton model, the ratio of proton to neutron structure functions is given by

$$\frac{F_2^{ep}(x)}{F_2^{en}(x)} = \frac{4\left(u+\bar{u}\right)+\left(d+\bar{d}\right)+\left(s+\bar{s}\right)}{\left(u+\bar{u}\right)+4\left(d+\bar{d}\right)+\left(s+\bar{s}\right)}$$

Show that this ration is bounded by

$$\frac{1}{4} \leq \frac{F_2^{ep}(x)}{F_2^{en}(x)} \leq 4$$

3. Consider the decay $\pi^-(p) \to \mu^-(k_1) + \overline{\nu}(k_2)$,

   (a) Show that the matrix element for this decay can be written as

$$M = G_F \frac{f_\pi}{\sqrt{2}} p^\mu \overline{u}(k_1) \gamma_\mu (1-\gamma_5) v(k_2)$$

   where G_F is the Fermi constant and f_π is the pion decay constant.

   (b) Compute pion decay rate for pion at rest and compare with the experimental data to determine f_π.

   (c) Compute the branching ratio

$$\frac{\Gamma(\pi^- \to e^-\overline{\nu})}{\Gamma(\pi^- \to \mu^-\overline{\nu})}$$

   (d) Use angular momentum conservation to illustrate that $\Gamma(\pi^- \to e^-\overline{\nu}) \ll \Gamma(\pi^- \to \mu^-\overline{\nu})$.

4. Consider the decay $W^-(p) \to e^-(k_1) + \overline{\nu}(k_2)$

   (a) Show that the matrix element can be written as

$$M = \frac{g}{2\sqrt{2}} \varepsilon^\mu(p)\, \overline{u}(k_1) \gamma_\mu (1-\gamma_5) v(k_2)$$

   where $\varepsilon^\mu(p)$ is the polarization of W^- and g is the weak gauge coupling constant.

   (b) Compute the total decay rate of W^-. Note the sum over polarization is of the form,

$$\sum_{spin} \varepsilon^\mu(p)\, \varepsilon^\nu(p) = \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M_W^2}\right)$$

5. Compute the differential and total cross sections for the following processes;

   (a) $\nu_\mu + e \longrightarrow \nu_\mu + e$

   (b) $\nu_e + e \longrightarrow \nu_e + e$