Quantum Field Theory

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Standard Model Phenomenology

Neutral Current Phenomenology

A set of new reactions are mediated by neutral Z boson– neutral current reactions. Their discoveries in 1970's lend a strong support to standard model.
Weak interaction of Z boson is of the form,

$$\mathcal{L}_Z = \frac{g}{\cos\theta_W} Z^\mu J^Z_\mu$$

where J^Z_μ is weak neutral current

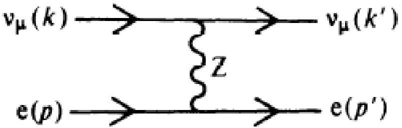
$$J^Z_\mu = \sum_i \bar{\psi}_i \gamma_\mu \left[T_3(\psi_i) - \sin^2\theta_W Q(\psi_i)\right]\psi_i$$

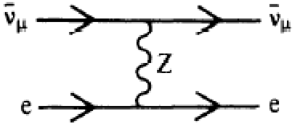
Here ψ_i is either left-or right-handed field and T_3 and Q are their weak isospin and electric charge,e.g. for u_L

$$T_3(u_l) - \sin^2\theta_W Q(u_l) = \frac{1}{2} - \sin^2\theta_W \frac{2}{3}$$

We can compute the cross sections for those reactions mediated by Z boson..

1. $\nu_\mu + e \to \nu_\mu + e$ and $\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e$
   The contributing diagrams are .





The differential and total cross section in the lab frame,

$$\frac{d\sigma\left(\nu_\mu e\right)}{dy} = \frac{8G_F^2}{\pi} m_e E_\nu \left(g_L^\nu\right)^2 \left[\left(g_L^e\right)^2 + \left(g_R^e\right)^2 (1-y)^2\right] \qquad \text{with} \quad y = \frac{E_e}{E_\nu}$$

and

$$\sigma\left(\nu_\mu e\right) = \frac{8G_F^2}{\pi} m_e E_\nu \left(g_L^\nu\right)^2 \left[\left(g_L^e\right)^2 + \frac{1}{3}\left(g_R^e\right)^2\right]$$

where

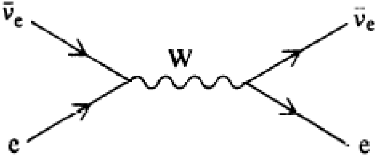
$$g_L^\nu = \frac{1}{2}, \qquad g_L^e = -\frac{1}{2} + \sin^2\theta_W, \qquad g_R^e = \sin^2\theta_W$$

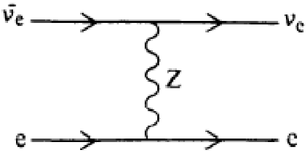
are the weak neutral couplings of ν and e.
Similarly,

$$\sigma\left(\bar{\nu}_\mu e\right) = \frac{8G_F^2}{\pi} m_e E_\nu \left(g_L^\nu\right)^2 \left[\frac{1}{3}\left(g_L^e\right)^2 + \left(g_R^e\right)^2\right]$$

2. $\nu_e + e \longrightarrow \nu_e + e$ and $\bar{\nu}_e + e \longrightarrow \bar{\nu}_e + e$

The contribution diagrams are





Here, there is also contribution from W-exchange diagram. The resulting cross sections are,

$$\sigma\left(\nu_e e\right) = \frac{8G_F^2}{\pi} m_e E_\nu \left(g_L^\nu\right)^2 \left[\left(1 + g_L^e\right)^2 + \frac{1}{3}\left(g_R^e\right)^2\right]$$

$$\sigma\left(\bar{\nu}_e e\right) = \frac{8G_F^2}{\pi} m_e E_\nu \left(g_L^\nu\right)^2 \left[\frac{1}{3}\left(1 + g_L^e\right)^2 + \left(g_R^e\right)^2\right]$$

Note that there are terms linear in g_L and its sign can be determined. This determines the weak mixing to be

$$\sin^2\theta_W \approx 0.23$$

There are other neutral current effects in $e^+e^- \longrightarrow \mu^+\mu^-$, $\nu p \longrightarrow \nu X$, $ep \longrightarrow ep$ reactions. These all have been measured and results agree with the theoretical qutie well

W and Z gauge bosons

The basic features of standard model is the existence of 3 massive gauge bosons, W^+, W^-, and Z.

1. Masses
   The masses of W and Z

$$M_W = \frac{1}{2}\left(\frac{e^2}{\sqrt{2}G_F}\right)^{\frac{1}{2}} \frac{1}{\sin\theta_W} = \frac{37.3 GeV}{\sin\theta_W}$$

$$M_Z = \left(\frac{e^2}{\sqrt{2}G_F}\right)^{\frac{1}{2}} \frac{1}{\sin 2\theta_W} = \frac{74.6 GeV}{\sin 2\theta_W}$$

From measurement of θ_W from the neutral current reactions, we can predict

$$M_W \approx 80\ GeV, \qquad M_Z \approx 90\ GeV \qquad \text{for} \qquad \sin^2\theta_W \approx 0.23$$

In the early 1980's, W and Z bosons are found in the experiments in CERN and their masses are found to be

$$M_W \approx 80.4\ \ GeV, \qquad M_Z \approx 91.2 GeV$$

The good agreement of with theoretical predictions lends another strong support for the stnadard model.

2. W decays

The weak interactions of charged gauge boson W is of the form,

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} W_\mu^+ [(\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\gamma^\mu(1-\gamma_5)] \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + (\bar{u}, \bar{c}, \bar{t})\gamma^\mu(1-\gamma_5)] U \begin{pmatrix} d \\ s \\ b \end{pmatrix}] + h.c.$$

The decay rate for $W^+ \to e^+ + \nu$ is

$$\Gamma_e = \Gamma(W^+ \to e^+ + \nu) = \frac{G_F}{\sqrt{2}} \frac{M_W^3}{6\pi} \approx 0.25 \ GeV$$

The same decay rate applies to other leptonic modes

$$W^+ \to \mu^+ \nu_\mu, \quad \tau^+ \nu_\tau$$

The hadronic decay modes can be described in terms of quarks

$$W^+ \to u\bar{d}, \quad u\bar{s}, \quad u\bar{b}$$

$$W^+ \to c\bar{d}, \quad c\bar{s}, \quad c\bar{b}$$

Use unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$, to get

$$\Gamma(W^+ \to ud, us, ub) = 3\Gamma_e$$

factor 3 comes from the color degree of freedom. Similarly $\Gamma(W^+ \rightarrow cd, cs, ub) = 3\Gamma_e$

$$\Gamma(total) = 9\Gamma_e$$

The branching ratio for electron mode is

$$B(W^+ \rightarrow e^+ + \nu) \approx \frac{1}{9} = 11.1\%$$

Experimental result is

$$B(W^+ \rightarrow e^+ + \nu) \approx (10.8 \pm 0.09)\%$$

The agreement seems to be quite good. Similarly,

$$B(W^+ \rightarrow hadrons) \approx \frac{6}{9} = 66.67\%$$

agrees with the measurement of 67.6%.

3. Z decays

One unique feature is that all Z decays conserve flavors. Of particular interest is the $Z \longrightarrow \nu\bar{\nu}$. These can be measured as the invisible width of Z boson. For each neutrino species

$$\Gamma\left(Z \longrightarrow \nu_i + \bar{\nu}_i\right) = \frac{G_F M_Z^3}{12\pi\sqrt{2}} = 0.161 \;\; Gev$$

From the measured invisible with of Z

$$\Gamma\left(Z \longrightarrow invisible\right) = 0.499 \pm 0.0015 \quad Gev$$

the number of light neutrinos is limited to 3.

Higgs particle

The most important ingredient of the Standard Model is the scalar particles. The left-over scalar, called the Higgs particle The LHC machine in CERN is built to look for such a particle. We now summarize important properties of Higgs particle.

1. $M_W = M_z \cos\theta_W$

   This follows from the doublet nature of the scalar fields and is well satisfied experimentally,

$$\rho = \frac{M_W^2}{M_z^2 \cos^2\theta_W} = 1.003 \pm .004$$

   For a general $SU(2) \times U(1)$ multiplets of Higgs particle $\phi_{T,Y}$ with weak isospin T and hypercharge Y, its contribution to ρ is

$$\rho = \frac{M_W^2}{M_z^2 \cos^2\theta_W} = \frac{\sum_{T,Y} |v_{T,Y}|^2 \left[T(T+1) - Y^2\right]}{2\sum_{T,Y} |v_{T,Y}|^2 Y^2/4}$$

   Here $v_{T,Y} = \langle 0 \left| \phi_{T,Y} \right| 0 \rangle$. Thus the constraint on Higgs in representation other than doublet is very severe.

2 Higgs couplings to fermions

The coupling of Higgs to fermions is

$$\mathcal{L}_Y = f_{ij}\bar{\psi}'_{iL}\phi\psi'_{jR} + h.c$$

Spontaneous symmetry breaking is generated by

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}, \qquad \text{with} \quad v \simeq 250 \quad Gev$$

and Yukawa coupling becomes

$$\mathcal{L}_Y = m_{ij}\bar{\psi}'_{iL}\psi'_{jR} + \frac{f_{ij}}{\sqrt{2}}h(x)\bar{\psi}'_{iL}\psi'_{jR} + h.c. \quad \text{with} \qquad m_{ij} = \frac{v}{\sqrt{2}}f_{ij}$$

Diagonalize the mass matrices by bi-unitary transformations,

$$UmV^\dagger = m_D, \qquad \text{where} \quad m_D \quad \text{is diagonal}$$

Explicitly, the mass term becomes

$$\mathcal{L}_m = \bar{\psi}'_{iL}m_{ij}\psi'_{jR} = \bar{\psi}'_L U^+ m_D V\psi'_R = \bar{\psi}_L m_D\psi_R = m_i\bar{\psi}_{Li}\psi_{Ri}$$

Here

$$\psi_L = U\psi'_L, \qquad \psi_R = V\psi'_R$$

are mass eigenstates. Since $m_{ij} \propto f_{ij}$, in the basis where mass matrix is diagonal, the Yukawa coupling is also diagonal, i.e.

$$\mathcal{L}_Y = m_i \bar{\psi}_{iL} \psi_{Ri} + \frac{m_i}{v} \eta(x) \bar{\psi}_{iL} \psi_{Ri} + h.c.$$

Higgs couplings is proportional to the mass of that fermion and coupling conserve flavors and parity. This means that Higgs will decay into the heavies particles allowed by the kinematics.
Coupling to gauge bosons

$$\mathcal{L}_{\phi VV} = g\eta(x)[M_W W^+_\mu W^{-\mu} + \frac{1}{2\cos\theta_W} M_Z Z^\mu Z_\mu]$$

also are proportional to masses.

3. Mass of Higgs particle

$$m_\eta = \sqrt{2\mu^2} = \sqrt{2\lambda} v \qquad v \approx 250 GeV$$

There is no information on Higgs self coupling λ.Thus m_η is not constrained and the search is going to be difficult.

Neutrino oscillations

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

When neutrinos are all massless, they are degenerate and all mixing angles can be removed by the redefinition of the neutrino fields. But if they are massive, then there will be neutrinno oscillations.

Suppose ν's are all massive, then they are linear combinations of mass eigenstates ν_1, ν_2, ν_3

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

If at time $t = 0$, ν_e is produced somehow, we can write

$$|\nu_e(0)\rangle = U_{e1} |\nu_1\rangle + U_{e2} |\nu_2\rangle + U_{e3} |\nu_3\rangle$$

The time evolution is controlled by the energy eigenvalues,

$$|\nu_e(t)\rangle = U_{e1} e^{-iE_1 t} |\nu_1\rangle + U_{e2} e^{-iE_2 t} |\nu_2\rangle + U_{e3} e^{-iE_3 t} |\nu_3\rangle$$

where

$$E_i^2 = p^2 + m_i^2$$

Assume $p >> m_i$ we get $E_i \approx p + \frac{m_i^2}{2p}$ and

$$E_i - E_j = \frac{(m_i^2 - m_j^2)}{2p}$$

Since the coefficients of the linear combination are time-dependent, the content of this beam will oscillate between ν_e, ν_μ, and ν_τ. Define the oscillation length l_{ij} as

$$l_{ij} = \frac{2\pi}{E_i - E_j} \approx \frac{4\pi p}{\left|m_i^2 - m_j^2\right|} = 2.5m[\frac{p(MeV)}{\Delta m^2 (eV)^2}]$$

This set the scale for the oscillation.
Consider the case of 2 netrino species. At $t = 0$, write

$$|\nu_e(0)\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu(0)\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

Consider the case of with $|\nu_e(0)\rangle$. The time evolution is,

$$|\nu_e(t)\rangle = \cos\theta e^{-iE_1 t}\,|\nu_1\rangle + \sin\theta e^{-iE_2 t}\,|\nu_2\rangle = e^{-iE_1 t}(\cos\theta\,|\nu_1\rangle + \sin\theta e^{-i\frac{2\pi}{l_{12}}t}\,|\nu_2\rangle)$$

The probability amplitude for finding the state ν_e at time t is,

$$\langle \nu_e | \nu_e(t) \rangle = e^{-iE_1 t}(\cos^2\theta + \sin^2\theta e^{-i\frac{2\pi}{l_{12}}x})$$

and

$$P(\nu_e \to \nu_e) = |\langle \nu_e | \nu_e(t) \rangle|^2 = 1 - 2\sin^2\theta\cos^2\theta(1 - \cos(\frac{2\pi x}{l_{12}}))$$

and probability to find other speciece, i.e. ν_μ is

$$P(\nu_e \to \nu_\mu) = 1 - P(\nu_e \to \nu_e) = 2\sin^2\theta\cos^2\theta(1 - \cos(\frac{2\pi x}{l_{12}}))$$

One of the most exciting development in last decade or so is the observation of neutrino oscillations in many different reactions and this establishes the existence of neutrino masses.