## A Subtle Periodic Table

- an unsolved mystery of our Mother Nature

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- A thousand years old question: What is our world made of?
- In ancient Greek, philosopher believed the building blocks are the "4 elements":

| season | element | humour | body fluid | location |
| :---: | :---: | :---: | :---: | :---: |
| Spring | air | sanguine | blood | heart |
| Summer | fire | choleric | "yellow bile" | liver |
| Autumn | earth | melancholic | "black bile" | spleen |
| Winter | water | phlegmatic | phlegm | (various) |



No, that's not enough!


Mendeleev first trained as a teacher in the Pedagogic Institute of St. Petersbug before earing his MS in 1856.

Textbook written between 1868-1870



- At that time, the experimentally determined atomic masses were not always accurate. Mendeleev reordered elements despite their accepted masses.
For example, he changed the weight of Beryllium from 14 to 9.
In all, he found 17 elements had to be moved to new positions.
- Even so, there are many elements missing at some positions.

From the gap, he predicted the existence and properties of unknown elements.

- Gallium (by a French, Gallia is Latin for France), Scandium (by a Scandinavian ), and Germanium (by a German) were found later to fit his prediction quite well.
- In all Medeleev predicted the existence of 10 new elements, of which seven were eventually discovered.
- After electron, proton, neutron and Quantum Mechanics were known, the periodic table can be easily understood.


## Modern Periodic Table

Table 4.1. Revised 2004 by C.G. Wohl (LBNL). Adapted from the Commission of Atomic Weights and Isotopic Abundances, "Atomic Weights of the Elements 1995 ," Pure and Applied Chemistry 68, 2339 (1996), and G. Audi and A.H. Wapstra, "The 1993 Mass Evaluation," Nucl. Phys. A565, 1 (1993). The atomic number (top left) is the number of protons in the nucleus. The atomic mass (bottom) is weighted by isotopic abundances in the Earth's surface. For a new determination of atomic masses, not weighted by abundances, see G. Audi, A.H. Wapstra, and C. Thibault, Nucl. Phys. A729, 337 (2003). Atomic masses are relative to the mass of the carbon-12 isotope, defined to be exactly 12 unified atomic mass units (u). Errors range from 1 to 9 in the last digit quoted. Relative isotopic abundances often vary considerably, both in natural and commercial samples. A number in parentheses is the mass of the longest-lived isotope of that element-no stable isotope exists. However, although Th, Pa, and U have no stable isotopes, they do have characteristic terrestrial compositions, and meaningful weighted masses can be given. For elements 110 and 111 , the numbers of nucleons $A$ of confirmed isotopes are given.

| $\begin{gathered} 1 \\ \mathrm{IA} \\ \hline \end{gathered}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} 18 \\ \text { VIIIA } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 \quad \mathrm{H}$ <br> Hydrogen <br> 1.00794 | $\begin{gathered} 2 \\ \text { IIA } \\ \hline \end{gathered}$ |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} 13 \\ \text { IIIA } \end{gathered}$ | $\begin{array}{r} 14 \\ \text { IVA } \\ \hline \end{array}$ | $\begin{aligned} & 15 \\ & \text { VA } \\ & \hline \end{aligned}$ | $\begin{array}{r} 16 \\ \text { VIA } \\ \hline \end{array}$ | $\begin{gathered} 17 \\ \text { VIIA } \\ \hline \end{gathered}$ | $2 \quad \mathrm{He}$ <br> Helium <br> 4.002602 <br> 10 |
| $3 \quad \mathrm{Li}$ <br> Lithium <br> 6.941 <br> 11 | 4 <br> Beryllium <br> 9.012182 | PERIODIC TABLE OF THE ELEMENTS |  |  |  |  |  |  |  |  |  | 5 <br> Boron <br> 10.811 | 6 <br> Carbon <br> 12.0107 | 7 N <br> Nitrogen  <br> 14.00674  | $8 \quad 0$ <br> Oxygen <br> 15.9994 | 9 <br> Fluorine <br> 18.9984032 <br> 17 | $10 \quad \mathrm{Ne}$ <br> Neon <br> 20.1797 <br> 18 Ar |
| $11 \quad \mathrm{Na}$ <br> Sodium <br> 22.989770 <br> 19 | $12 \quad \mathrm{Mg}$ <br> Magnesium <br> 24.3050 | $\begin{gathered} 3 \\ \text { IIIB } \end{gathered}$ | $\begin{gathered} 4 \\ \text { IVB } \end{gathered}$ | $\begin{gathered} 5 \\ \text { VB } \end{gathered}$ | $\begin{gathered} 6 \\ \text { VIB } \end{gathered}$ | $\begin{gathered} 7 \\ \text { VIIB } \end{gathered}$ | $8$ | $\begin{gathered} 9 \\ -\quad \text { VIII } \end{gathered}$ | $\begin{array}{r} 10 \\ -\quad \\ \hline \end{array}$ | $\begin{aligned} & 11 \\ & \mathrm{IB} \end{aligned}$ | $\begin{gathered} 12 \\ \text { IIB } \end{gathered}$ | $13 \quad \mathrm{Al}$ <br> Aluminum <br> 26.981538 | $14 \quad$ Si <br> Silicon <br> 28.0855 | 15 <br> Phosph. <br> 30.973761 | 16 S <br> Sulfur <br> 32.066 | $17 \quad \mathrm{Cl}$ Chlorine 35.4527 | $18 \quad \mathrm{Ar}$ <br> Argon <br> 39.948 |
| $19 \quad$ K <br> Potassium <br> 39.0983 | $20 \quad$ Ca <br> Calcium <br> 40.078 | 21 <br> Scandium <br> 44.955910 | $22 \quad \mathrm{Ti}$ <br> Titanium <br> 47.867 | $23 \quad$ V <br> Vanadium <br> 50.9415$\|$ | $24 \quad \mathrm{Cr}$ <br> Chromium <br> 51.9961 | $\begin{array}{\|lr} \hline 25 & \text { Mn } \\ \text { Manganes } \\ 54.938049 \\ \hline \end{array}$ | $\begin{gathered} \hline 26 \quad \mathrm{Fe} \\ \text { Iron } \\ 55.845 \\ \hline \end{gathered}$ | $27 \quad$ Co <br> Cobalt <br> 58.933200 | $\begin{array}{\|cc\|} \hline 28 \quad \mathrm{Ni} \\ \text { Nickel } \\ 58.6934 \\ \hline \end{array}$ | $29 \quad \mathrm{Cu}$ <br> Copper <br> 63.546 | $\begin{array}{\|c\|} \hline 30 \quad \mathrm{Zn} \\ \text { Zinc } \\ 65.39 \\ \hline \end{array}$ | $31 \quad \mathrm{Ga}$ <br> Gallium <br> 69.723 | $32 \quad \mathrm{Ge}$ German. 72.61 | $33 \quad$ As <br> Arsenic <br> 74.92160 | $34 \quad \mathrm{Se}$ <br> Selenium <br> 78.96 | $35 \quad \mathrm{Br}$ Bromine 79.904 | $36 \quad \mathrm{Kr}$ <br> Krypton <br> 83.80 <br> 24 |
| $37 \quad \mathrm{Rb}$ <br> Rubidium <br> 85.4678 <br> 55 | $38 \quad \mathrm{Sr}$ <br> Strontium <br> 87.62 | $39 \quad Y$ <br> Yttrium <br> 88.90585 | $40 \quad$ Zr <br> Zirconium <br> 91.224 | $\begin{array}{\|lr\|} \hline 41 & \mathrm{Nb} \\ \text { Niobium } \\ 92.90638 \\ \hline 7 & \hline \end{array}$ | $42 \quad$ Mo <br> Molybd. <br> 95.94 | $\begin{gathered} \hline 43 \quad \mathrm{Tc} \\ \text { Technet. } \\ 97.907215 \\ \hline 7 r \end{gathered}$ | $44 \quad \mathrm{Ru}$ <br> Ruthen. <br> 101.07 <br> 26 | 45 <br> Rhodium <br> 102.90550 | $46 \quad \mathrm{Pd}$ <br> Palladium <br> 106.42 | $47 \quad \mathrm{Ag}$ <br> Silver <br> 107.8682 | $48 \quad$ Cd <br> Cadmium <br> 112.411 | $49 \quad$ In <br> Indium <br> 114.818 | $\begin{array}{\|c} \hline 50 \quad \mathrm{Sn} \\ \text { Tin } \\ 118.710 \\ \hline \end{array}$ | 51 Sb <br> Antimony  <br> 121.760  | $52 \quad \mathrm{Te}$ <br> Tellurium <br> 127.60 <br> 84 | $53 \quad$ I <br> Iodine <br> 126.90447 | $54 \quad$ Xe <br> Xenon <br> 131.29 <br> 86 |
| $55 \quad$ Cs <br> Cesium <br> 132.90545 <br> 87 | $56 \quad$ Ba <br> Barium <br> 137.327 | $57-71$ <br> Lantha- <br> nides | $72 \quad \mathrm{Hf}$ <br> Hafnium <br> 178.49 <br> 104 | 73 <br> Tantalum <br> 180.9479 <br> 105 | $74 \quad$ W <br> Tungsten <br> 183.84 | $75 \quad \operatorname{Re}$ <br> Rhenium <br> 186.207 <br> 107 | $76 \quad$ Os <br> Osmium <br> 190.23 | $77 \quad$ Ir <br> Iridium <br> 192.217 <br> 109 | 78 Pt <br> Platinum  <br> 195.078  | 79Au <br> Gold <br> 196.96655 <br> 111 | $80 \quad \mathrm{Hg}$ <br> Mercury <br> 200.59 | $81 \quad$ TI <br> Thallium <br> 204.3833 | 82 Pb <br> Lead <br> 207.2 | $83 \quad \mathrm{Bi}$ <br> Bismuth <br> 208.98038 | $84 \quad$ Po <br> Polonium <br> $(208.982415)$ | 85 At <br> Astatine <br> $(209.987131)$ | $86 \quad \mathrm{Rn}$ <br> Radon <br> $(222.017570)$ |
| $87 \quad$ Fr <br> Francium <br> (223.019731) | $\left[\begin{array}{c}88 \quad \mathrm{Ra} \\ \text { Radium } \\ (226.025402)\end{array}\right]$ | $89-103$ <br> Actinides | $104 \quad \mathrm{Rf}$ <br> Rutherford. <br> (261.1089) | $\left.\begin{array}{\|l\|}105 \\ \text { Dubnium } \\ (262.1144)\end{array}\right]$ | $106 \quad \mathrm{Sg}$ <br> Seaborg. <br> $(263.1186)$ | 107 <br> Bohrium <br> $(262.1231)$ | $\begin{array}{cc} 108 \mathrm{Hs} \\ \text { Hassium } \\ (265.1306) \end{array}$ | $\left.\begin{array}{\|cc\|} \hline 109 & \mathrm{Mt} \\ \text { Meitner. } \\ (266.1378) \end{array} \right\rvert\,$ | $110 \quad$ Ds <br> Darmstadt. <br> $[269,271]$ | 111 <br> [272] |  |  |  |  |  |  |  |


| Lanthanide series | $57 \quad$ La Lanthan. 138.9055 | $\begin{array}{\|cc} 58 & \text { Ce } \\ \text { Cerium } \\ 140.116 \\ \hline \end{array}$ | $\|$59 Pr <br> Praseodym.  <br> 140.90765  | $\begin{gathered} 60 \quad \mathrm{Nd} \\ \text { Neodym. } \\ 144.24 \\ \hline \end{gathered}$ | $\begin{array}{\|c\|} 61 \quad \text { Pm } \\ \text { Prometh. } \\ (144.912745) \end{array}$ | $\begin{array}{\|cr} 62 & \text { Sm } \\ \text { Samarium } \\ 150.36 \\ \hline \end{array}$ | $\begin{array}{\|lr} 63 & \text { Eu } \\ \text { Europium } \\ 151.964 \\ \hline \end{array}$ | $\begin{gathered} 64 \quad \mathrm{Gd} \\ \text { Gadolin. } \\ 157.25 \\ \hline \end{gathered}$ | $65 \quad \mathrm{~Tb}$ <br> Terbium <br> 158.92534 | $\begin{array}{\|cc\|} \hline 66 & \text { Dy } \\ \text { Dyspros. } \\ 162.50 \\ \hline \end{array}$ | 67 Ho Holmium 164.93032 | $\begin{array}{\|cc\|} \hline 68 \quad \text { Er } \\ \text { Erbium } \\ 167.26 \\ \hline \end{array}$ | $\begin{aligned} & 69 \quad \text { Tm } \\ & \text { Thulium } \\ & \text { 168.93421 } \end{aligned}$ | $\begin{array}{cc} 70 & \mathrm{Yb} \\ \text { Ytterbium } \\ 173.04 \\ \hline \end{array}$ | $\begin{aligned} & 71 \quad \text { Lu } \\ & \text { Lutetium } \\ & 174.967 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Actinide series | $\begin{gathered} 89 \quad \text { Ac } \\ \text { Actinium } \\ (227.027747) \end{gathered}$ | $\begin{array}{\|l\|l} 90 \quad \text { Th } \\ \text { Thorium } \\ 232.0381 \end{array}$ | $\begin{aligned} & 91 \quad \mathrm{~Pa} \\ & \text { Protactin. } \\ & \text { 231.03588 } \\ & \hline \end{aligned}$ | $92 \quad$ U Uranium 238.0289 | $\left\|\begin{array}{lr}93 & \text { Np } \\ \text { Neptunium } \\ (237.048166)\end{array}\right\|$ | 94 Pu <br> Plutonium  <br> $(244.064197)$  | $\begin{gathered} 95 \quad \text { Am } \\ \text { Americ. } \\ (243.061372) \end{gathered}$ | $\begin{array}{ll} 96 \quad \mathrm{Cm} \\ \text { Curium } \\ (247.070346) \end{array}$ | $\left\lvert\, \begin{array}{lr} 97 & \mathrm{Bk} \\ \text { Berkelium } \\ (247.070298) \end{array}\right.$ | $\left\|\begin{array}{cc} 98 & \text { Cf } \\ \text { Californ. } \\ (251.079579) \end{array}\right\|$ | $\begin{gathered} 99 \quad \text { Es } \\ \text { Einstein. } \\ (252.08297) \end{gathered}$ | $\left\|\begin{array}{cc} 100 & \text { Fm } \\ \text { Fermium } \\ (257.095096) \end{array}\right\|$ | $\begin{array}{\|l\|} \hline 101 \quad \text { Md } \\ \text { Mendelev. } \\ (258.098427) \end{array}$ | $\begin{aligned} & 102 \text { No } \\ & \text { Nobelium } \\ & (259.1011) \end{aligned}$ | $\begin{array}{cc} 103 \quad \text { Lr } \\ \text { Lawrenc. } \\ (262.1098) \end{array}$ |


$J$ Thomson
electron, 1906


C Anderson positron, 1936


Rutherford proton, 1908 (chem)


Yukawa
pion theory, 1949

J. Chadwick neutron, 1935

C. Powell pion, 1950



In 1964, Murray Gell-Mann and George Zweig tentatively put forth the idea of quarks. They suggested that mesons and baryons are composites of three quarks or antiquarks, called up, down, or strange ( $u, d, s$ ) with spin $1 / 2$ and electric charges $2 / 3,-1 / 3,-1 / 3$, respectively (it turns out that this theory is not completely accurate). Since the charges had never been observed, the introduction of quarks was treated more as a mathematical explanation of flavor patterns of particle masses than as a postulate of actual physical object. Later theoretical and experimental developments allow us to now regard the quarks as real physical objects, even though they cannot be isolated.

## November Revolution in Physics

The world of physics was dazzled in November 1974 when two separate experiments at SLAC and at Brookhaven independently discovered the first of a new set of particle states, the J/Psi particle.
Burton Richter of the SLAC collaboration, and Sam Ting, of the Brookhaven group, received the 1976 Nobel Prize in Physics
"for their pioneering work in the discovery of a heavy elementary particle of a new kind."



## Standard Model

- Standard Model(SM) is the most successful theoretical understanding of the Mother Nature in human history ( with only 19 free parameters. )

$$
\begin{aligned}
S M= & \text { Quantum Mechanics }+ \text { Special Relativity }+ \text { Field theory } \\
& + \text { Gauge Symmetry }\left[\equiv S U(3)_{c} \times S U(2)_{L} \times U(1)\right] \\
& + \text { Matter Content }[\text { quarks, leptons }]+\text { Higgs Mechanism } .
\end{aligned}
$$

- Predicts that weak interaction is mediated by exchange of $W^{ \pm}$and $Z^{0}$ bosons.

S. Glashow


Abdus Salam


Steven Weinberg

## Baryons qqq and Antibaryons $\overline{\mathrm{q}} \overline{\mathrm{q}} \overline{\mathrm{q}}$ <br> Baryons are fermionic hadrons.

These are a few of the many types of baryons.

| Symbol | Name | Quark <br> content | Electric <br> charge | Mass <br> $\mathrm{GeV} / \mathrm{c}^{2}$ | Spin |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{p}$ | proton | uud | 1 | 0.938 | $1 / 2$ |
| $\overline{\mathbf{p}}$ | antiproton | $\overline{\text { ūū}} \overline{\mathbf{d}}$ | -1 | 0.938 | $1 / 2$ |
| $\mathbf{n}$ | neutron | udd | 0 | 0.940 | $1 / 2$ |
| $\Lambda$ | lambda | uds | 0 | 1.116 | $1 / 2$ |
| $\Omega^{-}$ | omega | $\mathbf{S S S}$ | -1 | 1.672 | $3 / 2$ |

## Mesons $\mathbf{q} \overline{\mathbf{q}}$

## Mesons are bosonic hadrons

These are a few of the many types of mesons.

| Symbol | Name | Quark <br> content | Electric <br> charge | Mass <br> $\mathrm{GeV} / \mathrm{c}^{2}$ | Spin |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\pi^{+}$ | pion | $\mathbf{u \overline { d }}$ | +1 | 0.140 | 0 |
| $\mathbf{K}^{-}$ | kaon | $\mathbf{s u}$ | -1 | 0.494 | 0 |
| $\rho^{+}$ | rho | $\mathbf{u} \overline{\mathbf{d}}$ | +1 | 0.776 | 1 |
| $\mathrm{~B}^{0}$ | B-zero | $\mathbf{d} \overline{\mathbf{b}}$ | 0 | 5.279 | 0 |
| $\eta_{\mathrm{c}}$ | eta-c | $\mathbf{c} \overline{\mathbf{c}}$ | 0 | 2.980 | 0 |

The subtle periodic table in the modern particle physics:

$$
\text { FERMIONS } \quad \begin{aligned}
& \text { matter constituents } \\
& \text { spin }=1 / 2,3 / 2,5 / 2, \ldots
\end{aligned}
$$

| Leptons spin =1/2 |  |  | Quarks spin =1/2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Flavor | $\begin{aligned} & \text { Mass } \\ & \mathrm{GeV} / \mathrm{c}^{2} \end{aligned}$ | Electric charge | Flavor | Approx. Mass $\mathrm{GeV} / \mathrm{c}^{2}$ | Electric charge |
| $V_{\mathrm{L}}{ }_{\text {l }}^{\substack{\text { lightest } \\ \text { neutrino* }}}$ | $(0-0.13) \times 10^{-9}$ | 0 | (U) up | 0.002 | 2/3 |
| (e) electron | 0.000511 | -1 | (d) down | 0.005 | -1/3 |
| $V_{M}{ }_{\text {M }} \begin{aligned} & \text { middle } \\ & \text { neutrino* }\end{aligned}$ | $(0.009-0.13) \times 10^{-9}$ | 0 | C charm | 1.3 | 2/3 |
| $\boldsymbol{\mu}$ muon | 0.106 | -1 | (S) strange | 0.1 | -1/3 |
| $\nu_{H} \begin{aligned} & \text { heaviest } \\ & \text { neutrino* }\end{aligned}$ | $(0.04-0.14) \times 10^{-9}$ | 0 | t top | 173 | 2/3 |
| $\tau^{\tau}$ tau | 1.777 | -1 | b bottom | 4.2 | -1/3 |

## Fermion masses

- Fermion masses in log scale
fermion masses

- Where comes the mass?


## Masses and the Higgs field

- The left-handed and right-handed fermions are coupled by Higgs boson and get their mass through nonzero VEV.

- Mathematically, the fermion mass term can be expressed as

$$
\mathcal{L}_{\text {Yukawa }}=f_{i j} \overline{\psi_{L i}} \psi_{R j} H+H . c .
$$

- A thought experiment:

If a left-handed fermion has mass, we can move fast enough to pass and find a right-handed partner.
Since we observe no right-handed neutrino $\Rightarrow$ neutrinos are massless in Standard Model.


## Fermion Mixing

- We have learnt that: the mixing among neutrinos are "Bi-LARGE" and only few mass matrix patterns can explain the data.
$U_{M N S}=\left(\begin{array}{ccc}e^{i \phi_{1}} & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23}\end{array}\right)\left(\begin{array}{ccc}c_{13} & 0 & s_{13} \\ 0 & e^{-i \delta+i \phi_{2}} & 0 \\ -s_{13} & 0 & c_{13}\end{array}\right)\left(\begin{array}{ccc}c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1\end{array}\right)$
$\theta_{12} \sim 33^{\circ}, \theta_{23} \sim 45^{\circ}, \theta_{13}<13^{\circ} ; \delta, \phi_{1}, \phi_{2}$ are still unknown.
- Compared to the SM quark sector:
$V_{C K M}=\left(\begin{array}{ccc}1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23}\end{array}\right)\left(\begin{array}{ccc}c_{13} & 0 & s_{13} \\ 0 & e^{-i \delta} & 0 \\ -s_{13} & 0 & c_{13}\end{array}\right)\left(\begin{array}{ccc}c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1\end{array}\right)$
$\theta_{12} \sim 13^{\circ}, \theta_{23} \sim 2^{\circ}, \theta_{13} \sim 0.2^{\circ} ; \delta \sim 65^{\circ}$.
- Puzzles!!


## Let's look back.

- Too many elements
$\Longrightarrow$ Periodic Table
$\Longrightarrow$ Atoms consist of electrons and nuclei
- Too many isotopes
$\Longrightarrow$ nuclei is made of protons and neutrons
- Too many hadrons
$\Longrightarrow$ quarks and $S U(3)_{c}$
- Too many redundant generations
$\Longrightarrow$ Preon and Hypercolor??


## Preon doesn't work!

In the 1980s, the preon was a very popular research topic. But it doesn't look promising anymore:

- No direct experimental evidence or hints of the existence of substructure of quarks or lepton.
Contact interaction search at LEP

$$
\Longrightarrow \Lambda_{p}>\mathrm{TeV}
$$

- The theory is difficult.
- Must be another Yang-Mills: Which group? Which representation? How to calculate?
- Why are quarks and leptons so light? Natural expectation is mass $\sim \Lambda_{p}>\mathrm{TeV}$. Chiral symmetry is the only known symmetry to protect large mass, no one knows how to make it work here.
- How to get the SM quantum number?
- Some generic bad predictions: exotic boson, quarks, and leptons..


## Other tries.

- Bigger symmetry group?

$$
S U(5) \rightarrow S U(8), S O(10) \rightarrow S O(10+4 k), E 6 \rightarrow E 8
$$

However, familon problem, predicts $K^{+} \rightarrow p i^{+}+f$

- Symmetry, or extra quantum number in the Yukawa sector: Structure Zeros, Froggatt-Nielsen, or the hybrid.
- Statistics:

Anarchy, Landscape..

## Geometry in extra Dimension?

- 5D fermion localizes at different position, $z_{i}$, in extra dimension $y \in[-\pi R, \pi R]$, $\psi_{i}(x, y)=g\left(z_{i}, y\right) \psi(x)$,

$$
\begin{array}{r}
g\left(z_{i}, y\right)=\frac{1}{\left(\pi \sigma^{2}\right)^{1 / 4}} \exp \left[-\frac{\left(y-z_{i}\right)^{2}}{2 \sigma^{2}}\right] \\
g\left(z_{1}, y\right) g\left(z_{2}, y\right)=\exp \left[-\frac{\left(z_{1}-z_{2}\right)^{2}}{4 \sigma^{2}}\right] g\left(\frac{z_{1}+z_{2}}{2}, y\right)
\end{array}
$$

- Exponential Yukawa hierarchy becomes linear displacement between left-handed and right-handed fermions in the fifth dimension.
- The following map can reproduce all quarks' masses and CKM mixings



## Intersecting brane?



It may provide a topological reason why we have 3 generations.



