Physics of Graphene:
Possibility of relativistic electronics and spintronics

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Acknowledgement

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Refs: cond-mat 0905-0251 (submitted to EPL)
Phys. Rev. B 79, 035405 (2009)
Phys. Rev. B 70, 205408 (2004)
Phys. Rev. B 68, 035432 (2003)
Phys. Rev. B 67, 024503 (2003)
Physica C 388-389, 45 (2003)

Outline:

- **Background and Introduction: Transport and magnetic properties**

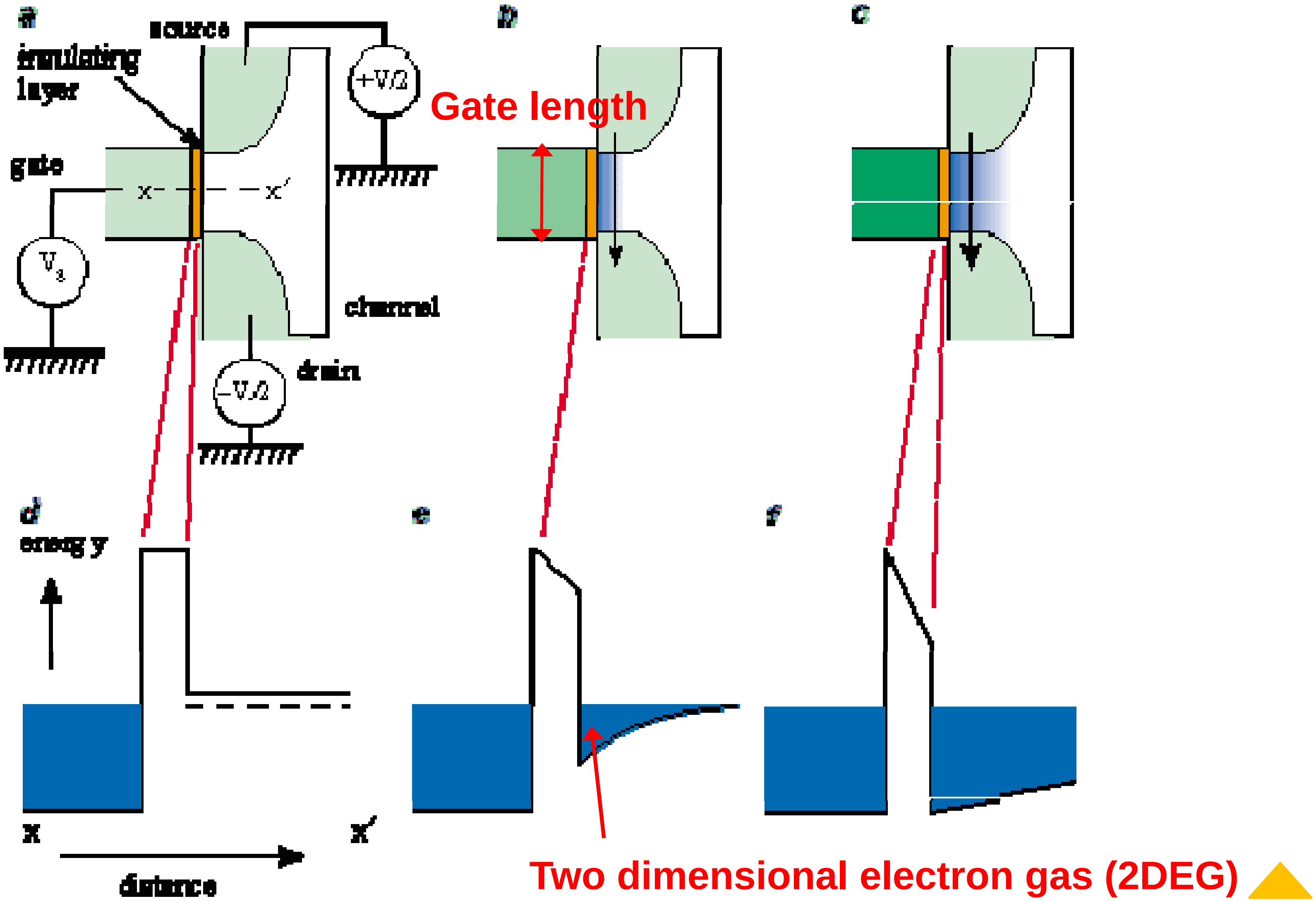
- **Novel magnetism associated with edge states and flat-band in nanoribbon**

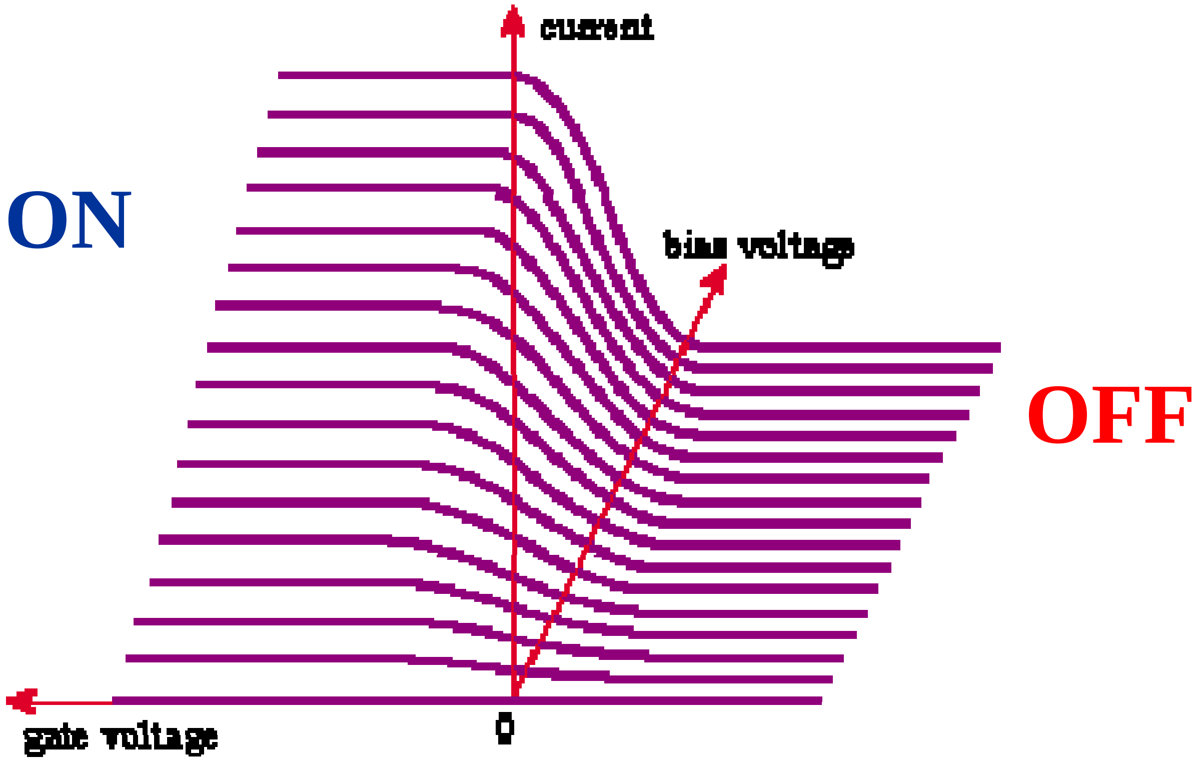
- **Impurity band due to point defects in graphene**

囊括凝體物理重要
平臺的兩個現代電
子科技之基礎元件

電子科技之基礎--MOSFET

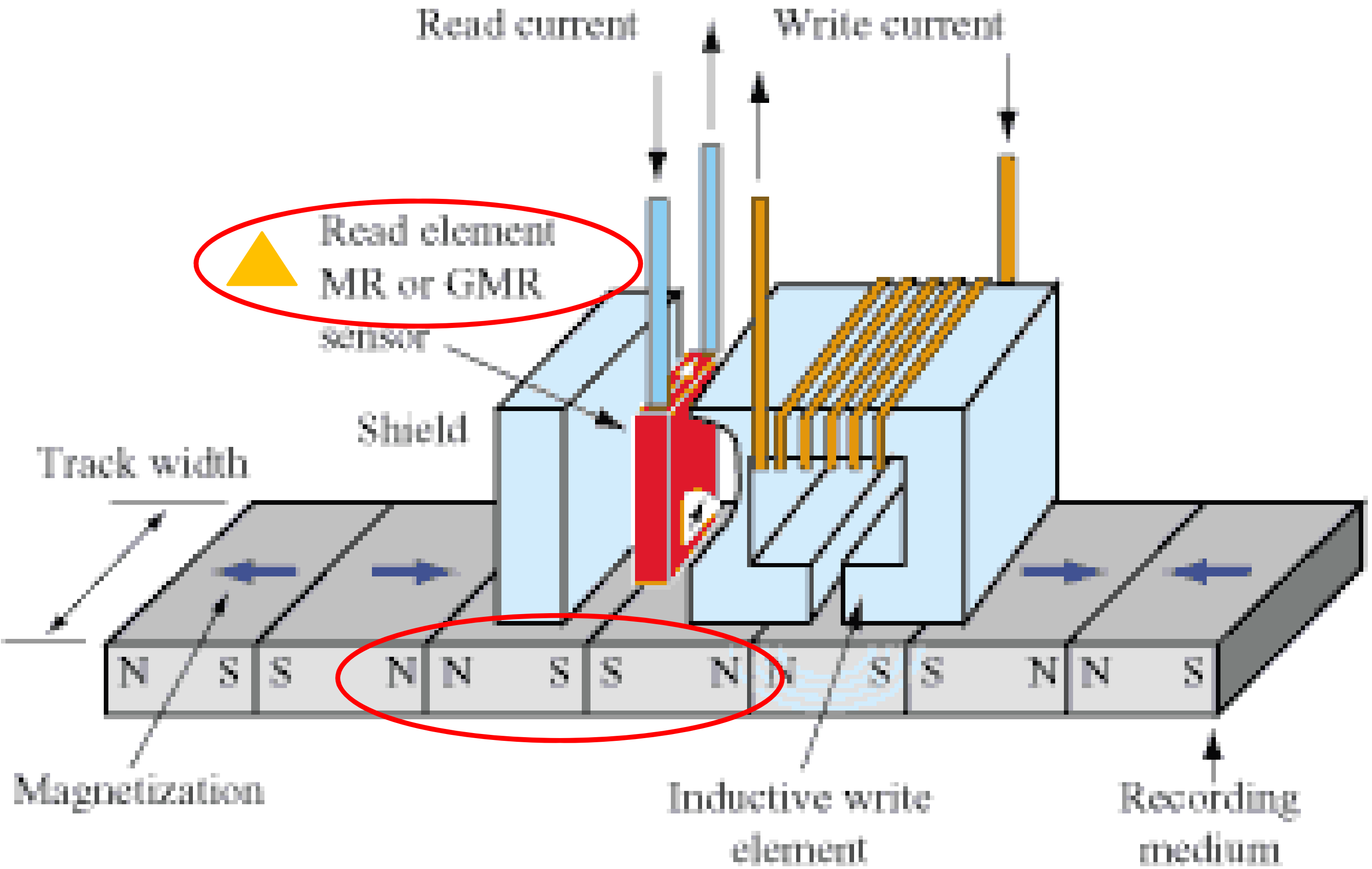
(metal-oxide-semiconductor field-effect transistor)





Fundamental unit of logical gates
(AND, OR, addition, …)

電子科技之基礎--磁記錄



Read current
Write current
Read element
MR or GMR
sensor
Shield
Track width
N S S N N S S N N S S N N S
Magnetization
Inductive write
element
Recording
medium

Basis : Nonrelativistic Quantum Mechanics

$$E = \frac{\hbar^2 k^2}{2m^*} (\sqrt{p^2 + m^{*2}c^4} = m^{*2}c^4 + \frac{p^2}{2m^*} + \ldots)$$

m^* = Effective mass

Transport:

Speed determined by mobility

$$m^* \vec{\upsilon} = e\vec{E} \cdot \tau \Rightarrow \vec{\upsilon} = \mu \vec{E}$$

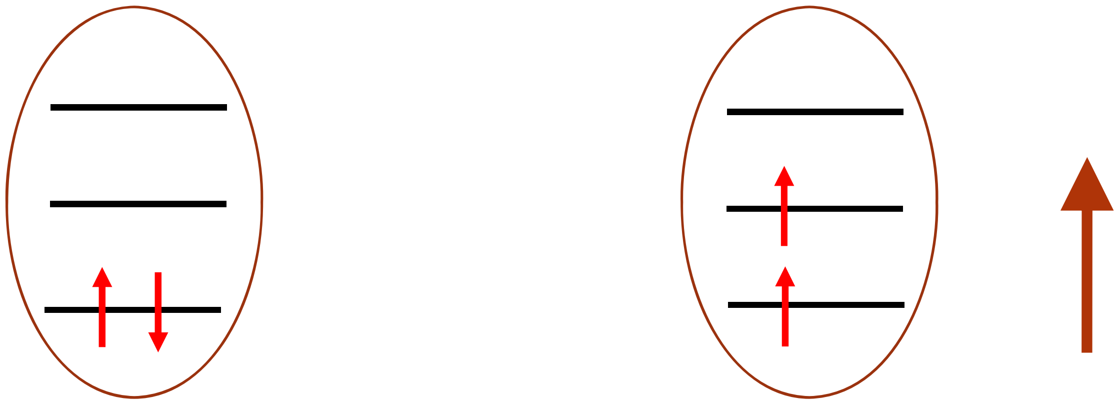
$$\mu = \frac{e\tau}{m^*} = \text{mobility}$$

$$\sigma = \frac{1}{\rho} = e\,(n_e \mu_e + n_h \mu_h)$$

Material	Electron effective mass	Hole effective mass
	Group IV	
Si (4.2K)	1.08 m_e	0.56 m_e
Ge	0.55 m_e	0.37 m_e
	III-V	
GaAs	0.067 m_e	0.45 m_e
InSb	0.013 m_e	0.6 m_e
	II-VI	
ZnO	0.19 m_e	1.21 m_e
ZnSe	0.17 m_e	1.44 m_e

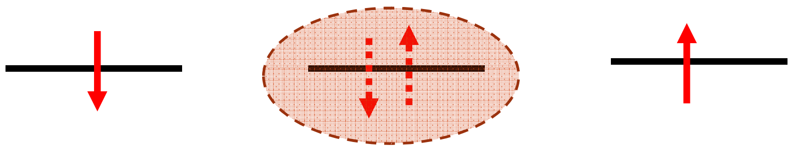
d-orbit (more localized) magnetism

Local moment: Pauli versus Coulomb



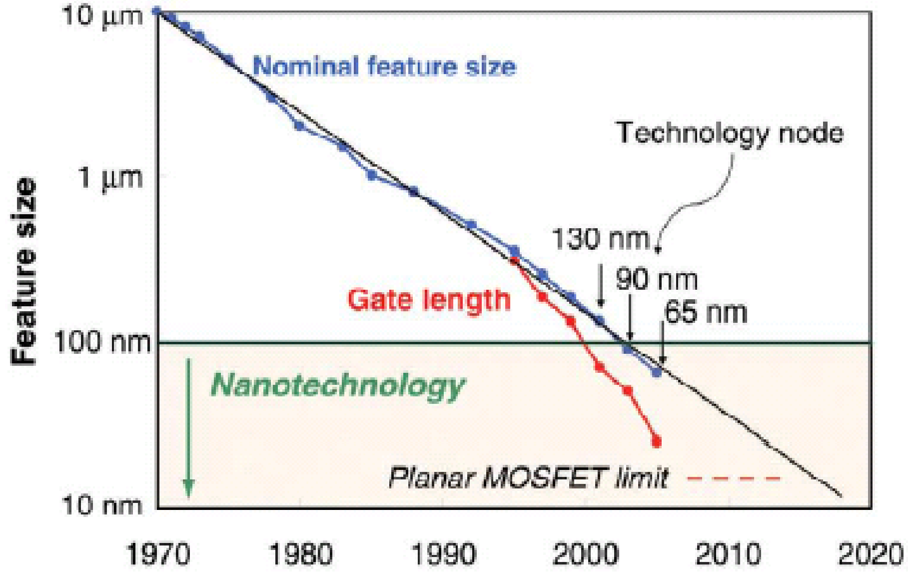
$(\uparrow\downarrow - \downarrow\uparrow)\cdot\phi_a(r_1)\phi_a(r_2)$ $\qquad$ $\uparrow\uparrow\cdot[\phi_a(r_1)\phi_b(r_2)-\phi_b(r_1)\phi_a(r_2)]$

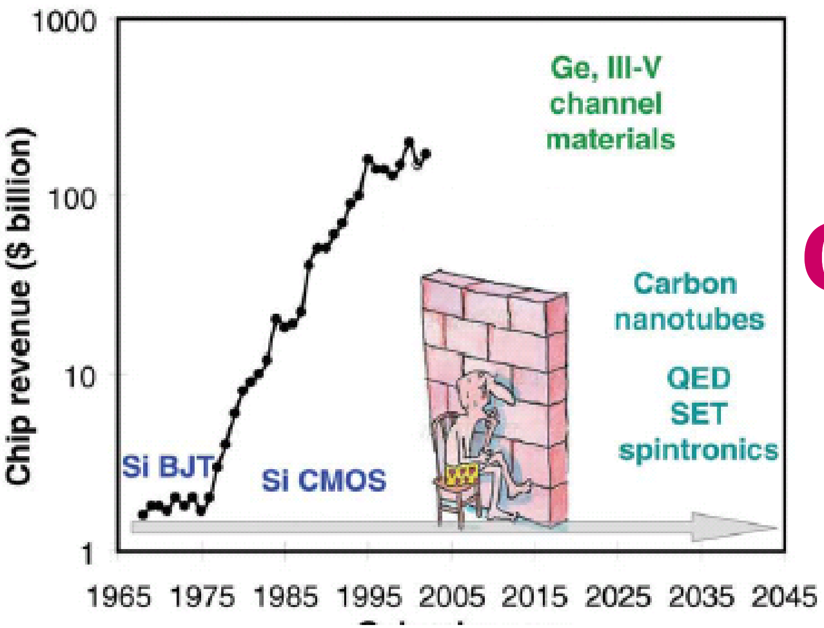
Exchange interactions: $J\vec{S}_i\cdot\vec{S}_j$



Background for search new platform

Scaling limit of Si MOSFET
& superparamagnetism

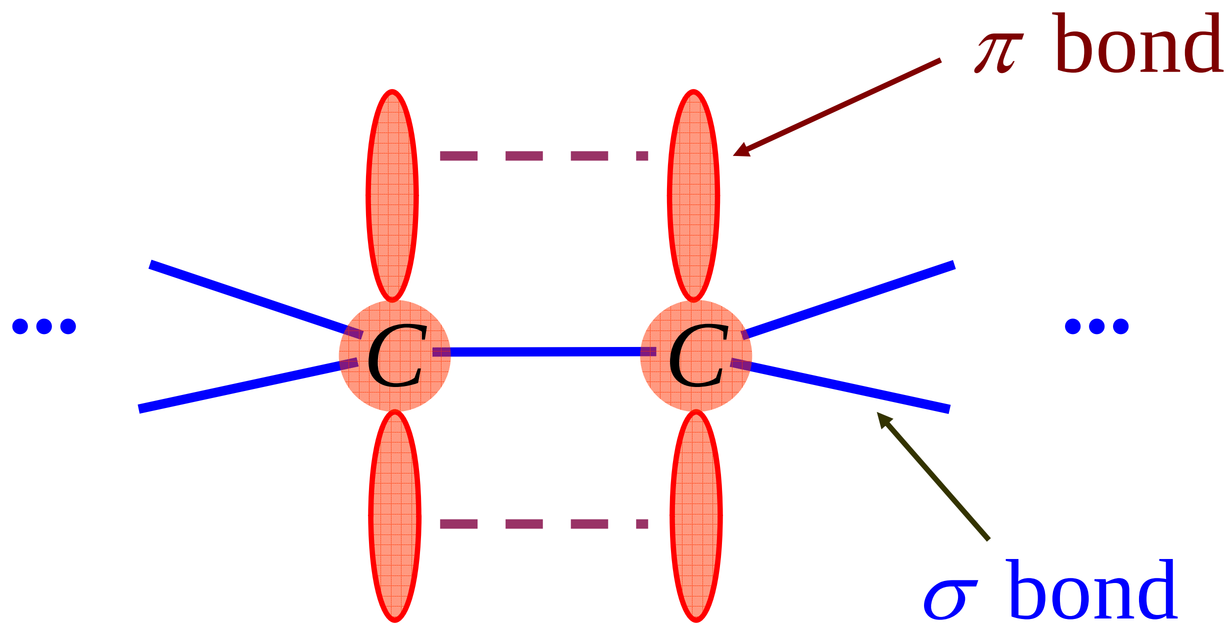




Carbon era?

Thompson and Parthasarathy, Materialstoday 9, 20, 2006

Element of Carbon Network

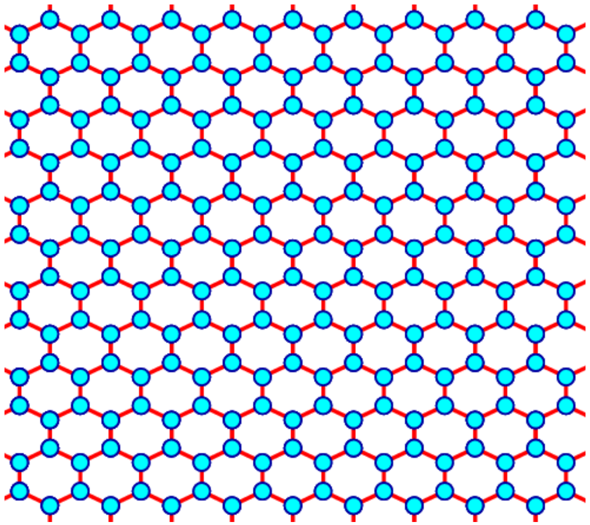


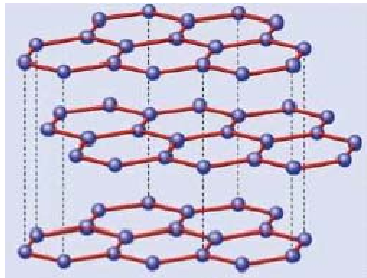
Carbon $1S^2\,\underbrace{2S^2\,2P^2}$

4 electrons in σ bonds (SP^2) + π bond or SP^3

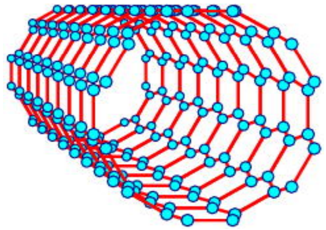
Diversity of carbon forms

graphene



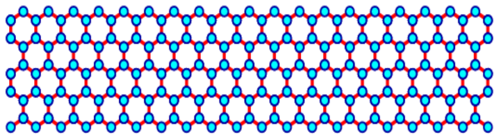


graphite

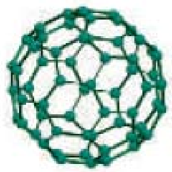


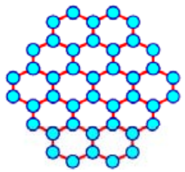
Nanotube

S. Iijima (1991)



Nanoribbon





Nanoparticle & buckyball

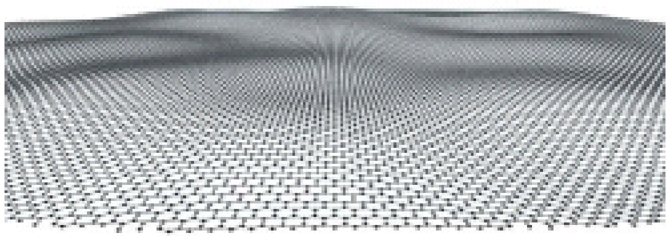
R. Smalley, R. Curl,
H. Kroto (1985)

Unexpected realization of graphene sheet
(⟺ Mermin-Wagner theorem)

mechanically exfoliated graphene sheets

AFM image of single-layer graphene on SiO_2
K.S. Novoselove et al., Science 306, 666 (2004)

Spontaneous ripples in free-standing graphene

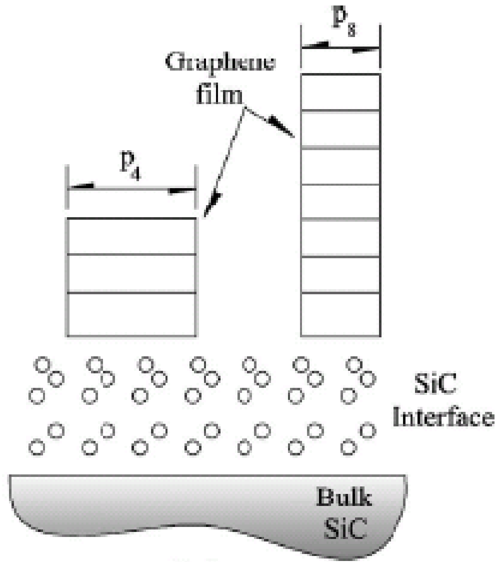


Ripples are of order 2–20 Å high and 20–200 Å wide;
Meyer et al., Nature 446, 60, (2007);
Fasolinao et al., Nature Materials 6, 858 (2007)

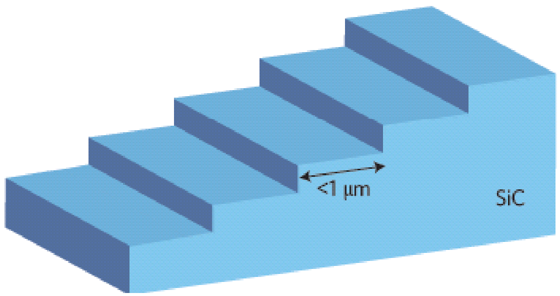
Epitaxial growth of graphene

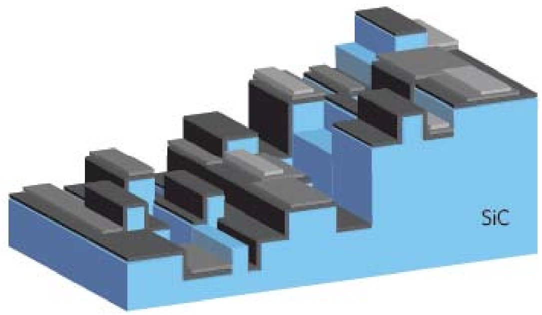
Epitaxial graphene grown on SiC(0001) surfaces

One often ends up with multi-layers of graphene with small grains 30nm-200nm

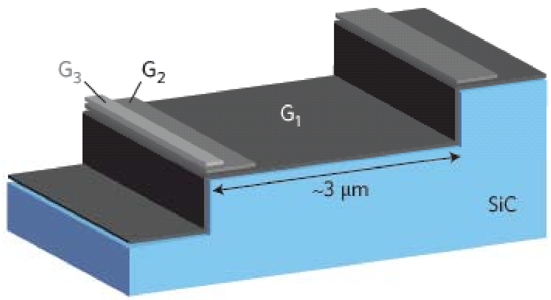


Recent progress:





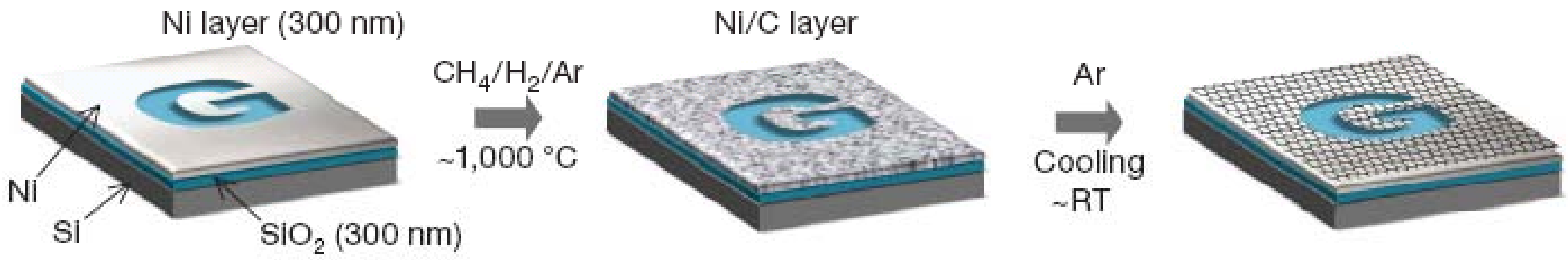
UHV



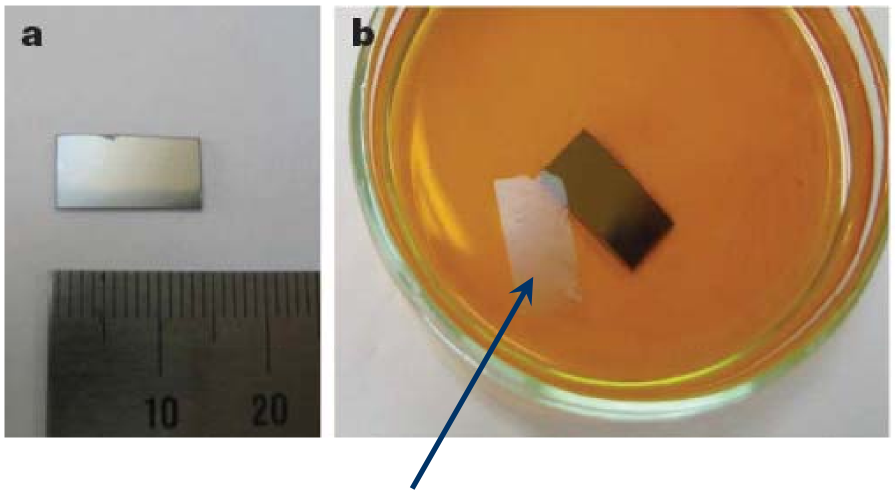
In the presence of pressurized Ar

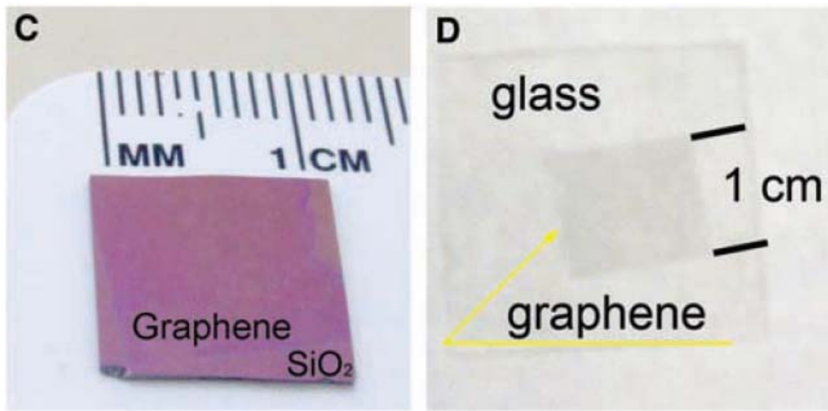
K.V. Emvsev et al., Nature Materials 8, 203 (2009)

CVD graphene on metal substrates



Etching and transfer



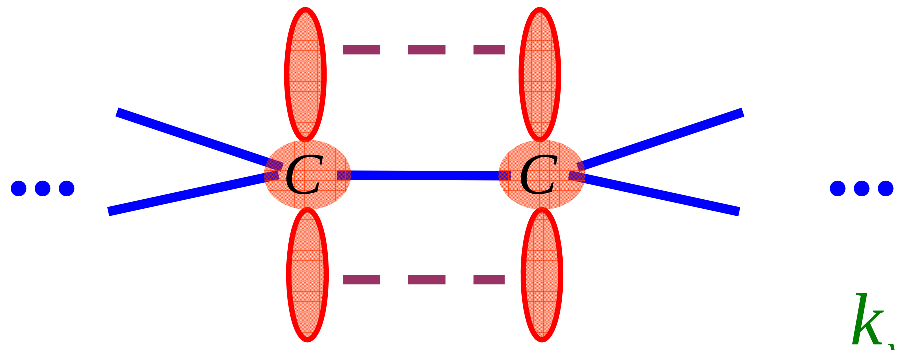


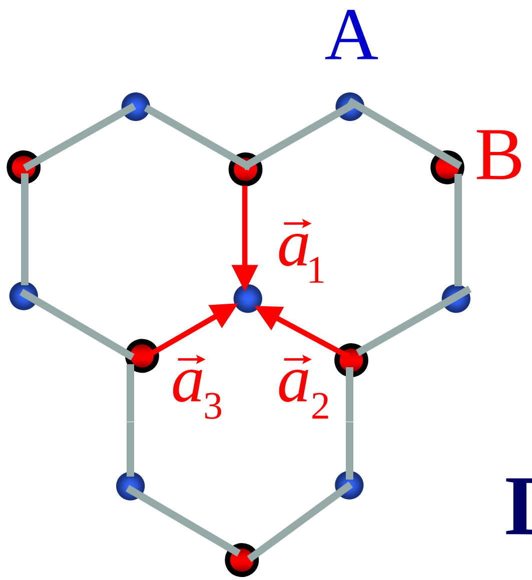
Cu: Li et al., Science 324, 1312 (2009)

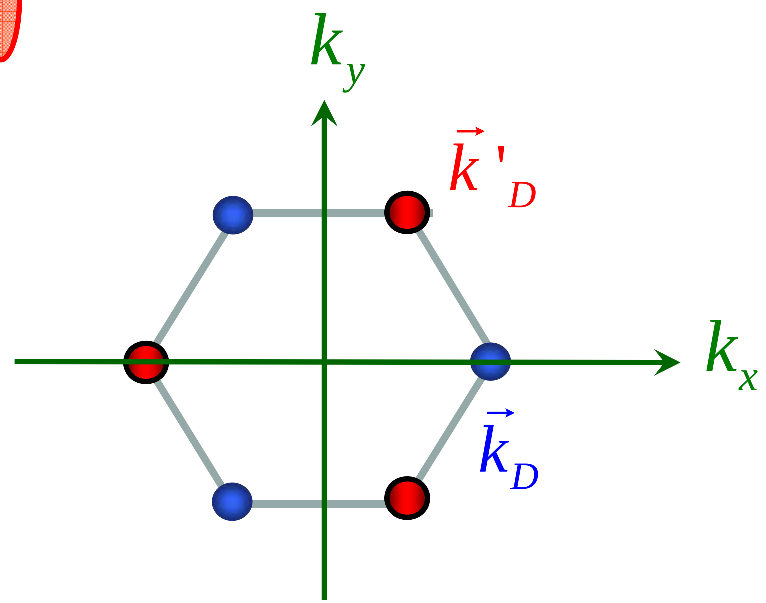
Floating graphene after Ni being etched

Ni: Kim et al., Nature 457, 706 (2009)

Parent spectrum
Two dimensional Dirac Fermions





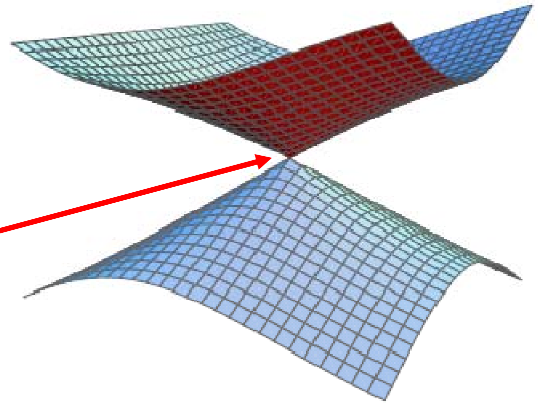


Dirac points

$$e^{i\vec{k}_D\cdot\vec{a}_n} = 1,\ e^{i2\pi/3},\ e^{i4\pi/3} \Leftrightarrow \sum_n e^{i\vec{k}_D\cdot\vec{a}_n} = 0$$

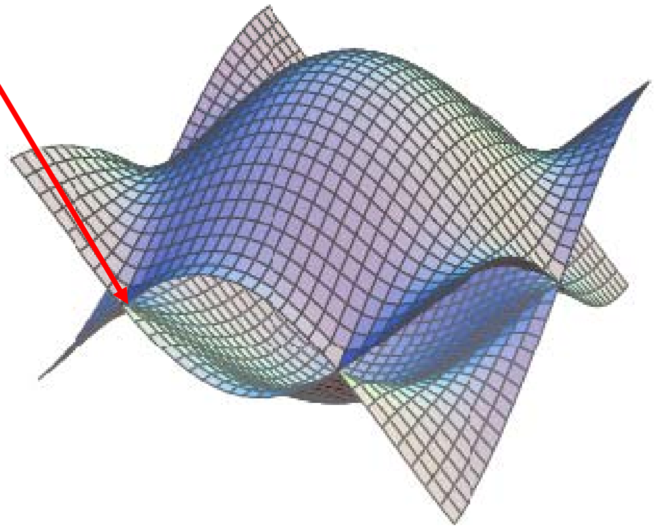
Quasi-Dirac Fermions

$E = \hbar \upsilon k$ with $m = 0$



Dirac points

New playground for testing
relativistic QM and QED!



Not exactly Dirac Fermions

$\upsilon \approx c / 300$

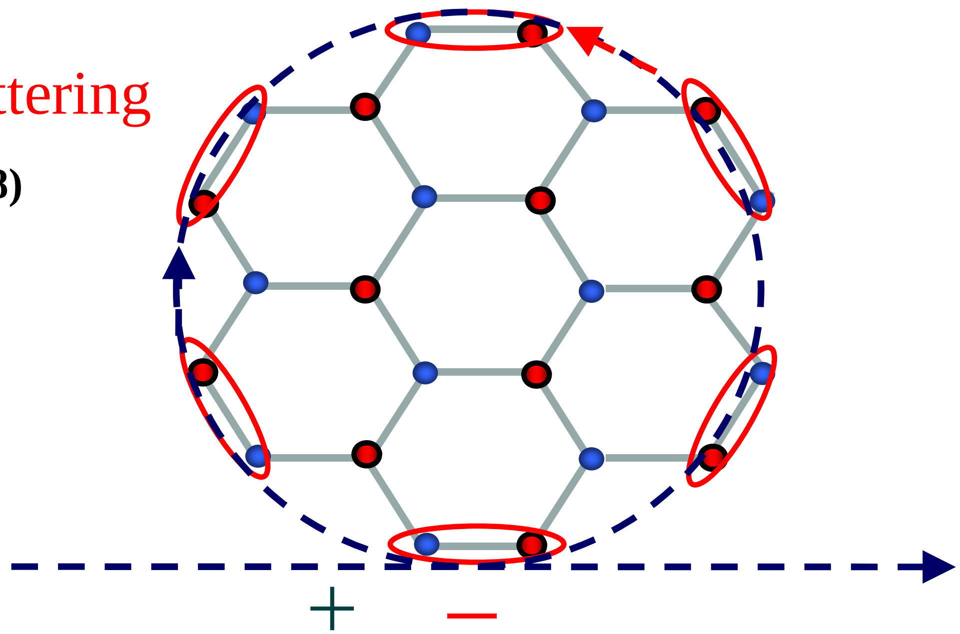
$k_D \neq 0$ and inter-Dirac point scatterings!

Expectation

$$m^{*}\vec{\upsilon} = e\vec{E}\cdot\tau \Rightarrow \vec{\upsilon} = \mu\vec{E}$$

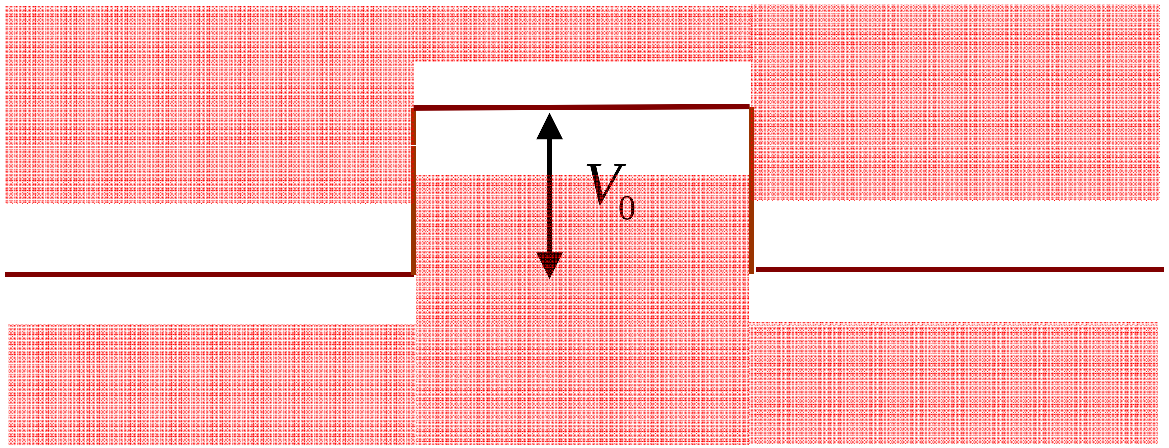
- $m^{*} = 0$
- Berry phase & absence of back scattering

T. Ando et al., J. Phys. Soc. Jap. 67, 2857 (1998)



$\Rightarrow$ huge μ and high speed

Potential complication: Klein Paradox



$$T \to 1 \text{ as } V_0 >> m_0c^2$$

V_0: repulsive for electrons, attractive for positrons

No confinement for electrons

On/off ratio is reduced in graphene FET

Super-Qualities

- m* = 0 expect huge mobility

Carrier mobility: 200000 $cm^2/V.s$

(Geim, 2008, 300K, $n \approx 10^{13} cm^{-2}$)

Ballistic transport at micronscale

Epitaxial graphene: 2000 $cm^2/V.s$ **(27K)** $\lambda_\phi \geq 1 \mu m$

CVD graphene: 4050 $cm^2/V.s$ **(room temp)**

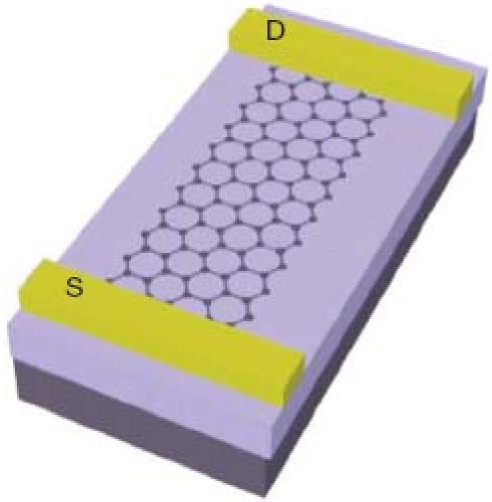
Si 1500 $cm^2/V.s$ **high speed GaAs 8500** $cm^2/V.s$

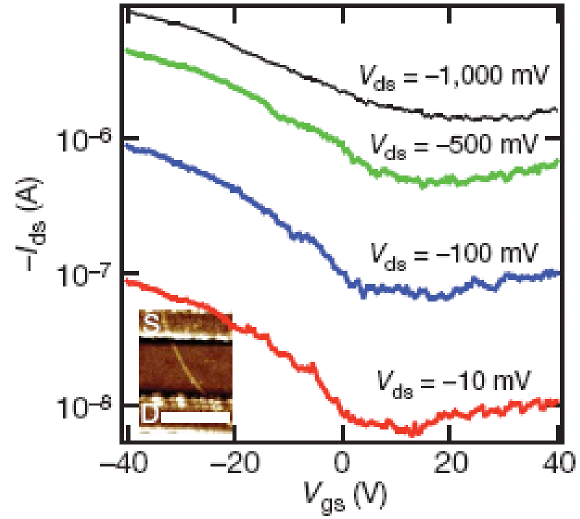
InSb (undoped) 77000 $cm^2/V.s$

- Thermal conductivity (room temp)

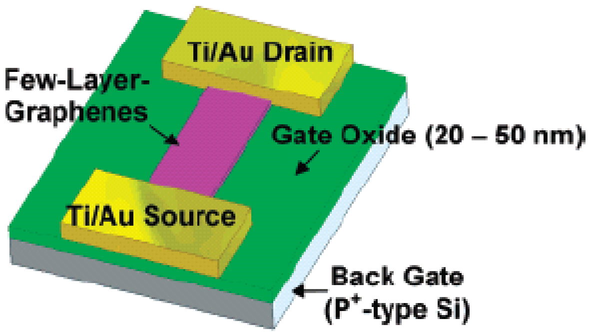
$\approx 5 \times 10^3 Wm^{-1}K^{-1} \sim$ 10 × Cu or Al

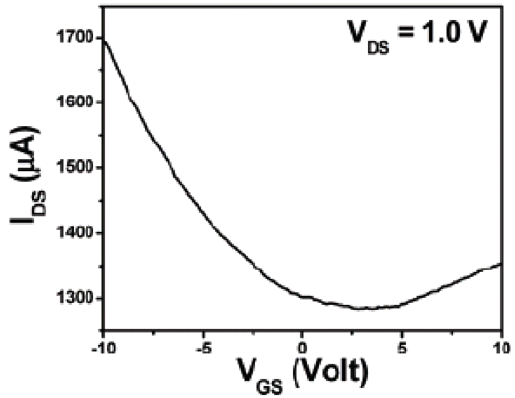
Room temperature Graphene FET

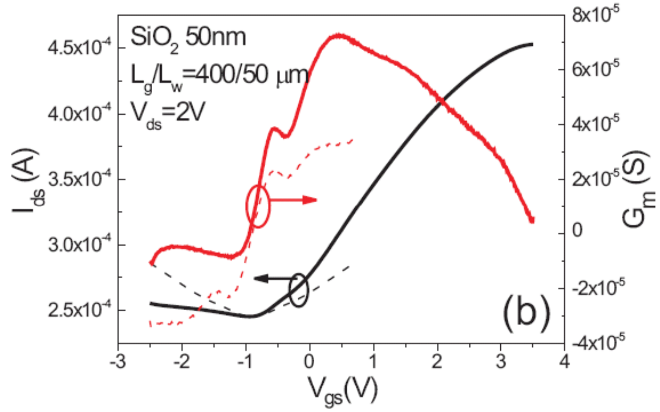




Nanoribbon, Hongjie Dai's group, Science 319, 1229 (2008) ; Nature 458, 872(2009)







X. Liang et al. , Nano Letters 7, 3840 (2007); Y.Q. Wu, APL 92, 092102 (2008) Kedzierski, IEEE Electron Dev. 55, 2078 (2008)

RF performance

Technology	**f_T (GHz)**
InP	22 (GHz-µm)
ITRS Bulk NMOS	9 (GHz-µm)
SOI (90nm)	11 (GHz-µm)
Graphene made from SiC[1]	10 (GHz-µm)
Mechanically exfoliated Graphene (500nm)[2]	14.7 (GHz)
Carbon nanotube transistor[3] (3µm)	50 (GHz)

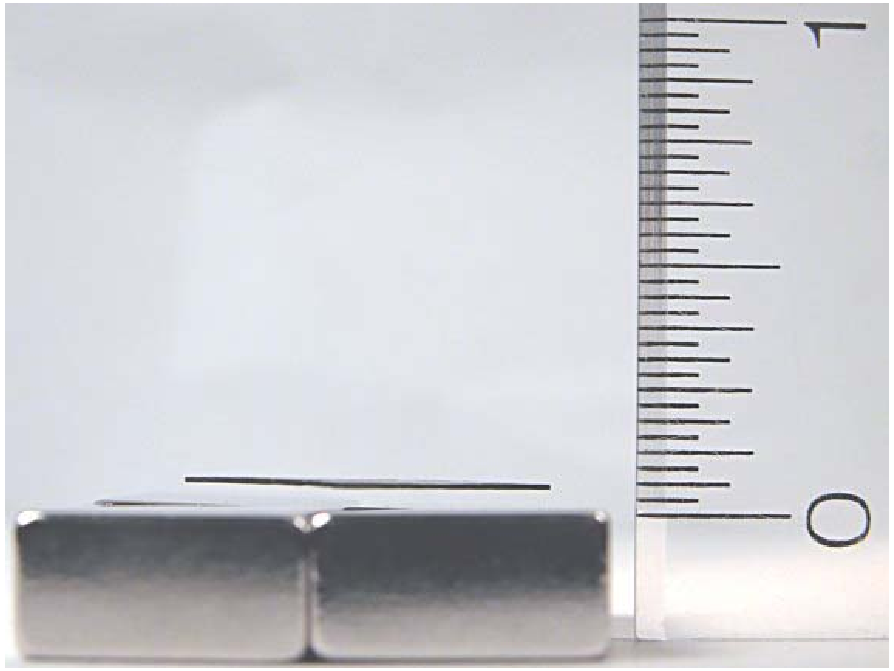
1. **J.S. Moon et al., ECS Transactions 19, 35 (2009)**
2. **I. Meric et al., IEEE Electron Device Meeting (2008)**
3. **S. Rosenblatt et al., Appl. Phys. Lett. 87, 1531111 (2005)**
   Theoretical value: THz (L=20nm W=0.69Nm) , L.C. Castro et al, IEEE Trans. Nanotech. 4, 699 (2005)

Magnetism?

d0 magnetism ?

Magnetic levitation at room temperature

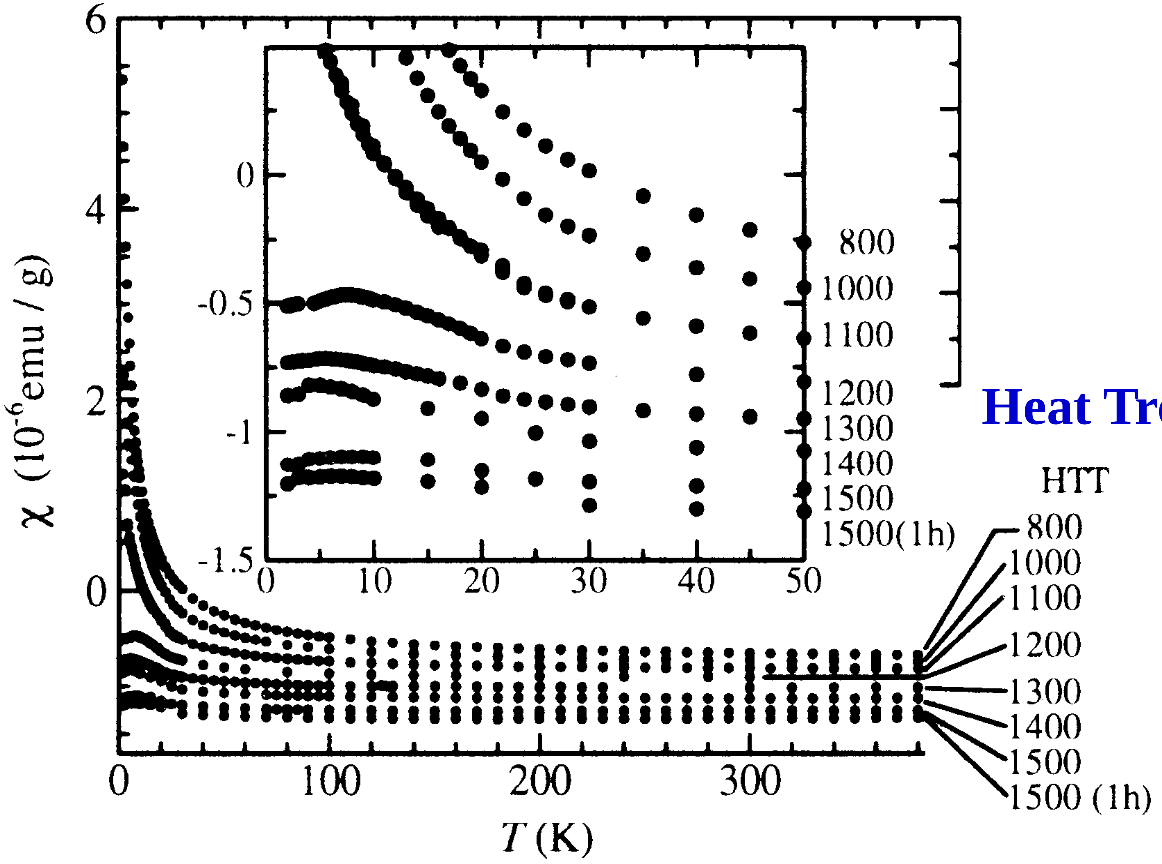
Graphite: two of the strongest diamagnetic materials
(the other one is bismuth)



Pyrolytic Graphite
http://cgi.ebay.com.hk/ws/eBayISAPI.dll?ViewItem&item=180186727424

Why magnetism?

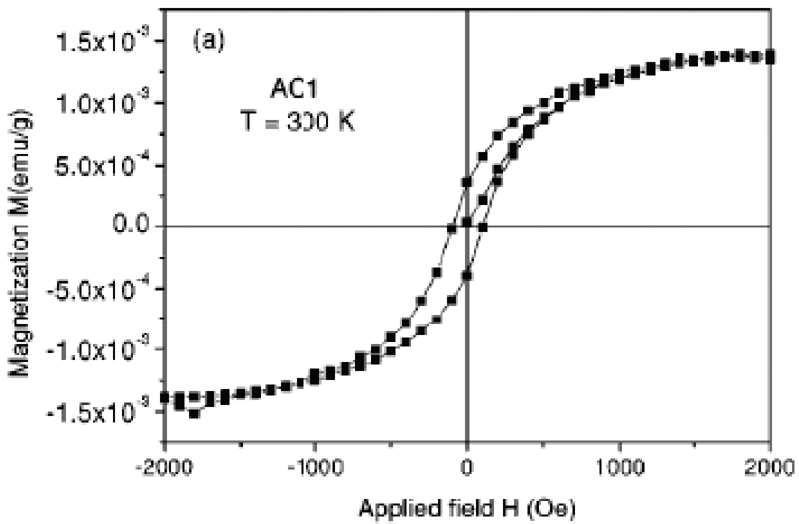
Evidences of Magnetic Caron?

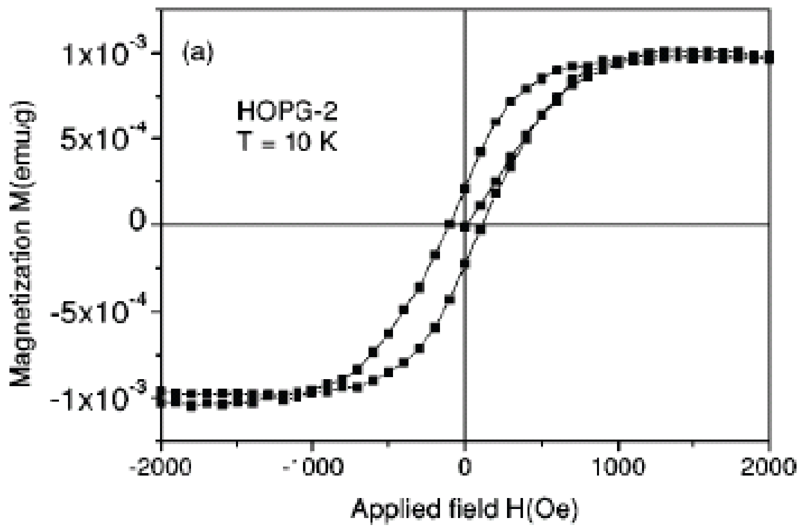


Heat Treatment Temp

Shibayama et al. Phys. Rev. Lett. 84, 1744(2000)

Weak ferromagnetism





Mechanism:

Magnetic impurity?

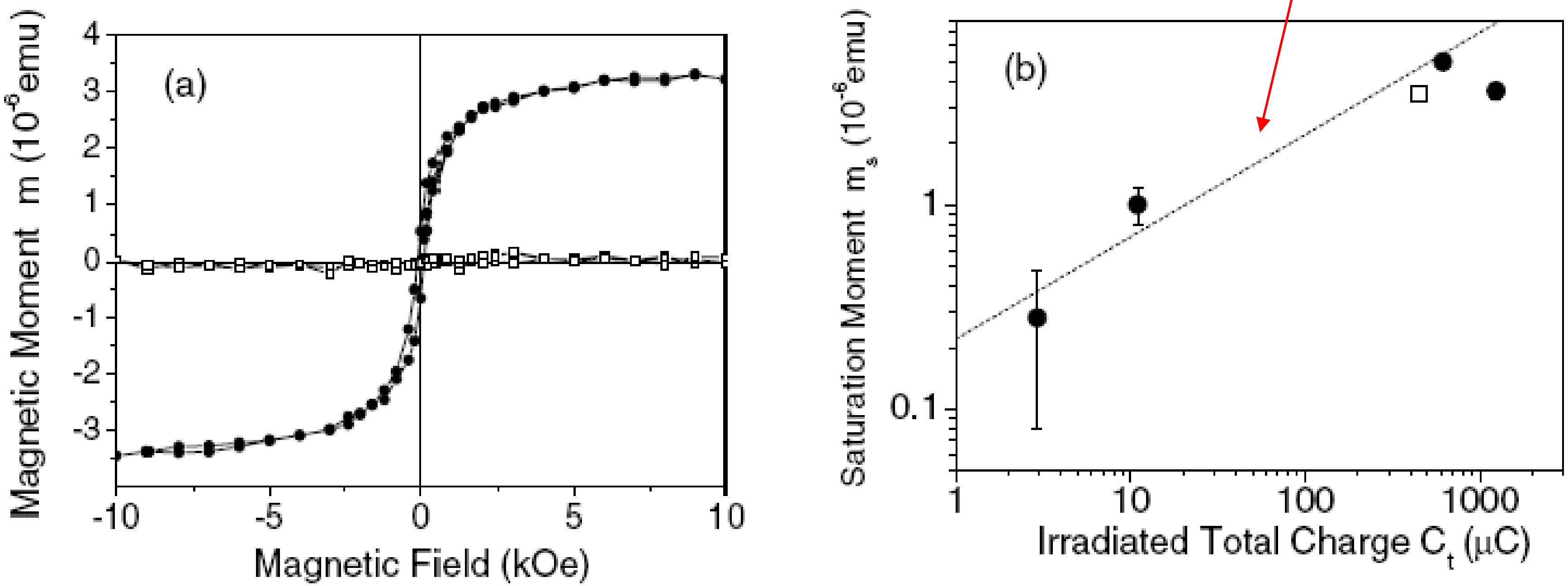
Defects?

Triplet excitons condensate?

Esquizani et al. Phys. Rev. B 66, 024429 (2002)

Defects induced weak ferromagnetism

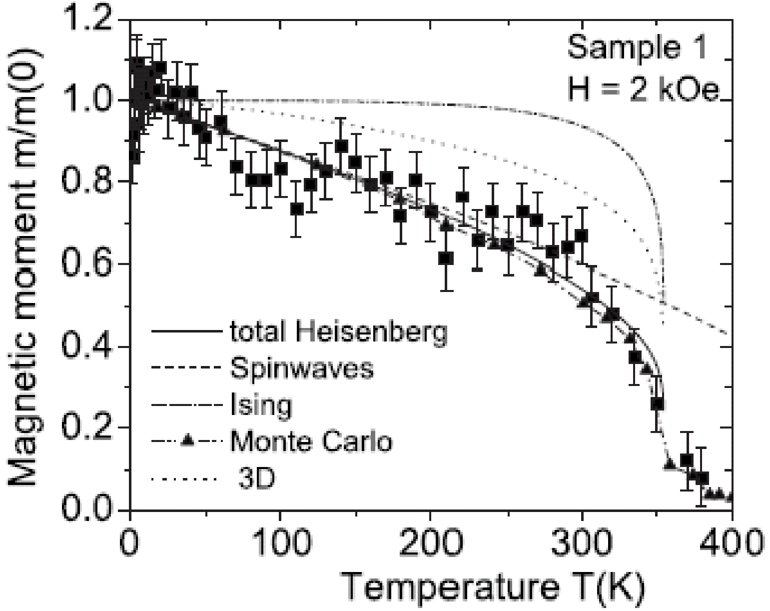
$$m \approx \sqrt{C_t}$$

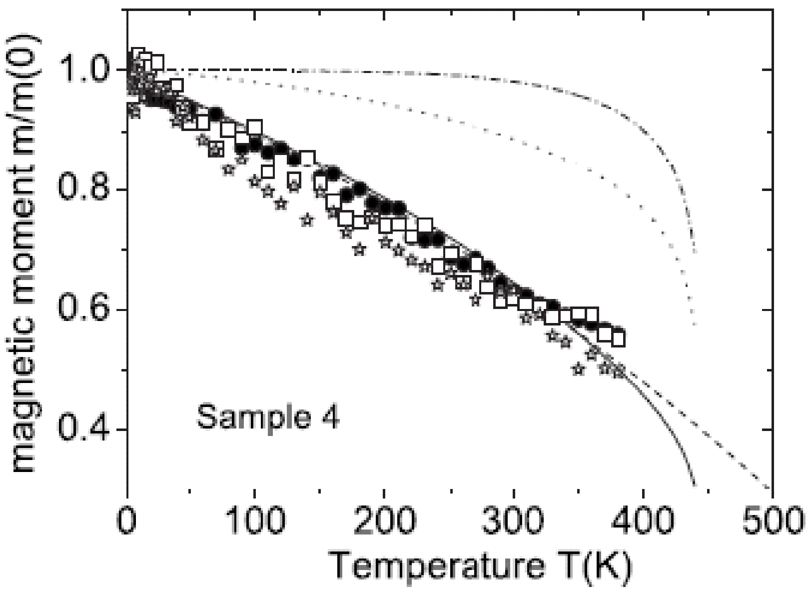


Magnetism persists to the case when only 0.1% carbon atoms gets shifted

Esquinazi et al Phys. Rev. Lett. 91, 227201, 2001

Unequivocal linear temperature behavior

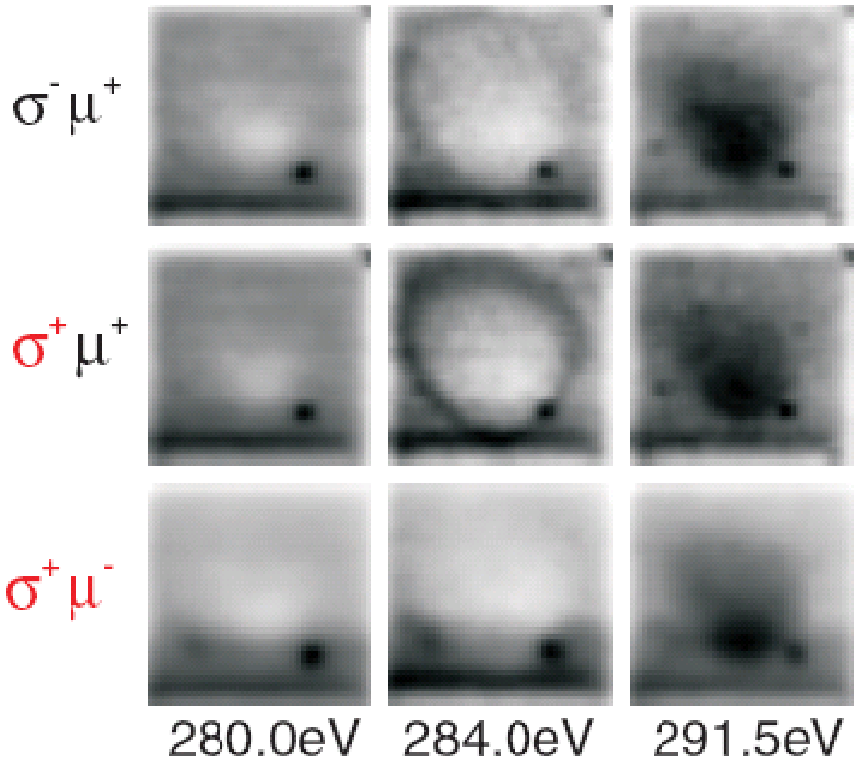


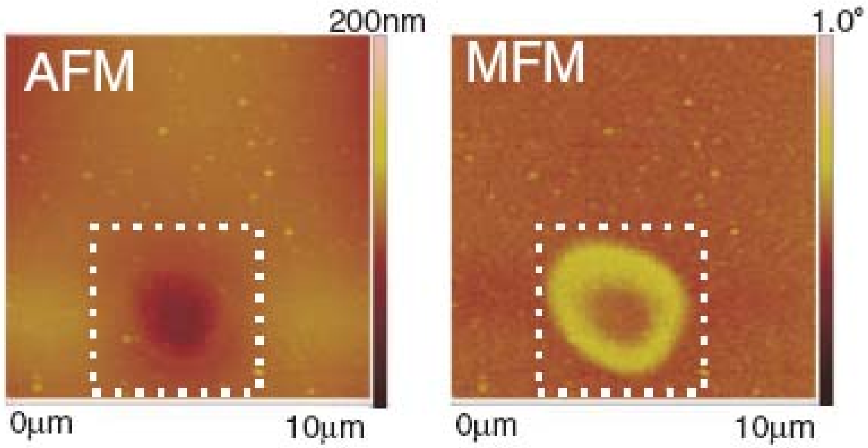


Barzola-Quiquia et al Phys. Rev. B 76, 161403, 2007

π electrons ferromagnetism: spectroscopic evidence

X-ray circular dichroism (XMCD)





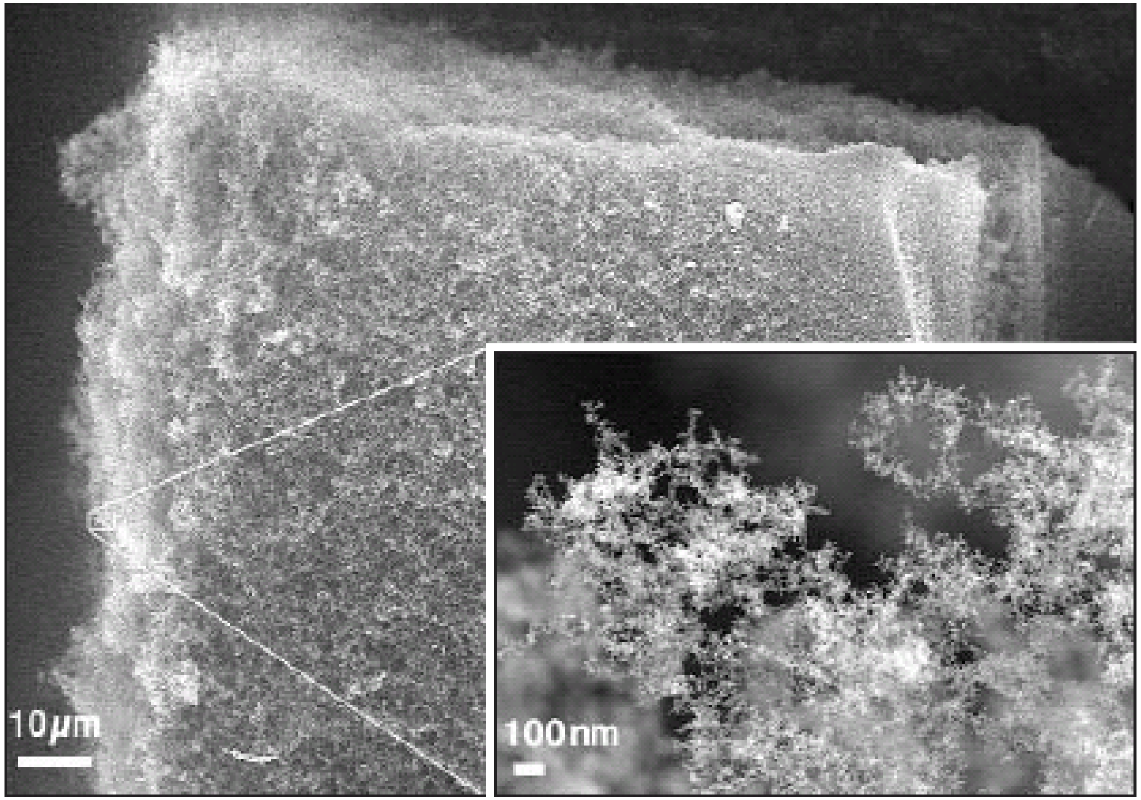
π σ

weak ferromagnetism:

$5\times10^{-4} - 1\times10^{-3}\,\mu_B$ per atom

Ohldag et al., PRL 98, 187204 (2007).

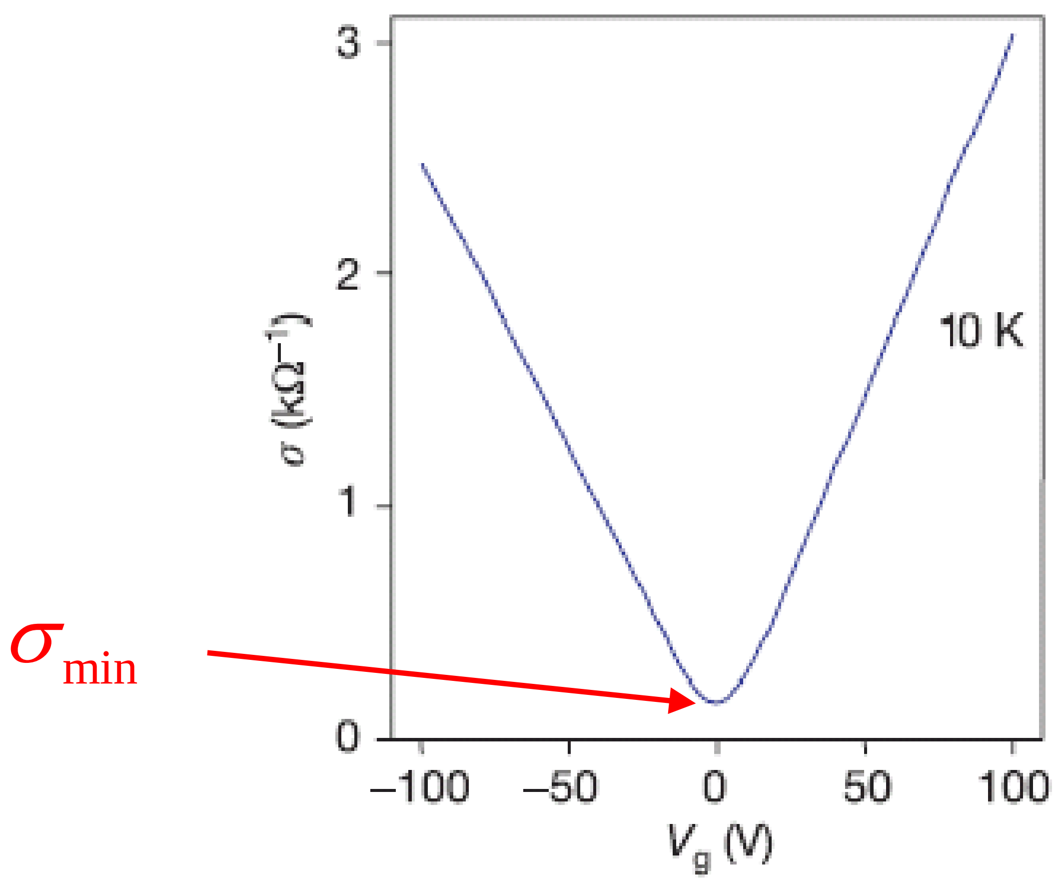
Extreme form: Carbon foam



Attractive. One of the lightest substances ever made, nanoscale froth condensed from superheated carbon atoms is also magnetic at room temperature for a few hours.

Extensive tests show that magnetic impurity can only account for 20% of magnetism, Science 304, 42 (2004).

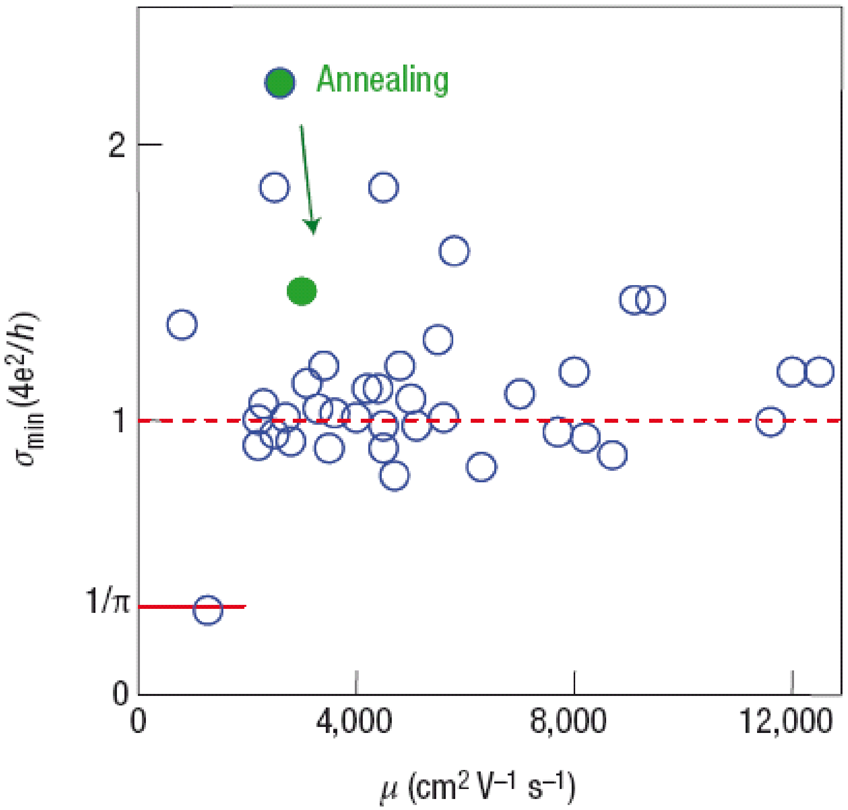
Other evidences for roles of defects

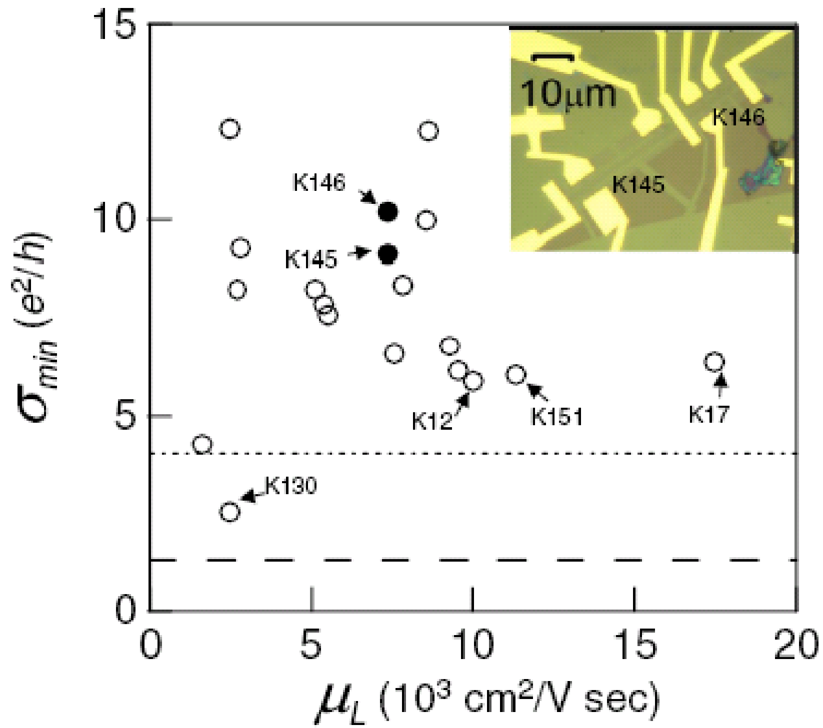


Absence of strong localization!

Novoselov et al. Nature 438, 197 (2005)

Universal ?

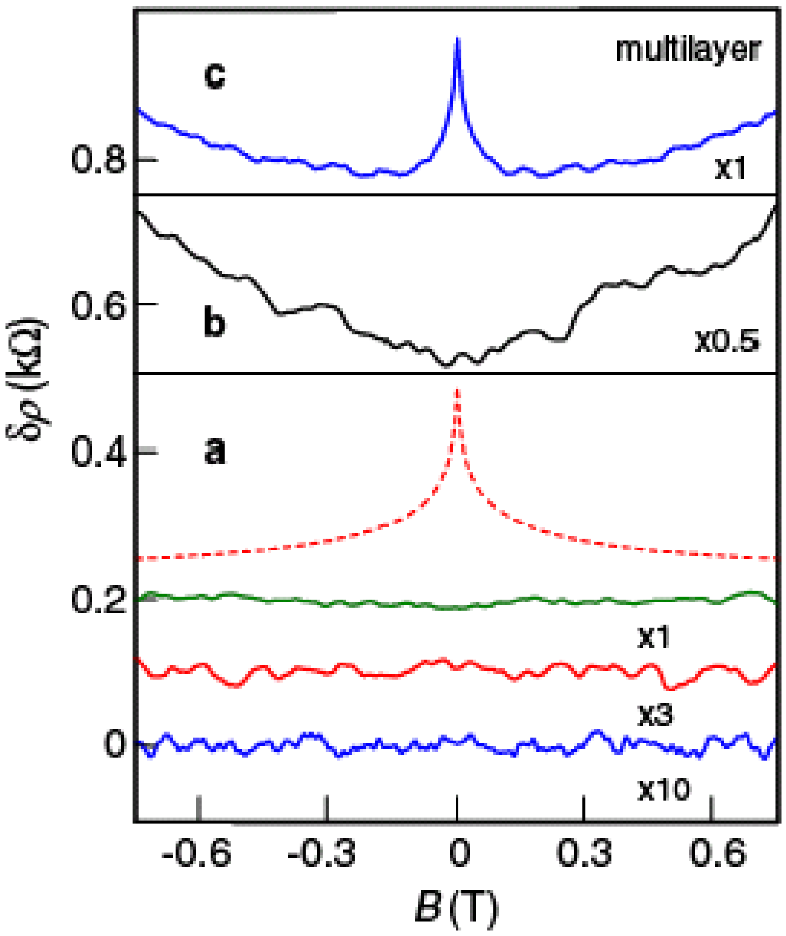




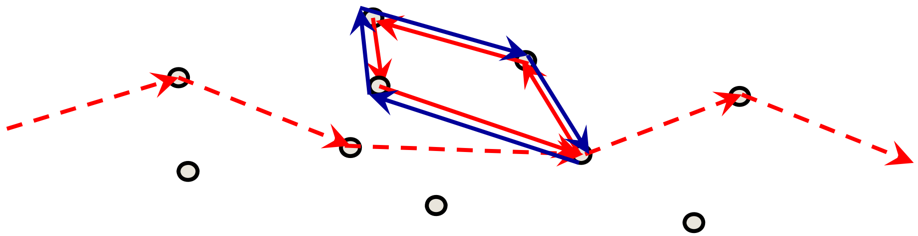
Geim and Novoselov,
Nature Mater. 6, 183 (2007)

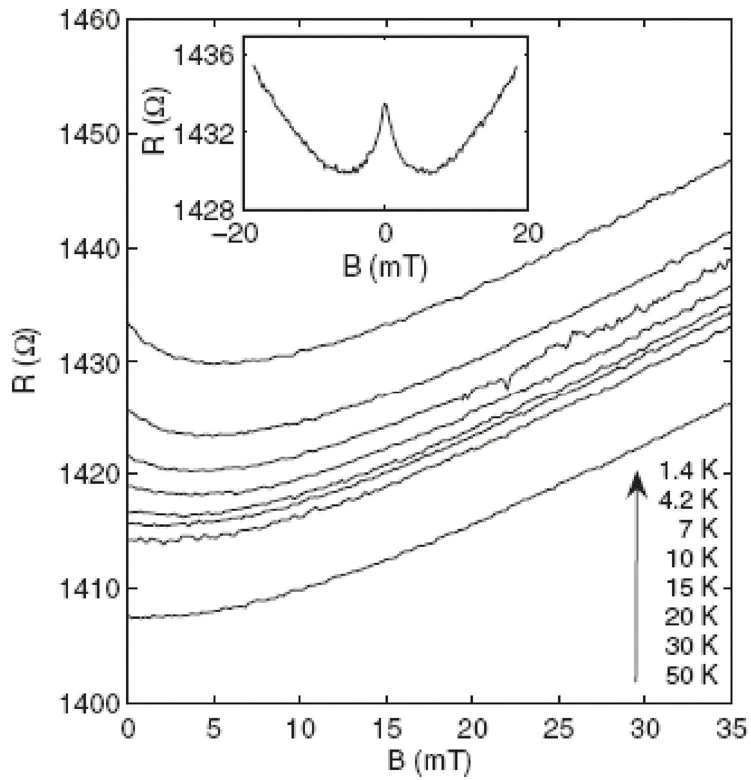
Tan et al., PRL 99, 246803 (2007)

Weak localization and weak antilocalizaion



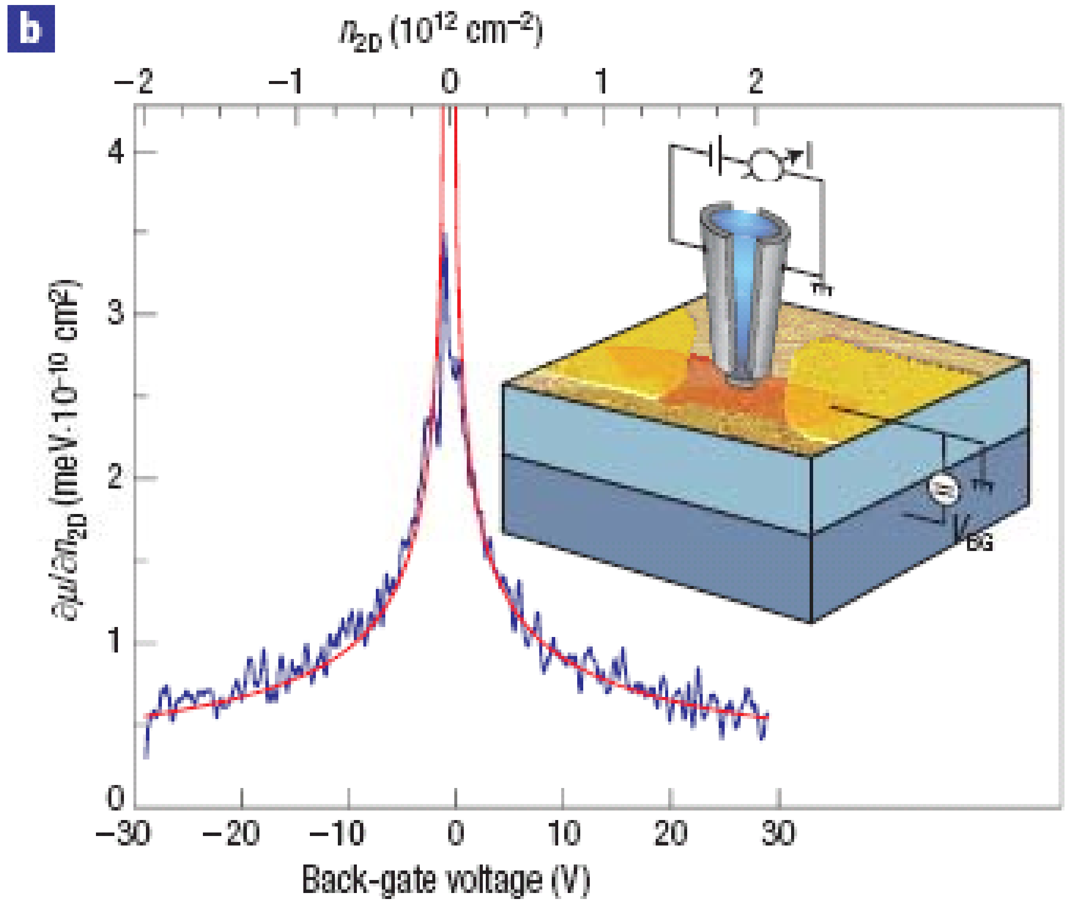
In some samples:
Morozov et al., PRL 97, 016801 (2006)





Wu et al., PRL 98, 136801 (2007)

Inverse compressibility



$$n = \int_{-\infty}^{\mu} D(E)dE \Rightarrow 1 = D(\mu)\frac{\partial \mu}{\partial n}, \quad \frac{\partial \mu}{\partial n} \propto \frac{1}{D(\mu)}$$

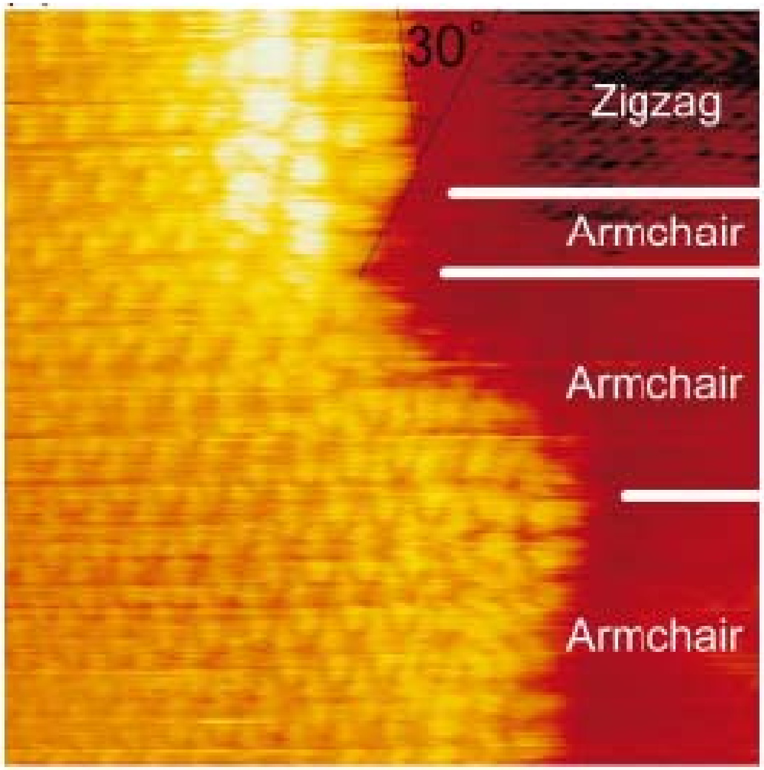
Martin et al., Nature Phys. 4, 144 (2008)

Outline:

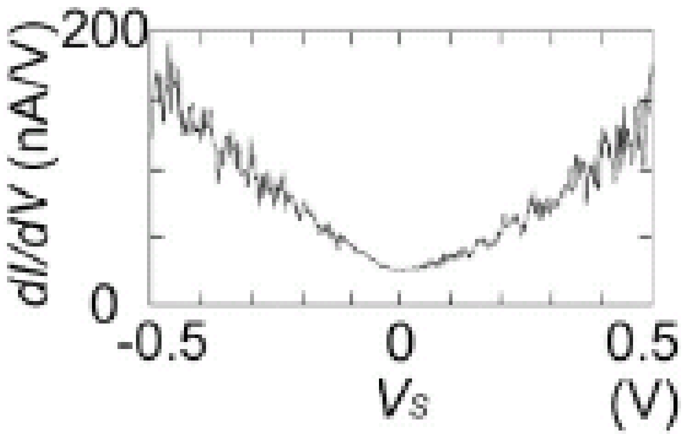
- Background and Introduction: Transport and Magnetic Properties
- **Novel magnetism associated with edge states and flat-band in nanoribbons**
- Impurity band due to point defects in graphene

Peculiar density of states in edges of graphene

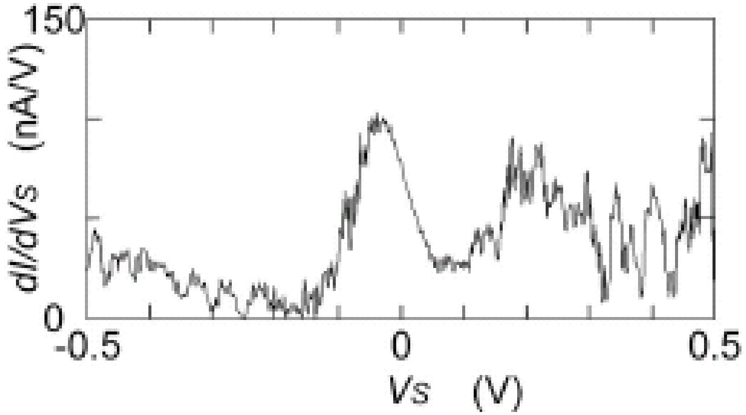
STM measurement



Kobayashi et al, PRB 71, 193406 (2005)



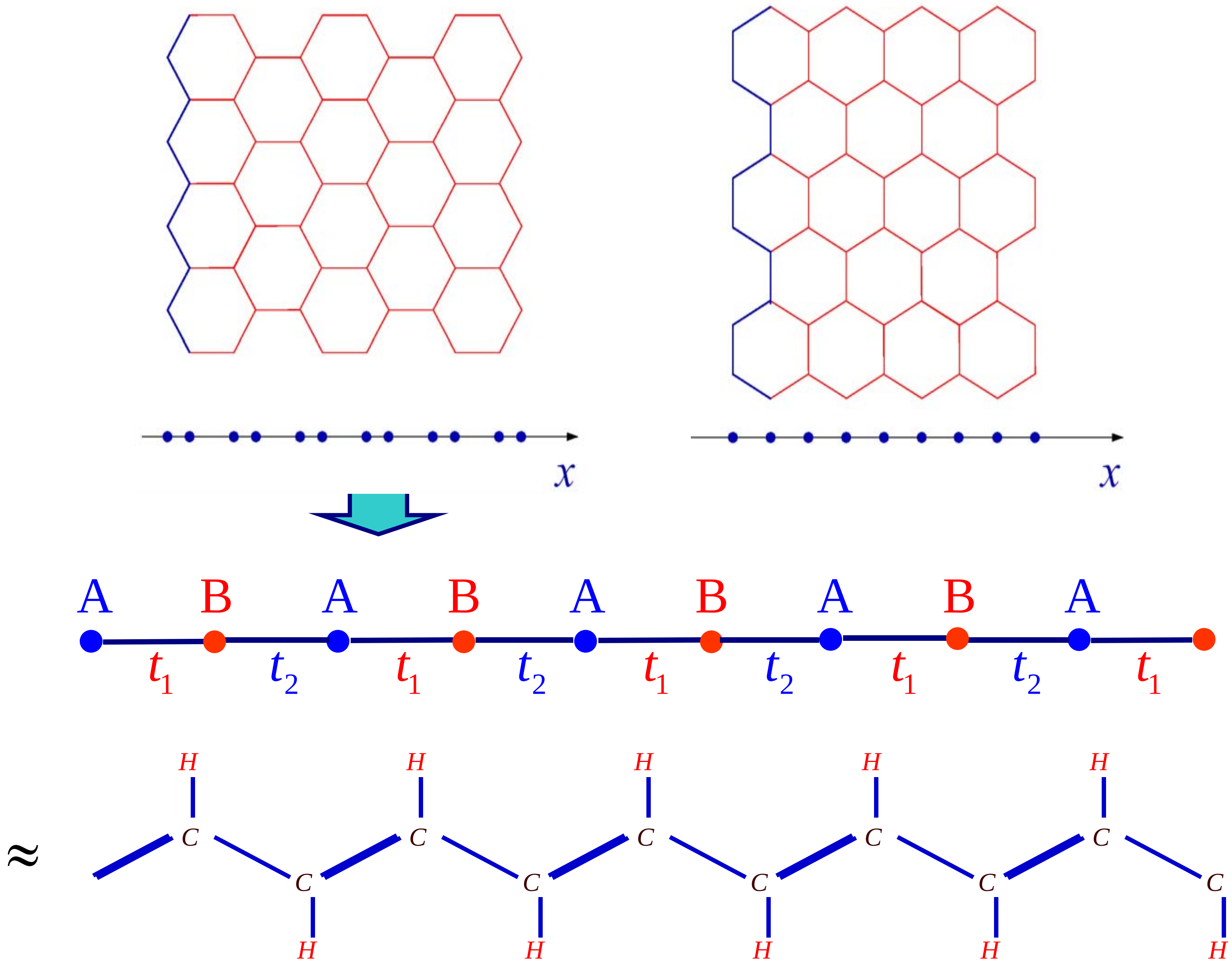
Typical SIS for armchair edge



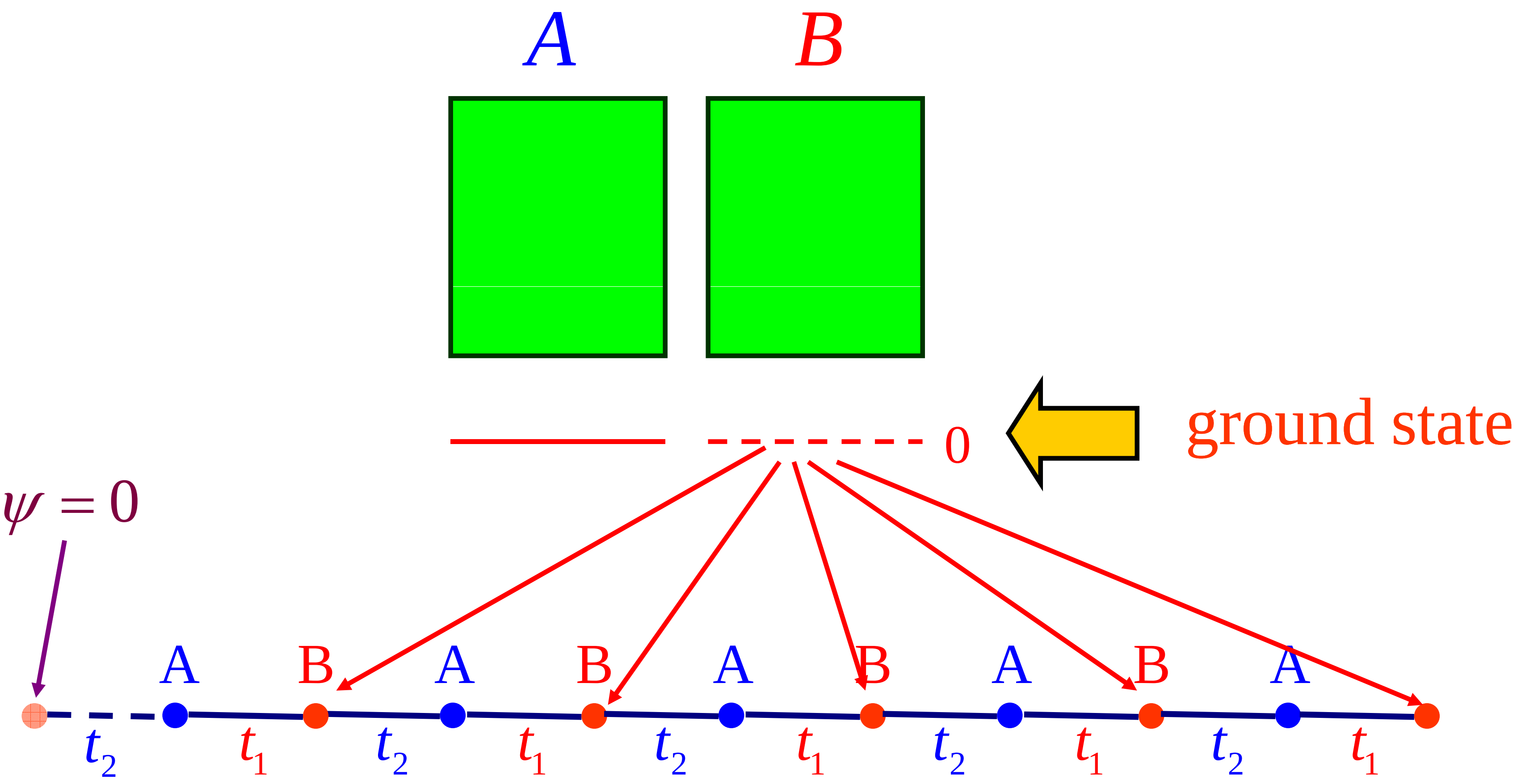
Typical SIS for zig-zag edge

How do we understand it?

Fourier transform along the interface



SUSY Quantum Mechanics and zero-energy state



$$t_1\psi_A^{n-1} + t_2\psi_A^{n+1} = E\psi_B^n = 0 \Rightarrow \quad \psi_A^{n+1} = -(t_1/t_2)\psi_A^{n-1}$$

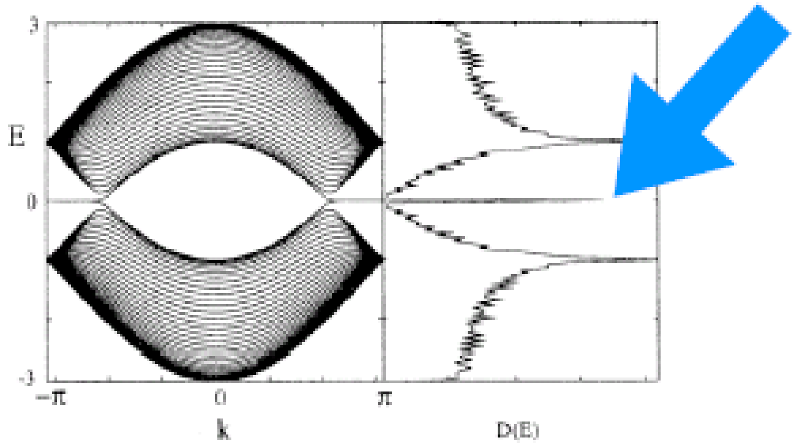
$$t_2\psi_B^n + t_1\psi_B^{n+2} = E\psi_A^{n+1} = 0 \Rightarrow \quad E = 0$$

Huang et al. Phys. Rev. B 70, 205408 (2004)

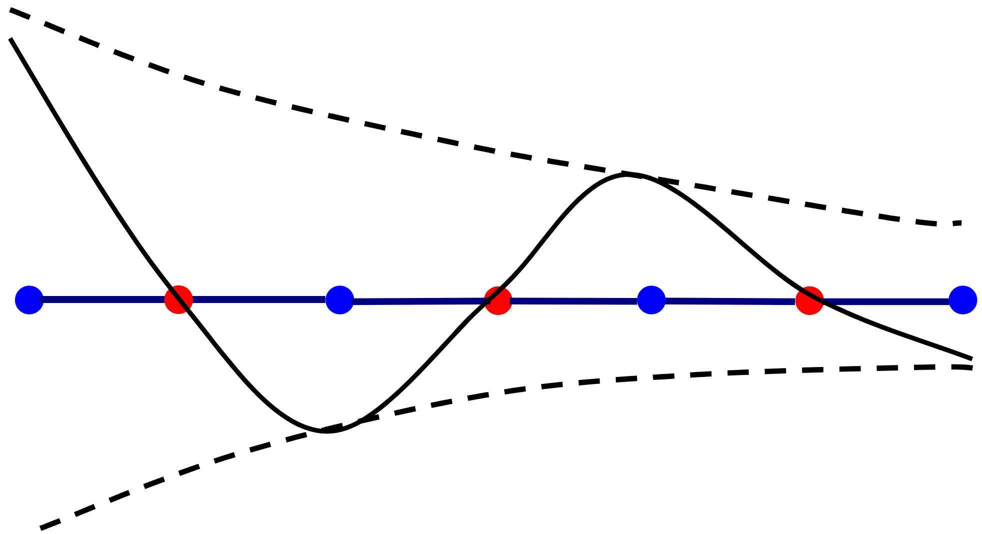
Localization of the zero-energy state

$$\psi = \left(-\frac{t_1}{t_2} \right)^n$$

DOS for *zigzag* nanoribbon



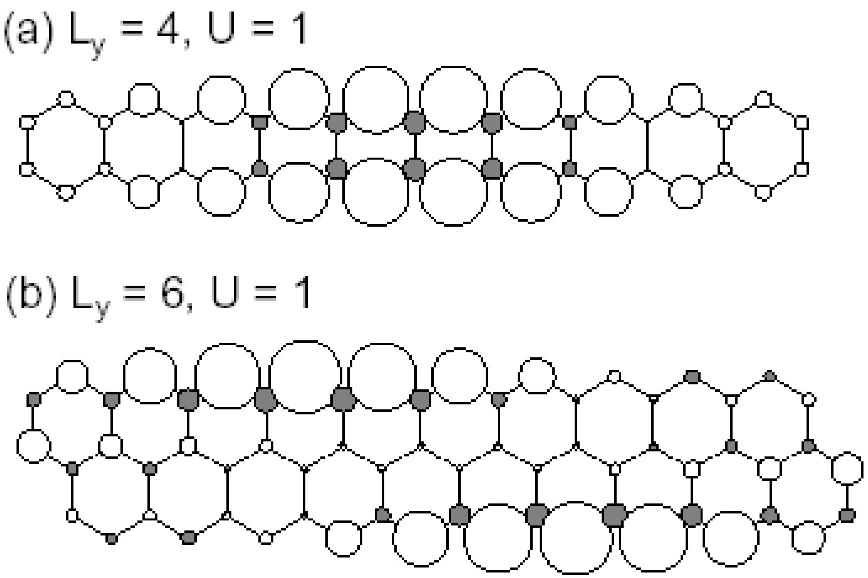
For t1<t2:

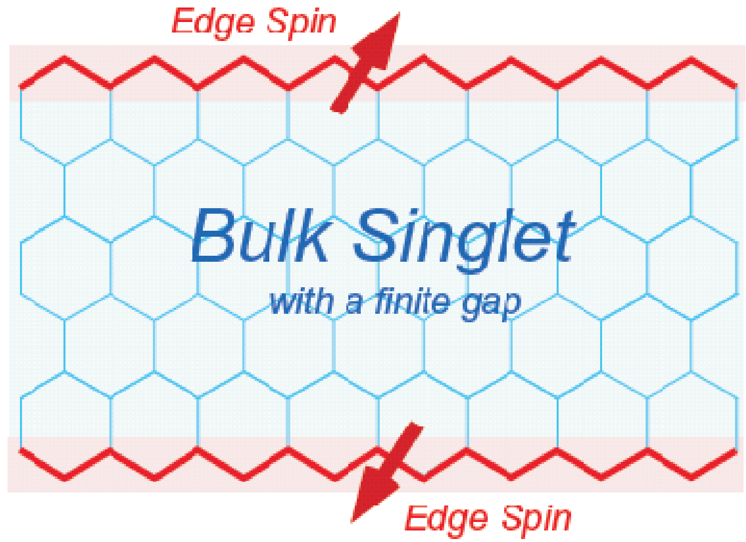


Interacting electrons in zigzag ribbons

M=1

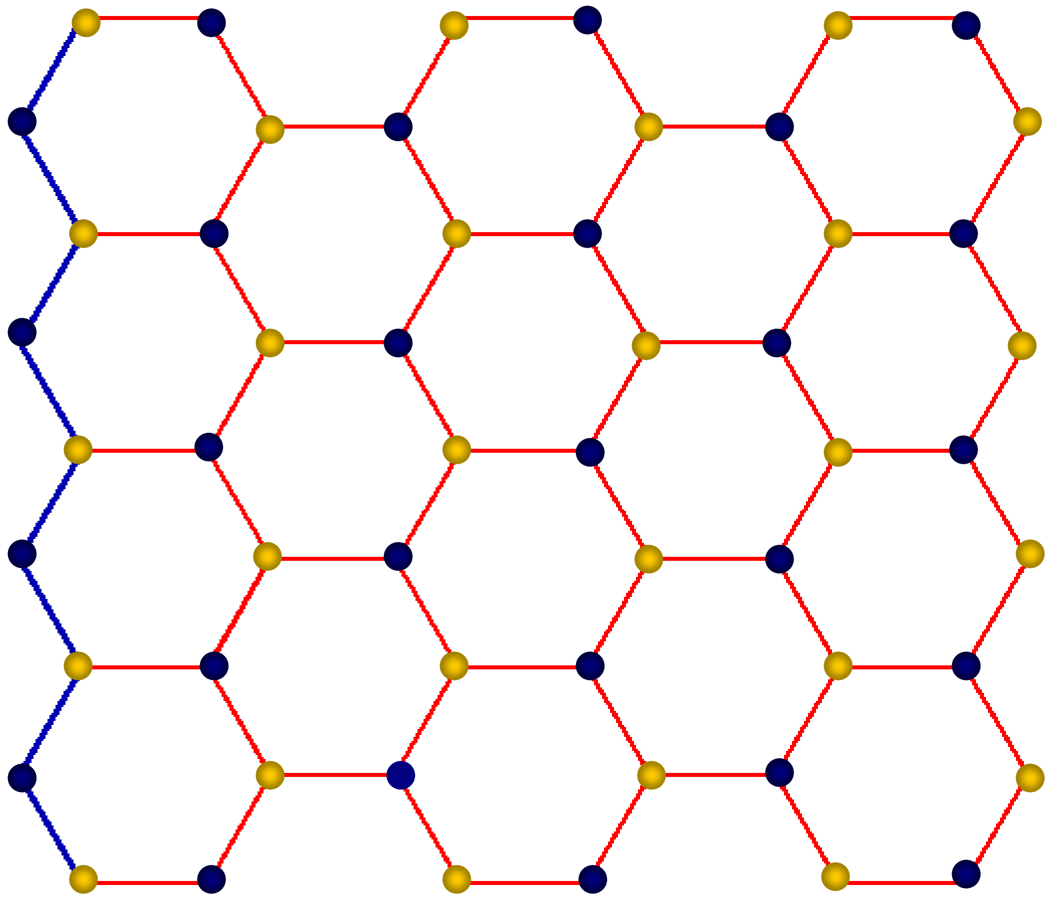
$$H = -t\sum_{<i,j>\sigma} C^{+}_{i,\sigma} C_{j,\sigma} + H.c. + U\sum_{i} n_{i\uparrow} n_{i\downarrow}$$



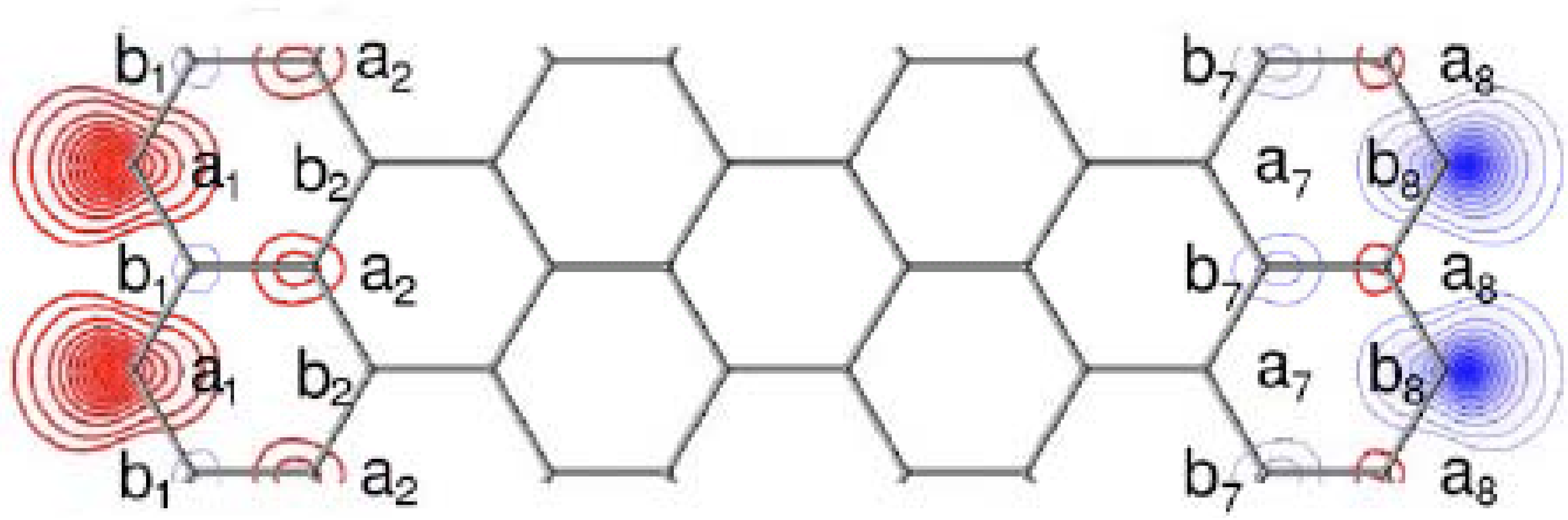


Hikihara, Hu, Lin, and Mou, PRB 68, 035432 (2003)

Paramagnetism as a result of bi-partite nature

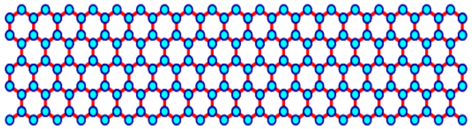


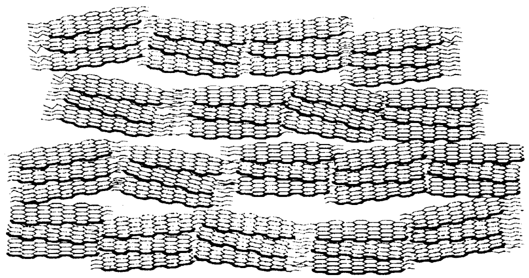
Calculation by Local Spin Density Approximation

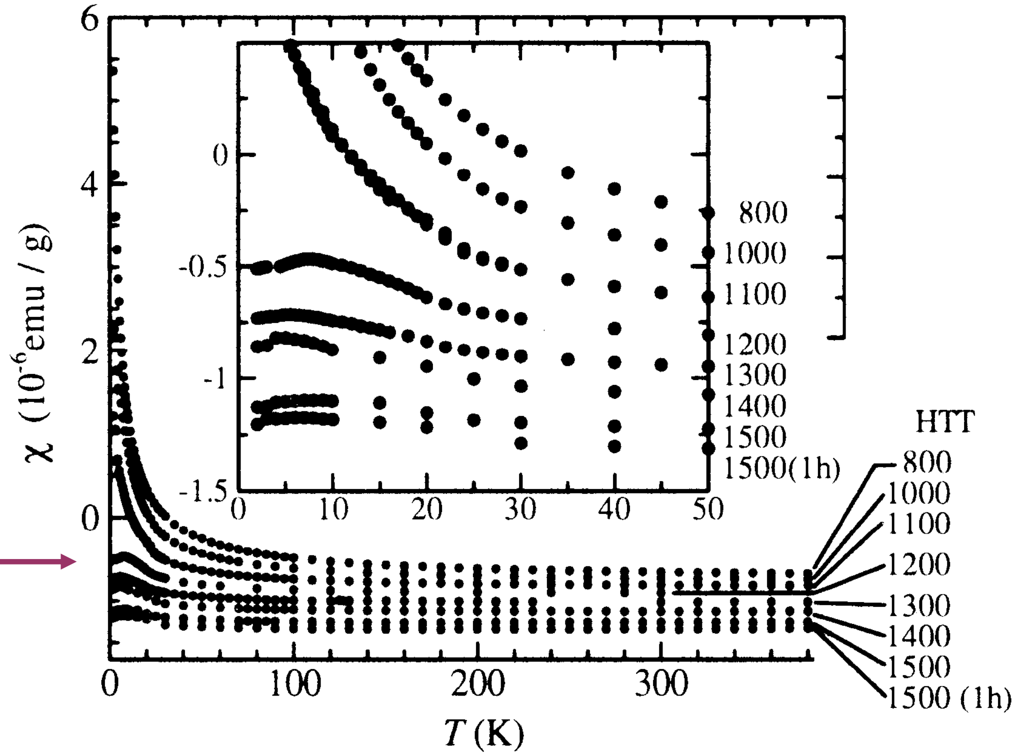


H. Lee et al., PRB 72, 174431 (2005)

Possible origin of Curie-like behavior



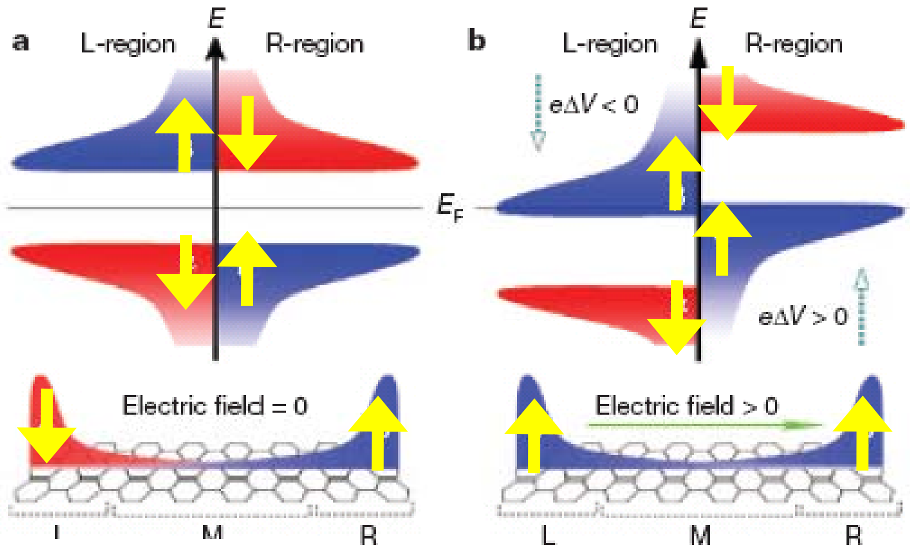


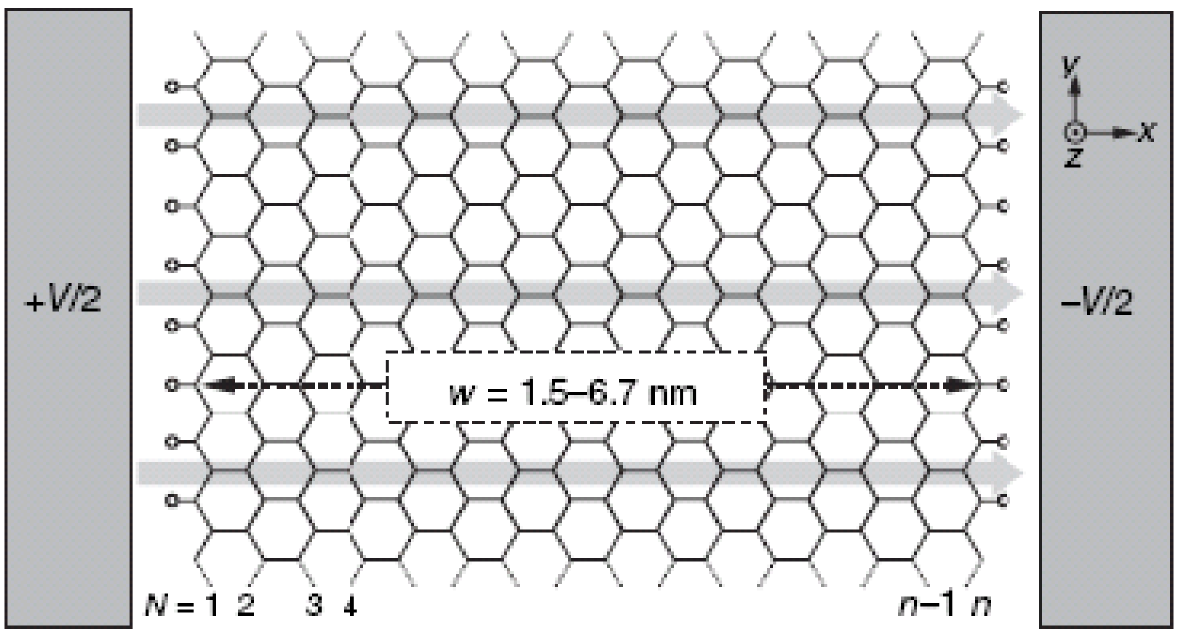


Shibayama et al. Phys. Rev. Lett. 84, 1744(2000)

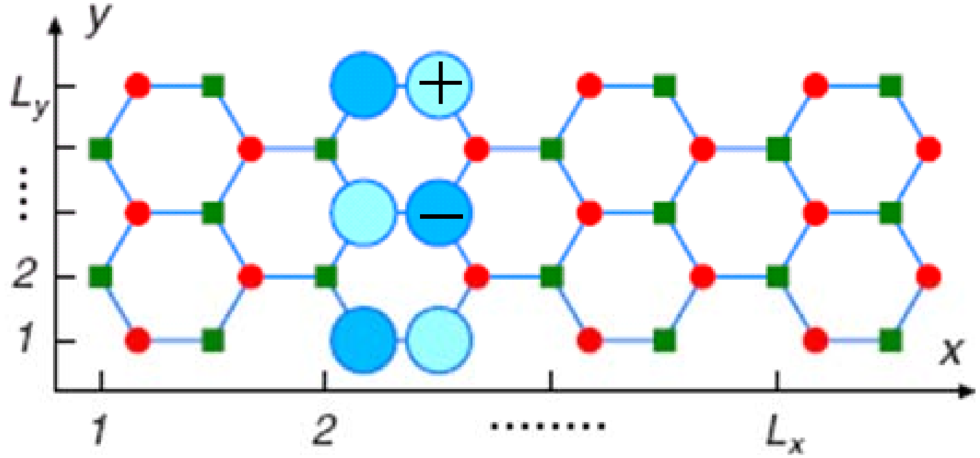
Proposed half-metallic application for spintronics

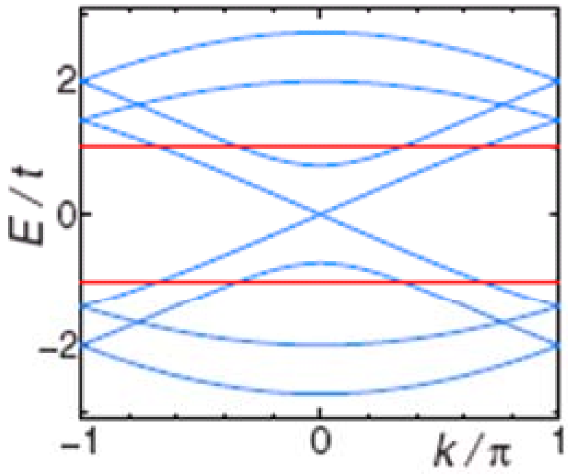
Son et al., Nature 444, 347 (2006)





Nanostructure confinement and ferromagnetism in armchair nanoribbon





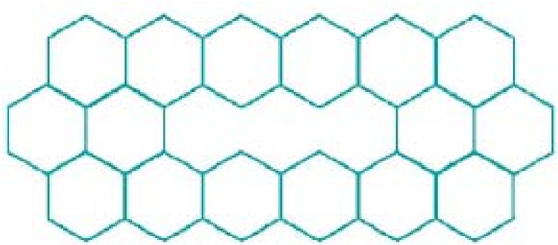
HH Lin et al., Phys. Rev. B 79, 035405 (2009)

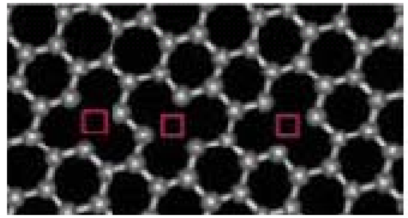
Outline:

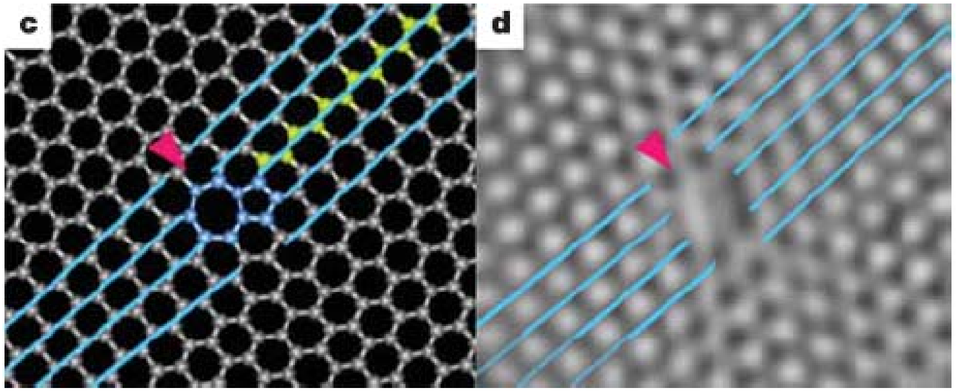
- Background and Introduction: Transport and Magnetic properties
- Novel magnetism associated with edge states and flat-band in nanoribbons
- **Impurity band due to point defects in graphene**

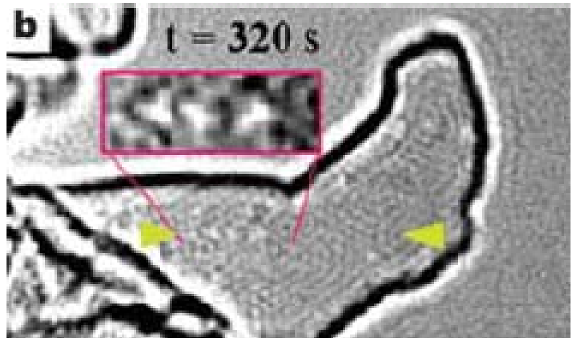
Many possible defects on graphene

Vanacies, Pentagon-heptagon pair, adatoms, cracks…







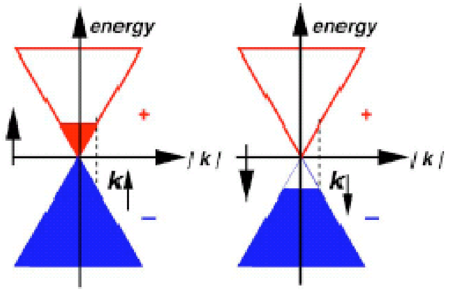


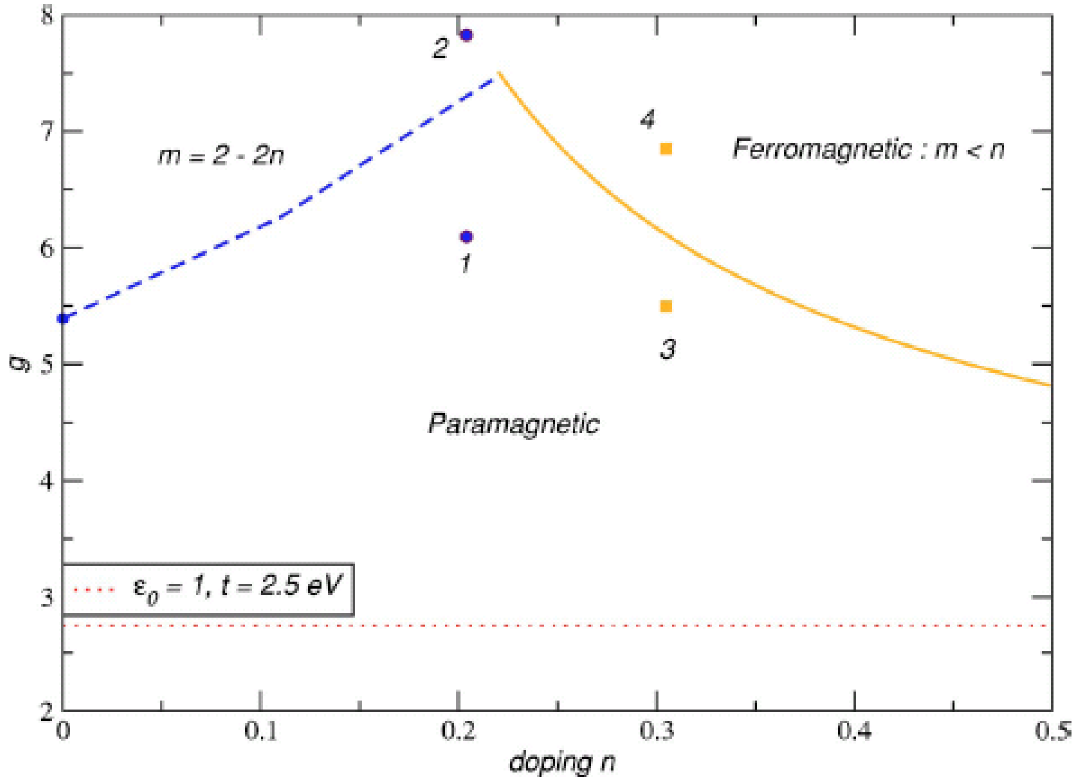
Stone and Wales, Nature 430, 870, 2004

What is the best candidate for ferromagnetism?

Ferromagentism of Dirac band

$$g = \frac{e^2 / \varepsilon_0}{\hbar \upsilon_F}$$

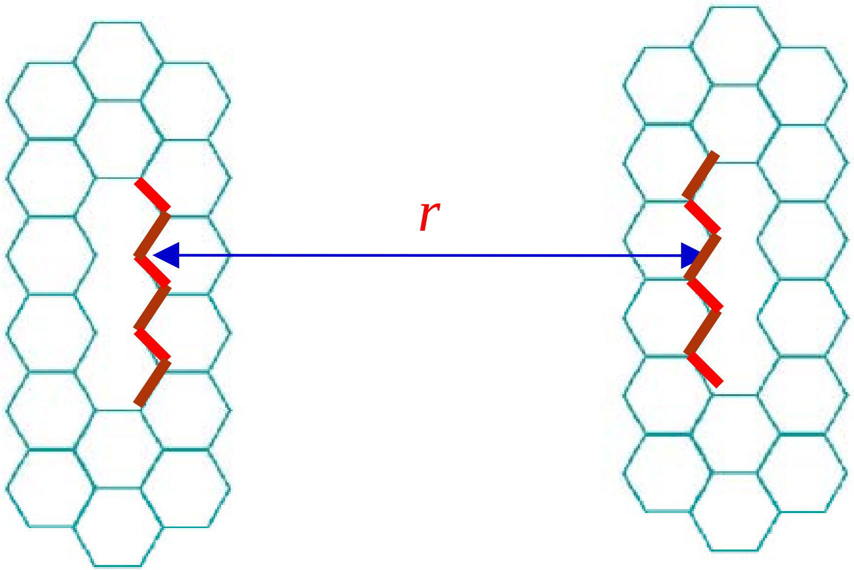




Peres, Guinea, and Neto, Phys. Rev. B 72, 174406 (2005)

Large nonspherical-defects
(cracks…, real localized states in zig-zag edges)
-- carrier mediated ferromagnetism

RKKY on graphene: AB antiferromagnetic ↑ ↓
AA ferromagnetic ↑ ↑

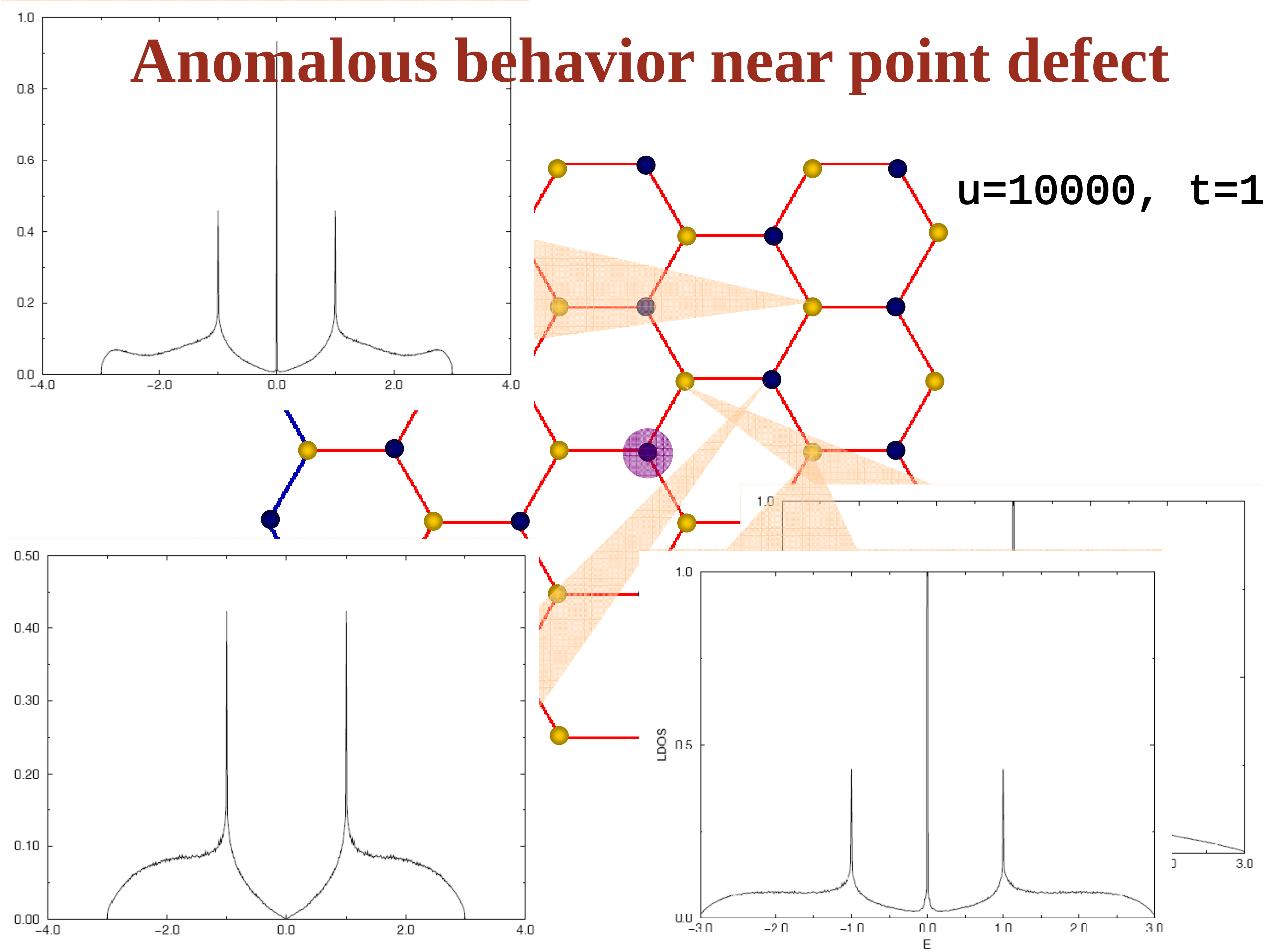


Brey at al., Phys. Rev. Lett. 99, 116802 (2007); Saremi, Phys. Rev. B 76, 184430 (2007)

Plausible candidates:
disk-like or point defects

Anomalous behavior near point defect

u=10000, t=1



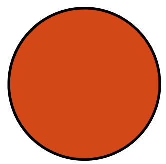
The zero-energy state was known for Dirac Hamiltonian

$$G(r,r') = G_0(r,r') + G_0(r,0)uG_0(0,r') + G_0(r,0)uG_0(0,0)uG_0(0,r') + \cdots$$

$$= G_0(r,r') + G_0(r,0)\frac{1}{1/u - G_0(0,0)}G_0(0,r')$$

Perturbative approach: Balatsky et al., Phys. Rev. B 51, 15547 (1995); Pereira et al. Phys. Rev. Lett 96, 036801 (2006) …….

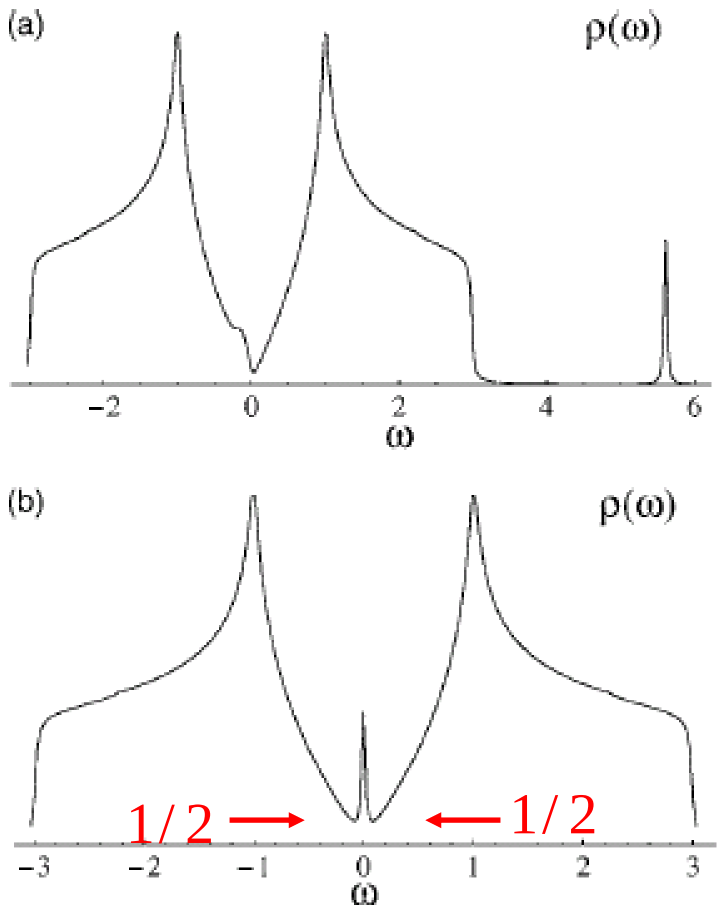
Continuum limit: point defect $\Longrightarrow$ disk like potential



$$E = 0,\ \ \psi \approx \frac{1}{\sqrt{r}}$$

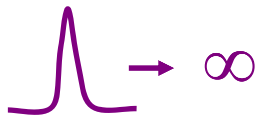
Dong et al., Phys. Rev. A 58, 2160 (1998)

Apparent particle-hole symmetry



$$u = 2.5eV$$

$$u = \infty$$

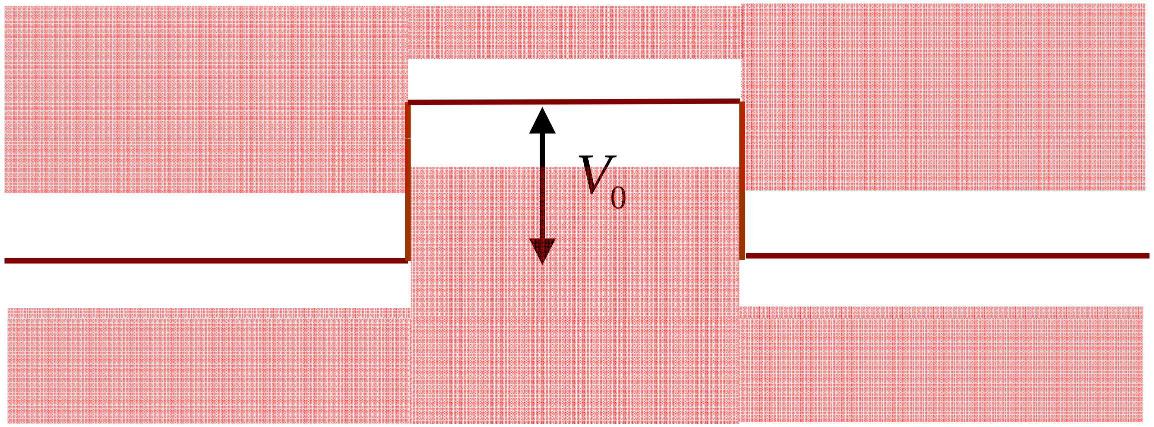


Spatially averaged density of state

Bena and Kivelson, Phys. Rev. B 72, 125432, 2005

Reflection of anomalous behaviors at zero energy

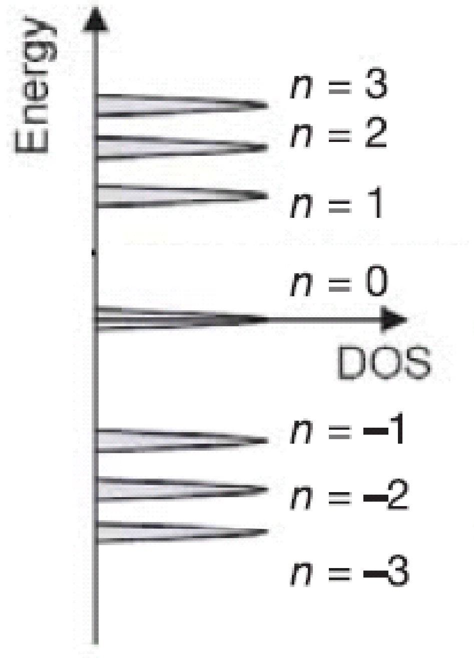
Klein paradox



$T \to 1$ as $V_0 >> m_0 c^2$

Strong diamagnetism

$$\chi = -\frac{a^2 t^2 e^2}{2\pi\hbar^2}\delta(\varepsilon_F)$$



Investigations on magnetism of point-like defects

- Local moments are established near point defects

- Conflicting reports on long-range order (ferromagnetism or antiferromagentism ?)

Semimetal

Herbut, Phys. Rev. Lett 97, 146401 (2006) (RG + Hubbard model)

Antiferromagnetism (Hubbard U + LDA)

Yazyev and Helm, Phys. Rev. B 75, 125408 (2007)

(1st principle); Brey et al. Phys. Rev. Lett. 99, 116802 (2007)

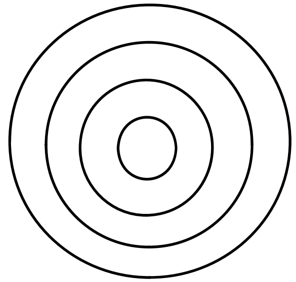
Ferromagnetism

Vozmediano et al. Phys. Rev. B 72, 155121 (2005);
Yazyev, Phys. Rev. Lett. 101, 037203 (2008) (Stacking

along c-axis is responsible for ferromagnetism)

Our approach: constructing exact wavefunction

$$(H_0 + u\delta_{r,0})\psi = E_0\psi$$



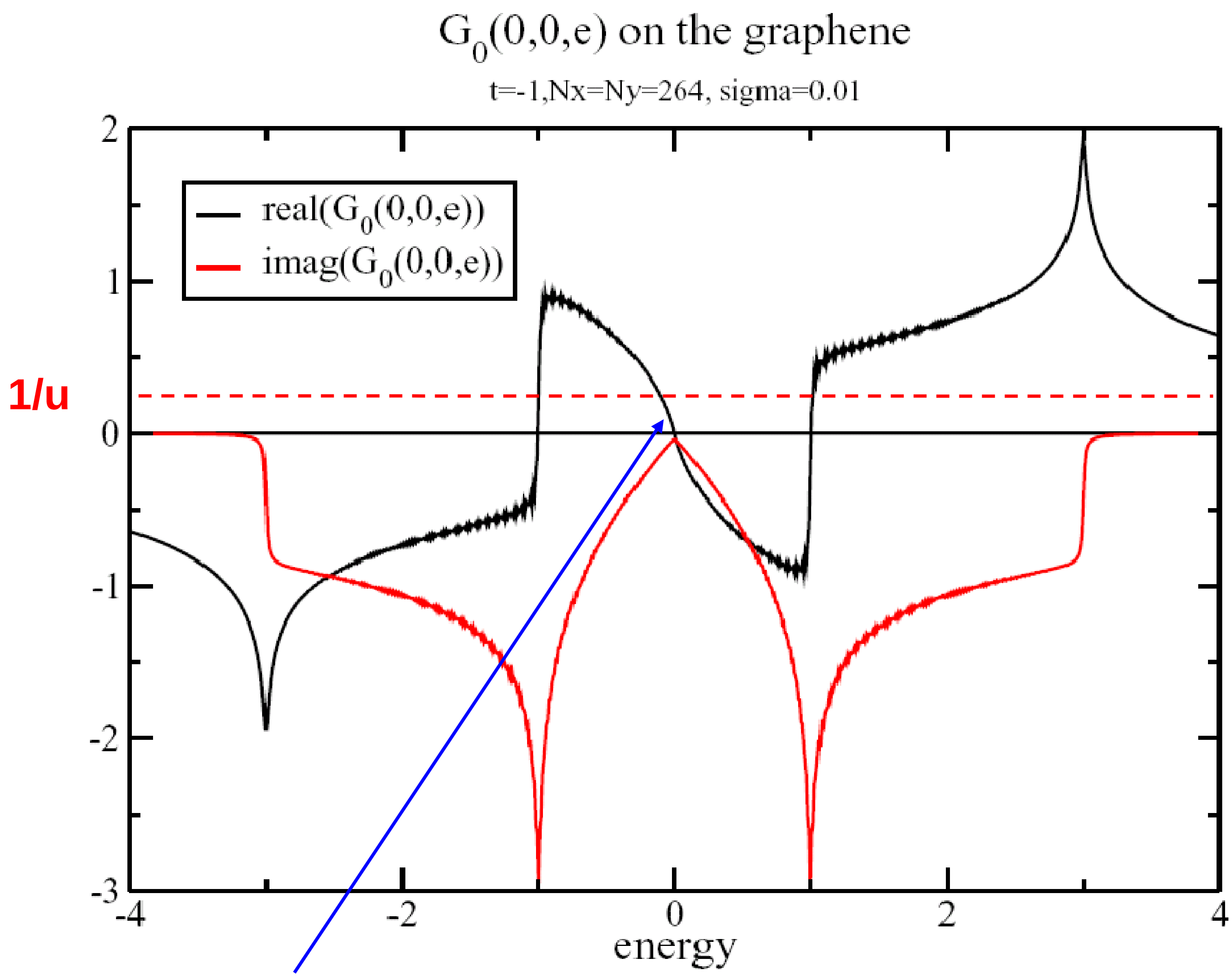
Huygens' principle:

$\psi = A\,\mathrm{Re}\,G_0(r,0,E_0)$ (BC: $\psi \to 0$)

Check: Using $(E - H_0)G_0(r,r',E) = \delta_{r,r'}$

$$(E - H_0)\,\mathrm{Re}\,G_0(r,0,E) = \delta_{r,0} = u\delta_{r,0}\,\mathrm{Re}\,G_0(r,0,E)$$

Self-consistent condition: $\dfrac{1}{u} = \mathrm{Re}\,G_0(0,0,E_0) \equiv g_0(0,0,E_0)$



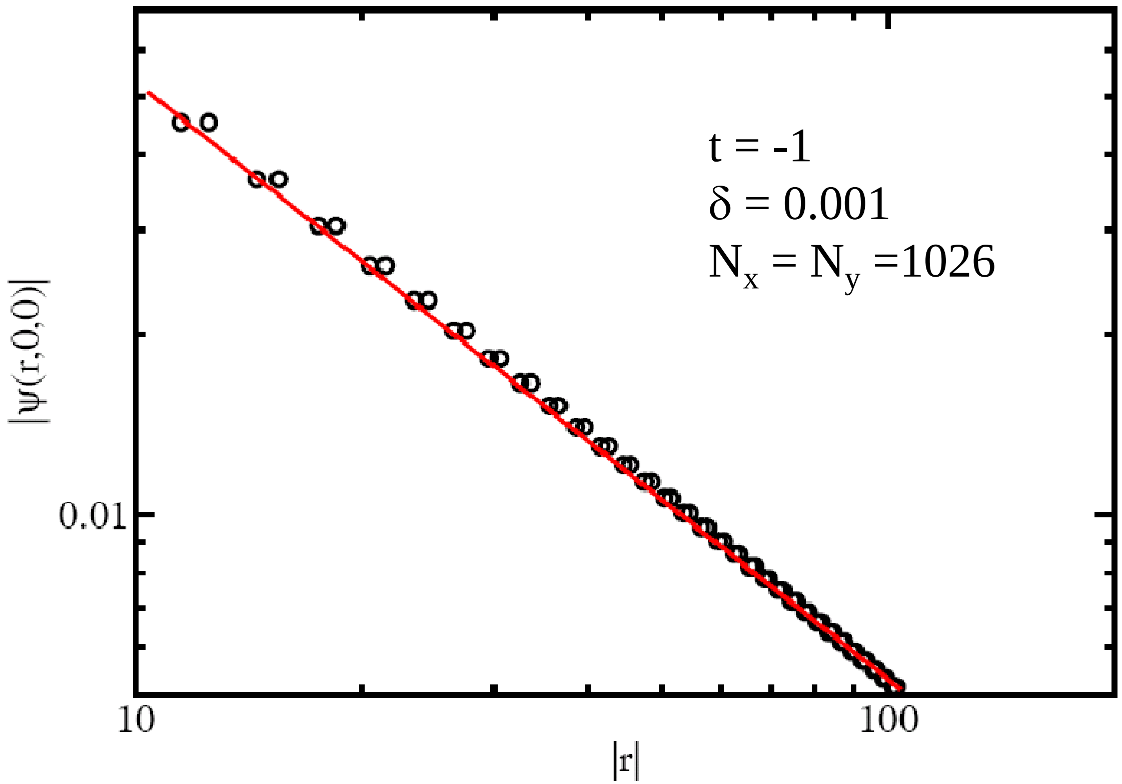
$$\mathrm{Re}\,G(0,0,E) \approx -\gamma E$$

$$\frac{1}{u} = \mathrm{Re}\,G_0(0,0,E_0) = -\gamma E_0 \quad \therefore E_0 \to 0 \text{ as u} \to \infty$$

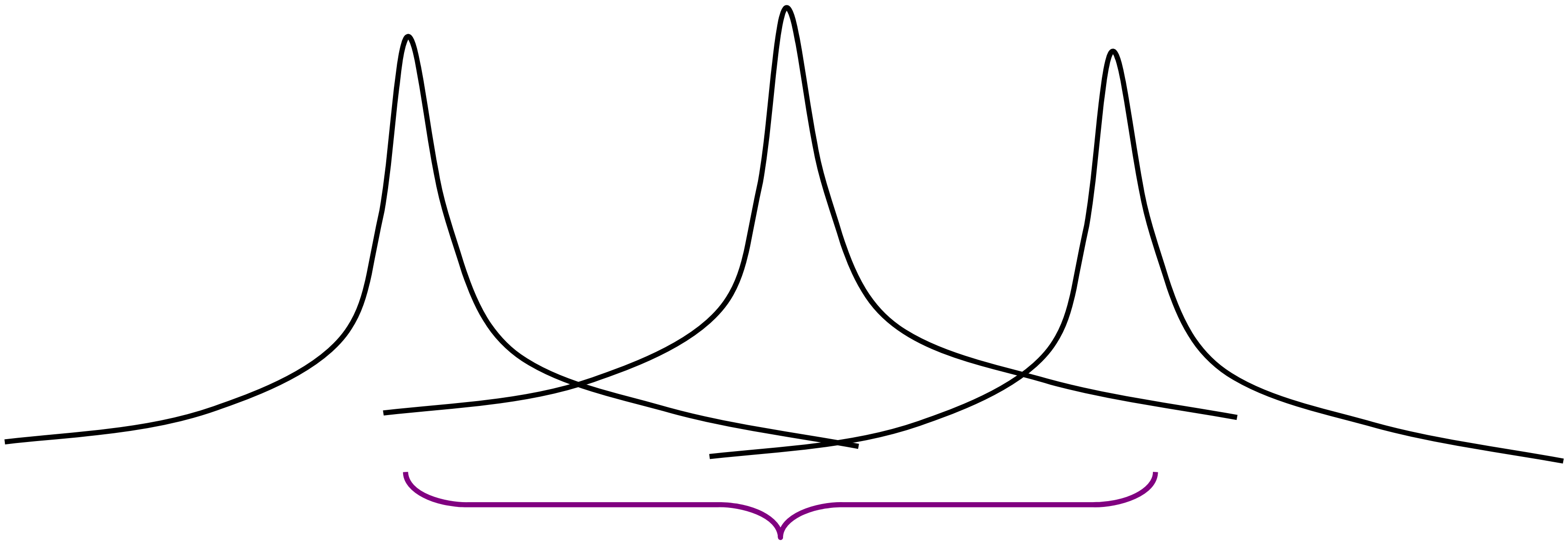
Semi-localization

$$\psi = A\,\mathrm{Re}\,G_0(r,0,E_0)$$

$$G_0(\vec{r},0,0) \propto \mathrm{Re}\sum_D \int \frac{d\theta}{2\pi}\int_\lambda k\,dk\,e^{ikr\cos\theta}\,\frac{e^{i\phi_{\vec{k}}}}{-\upsilon k}\,e^{i\vec{k}_D\cdot\vec{r}} = \frac{1}{r}\frac{2}{\upsilon}\left[1-J_0(k_D r)\right]\sin\frac{4\pi x}{3a}$$



Defects are strongly coupled
-- direct exchange dominates



overlapping is strong!

sensitive to boundary and size!

Multi-defects state of our construction

$$\psi = \sum_i A_i \operatorname{Re} G_0(r, r_i, E)$$

Self-consistent condition:

$$(E - H_0)\psi = \sum_i A_i \delta_{r,r_i} = u \sum_i \delta_{r,r_i} \psi = u \sum_i \delta_{r,r_i} \sum_j A_j \operatorname{Re} G_0(r_i, r_j, E)$$

$$\det \begin{pmatrix} 1/u - g_{11} & -g_{12} & . & . & -g_{1N} \\ -g_{21} & 1/u - g_{22} & . & . & -g_{2N} \\ . & . & . & . & . \\ . & . & . & . & . \\ -g_{N1} & -g_{NN-1} & . & . & 1/u - g_{NN} \end{pmatrix} = 0$$

Random matrix and Impurity band

$u \to \infty$ and $\mathrm{Re}\, G(0,0,E) \approx -\gamma E$ and for $E \approx 0$

$$\det(\alpha E 1\text{-h}) = \det \begin{pmatrix} \gamma E & -g^0_{12} & . & . & -g^0_{1N} \\ -g^0_{21} & \gamma E & . & . & -g^0_{2N} \\ . & . & . & . & . \\ . & . & . & . & . \\ -g^0_{N1} & -g^0_{NN-1} & . & . & \gamma E \end{pmatrix} = 0$$

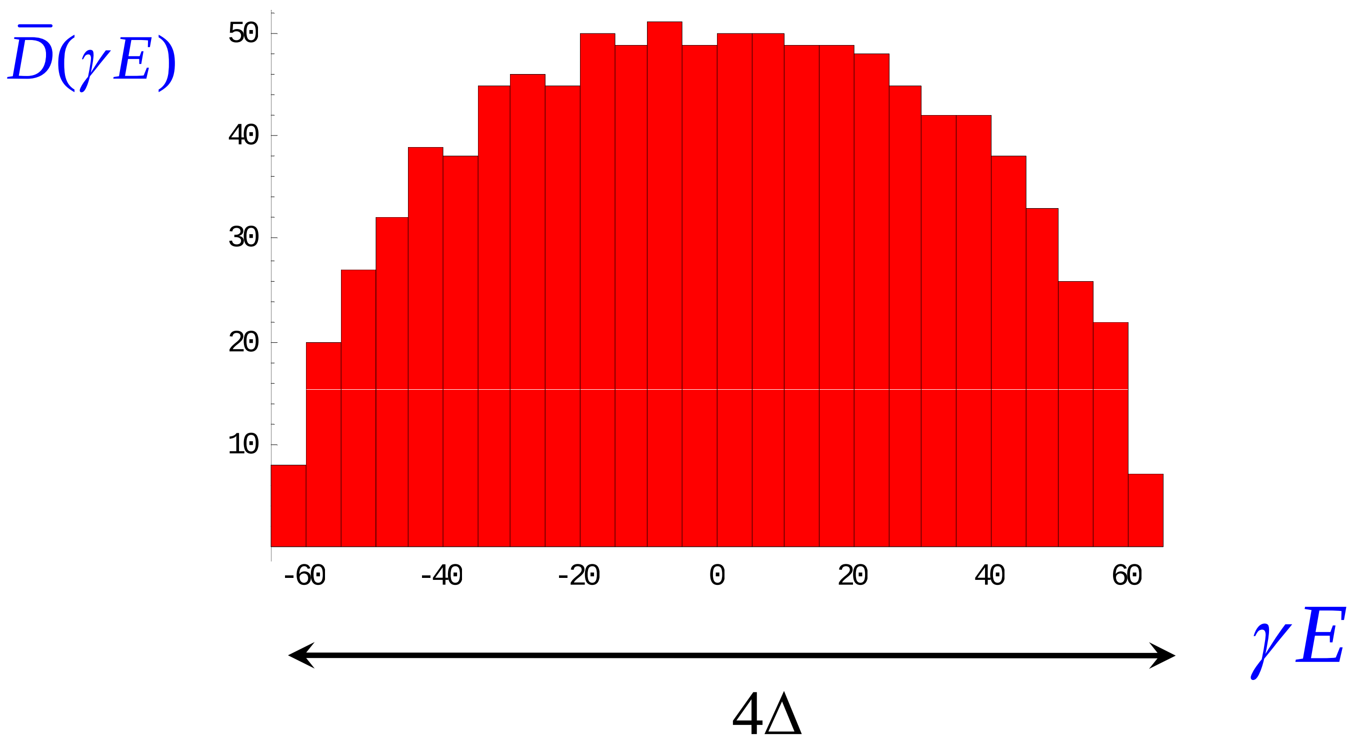
$\because r_i$ positions are random, g^0_{ij} are random

Mapping $\Rightarrow$ eigenvalues of random matrix

Generalized Central Limit Theorem and Wigner Semi-Circle Law

$$h = \begin{pmatrix} 0 & -g^0{}_{12} & . & . & -g^0{}_{1N} \\ -g^0{}_{21} & 0 & . & . & -g^0{}_{2N} \\ . & . & . & . & . \\ . & . & . & . & . \\ -g^0{}_{N1} & -g^0{}_{NN-1} & . & . & 0 \end{pmatrix}$$

$$= g^0{}_{21} \begin{pmatrix} 0 & -1 & . & . & 0 \\ -1 & 0 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & 0 \end{pmatrix} + \ldots + g^0{}_{1N} \begin{pmatrix} 0 & 0 & . & . & -1 \\ 0 & 0 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ -1 & 0 & . & . & 0 \end{pmatrix} \ldots$$

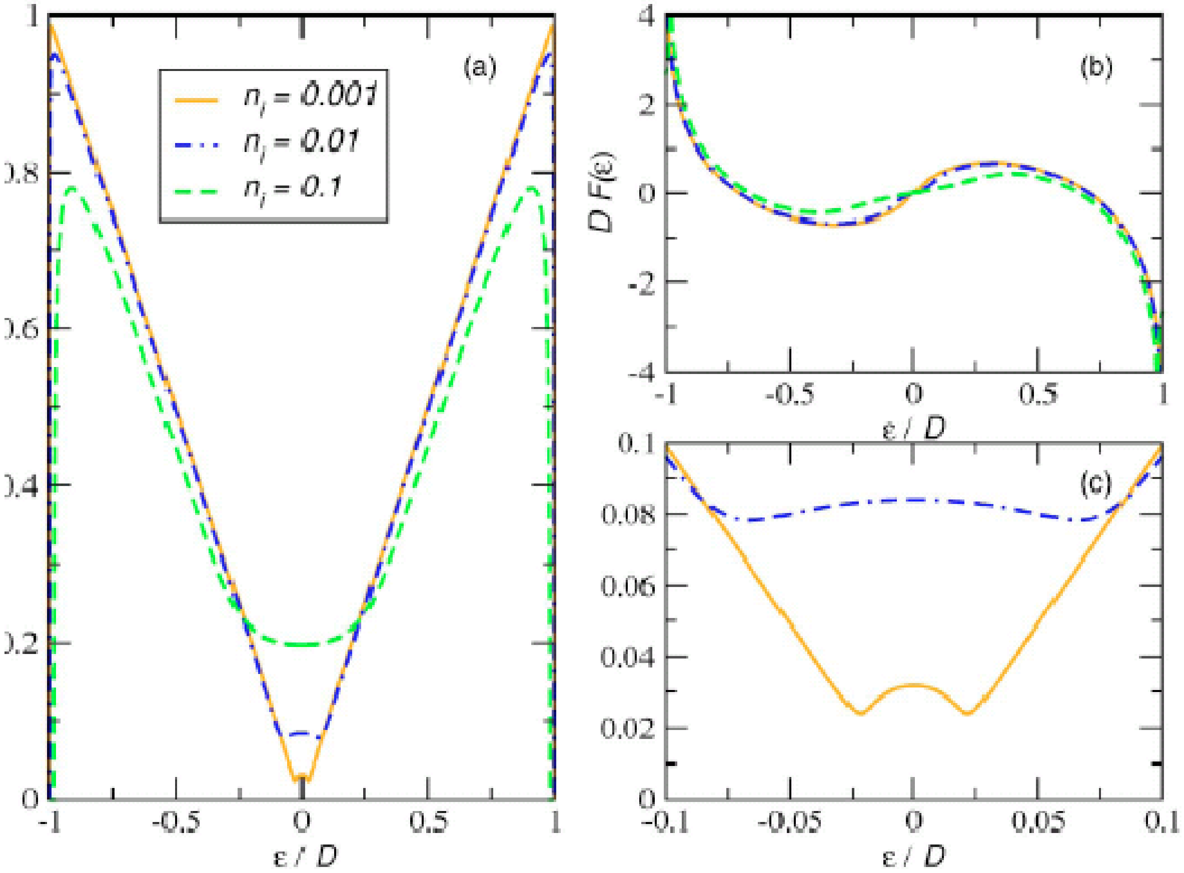


$$\bar{D}(\gamma E) = \frac{N}{2\pi\Delta^2}\sqrt{4\Delta^2 - \gamma^2 E^2}$$

(Density of state per site $= \dfrac{n_I \gamma}{2\pi\Delta^2}\sqrt{4\Delta^2 - \gamma^2 E^2}$)

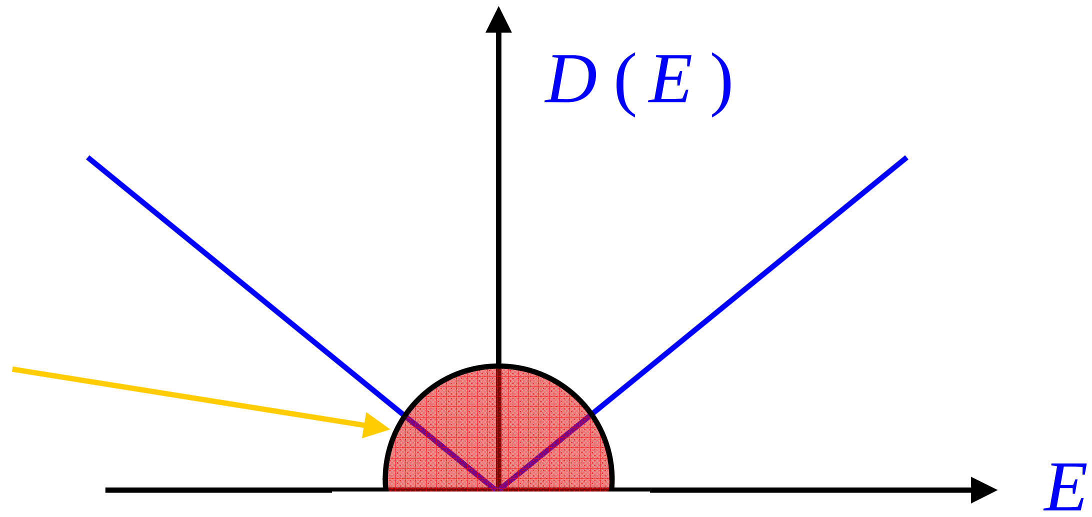
$\Delta^2 = N_I \sigma^2$, N_I = number of defects $\quad \sigma^2 = \langle (g_{ij}^0)^2 \rangle$

Reconstruction of Dirac band in self-consistent Born approximation



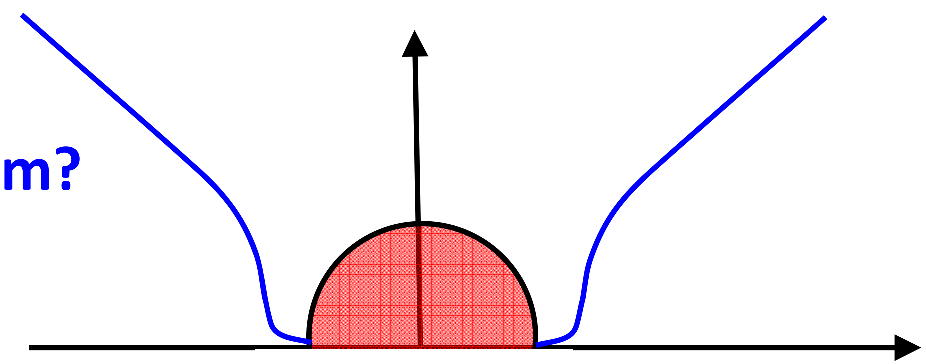
Peres et al., Phys. Rev. B 73, 125411 (2006)

Number of states
are not matched



Dirac band must be reconstructed

Ferromagnetism?



Does defects support ferromagnetism?

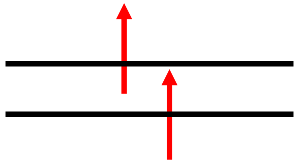
Two defects as an example:

Two defect states

$\varepsilon_a(r_1 - r_2)$

$\varepsilon_b(r_1 - r_2)$

Hund's rule: $\varepsilon_{tr} = \varepsilon_a + \varepsilon_b + C_{ab} - J_{ab}$

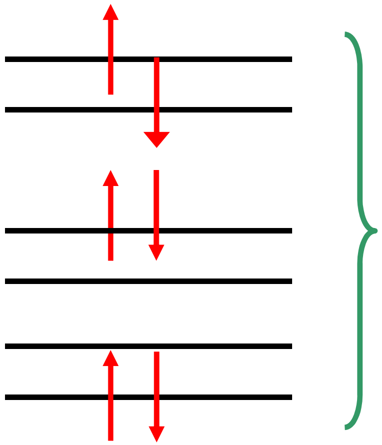


triplet

$$\varepsilon_s = \varepsilon_a + \varepsilon_b + C_{ab} + J_{ab}$$

$$2\varepsilon_a + C_{aa}$$

$$2\varepsilon_b + C_{bb}$$

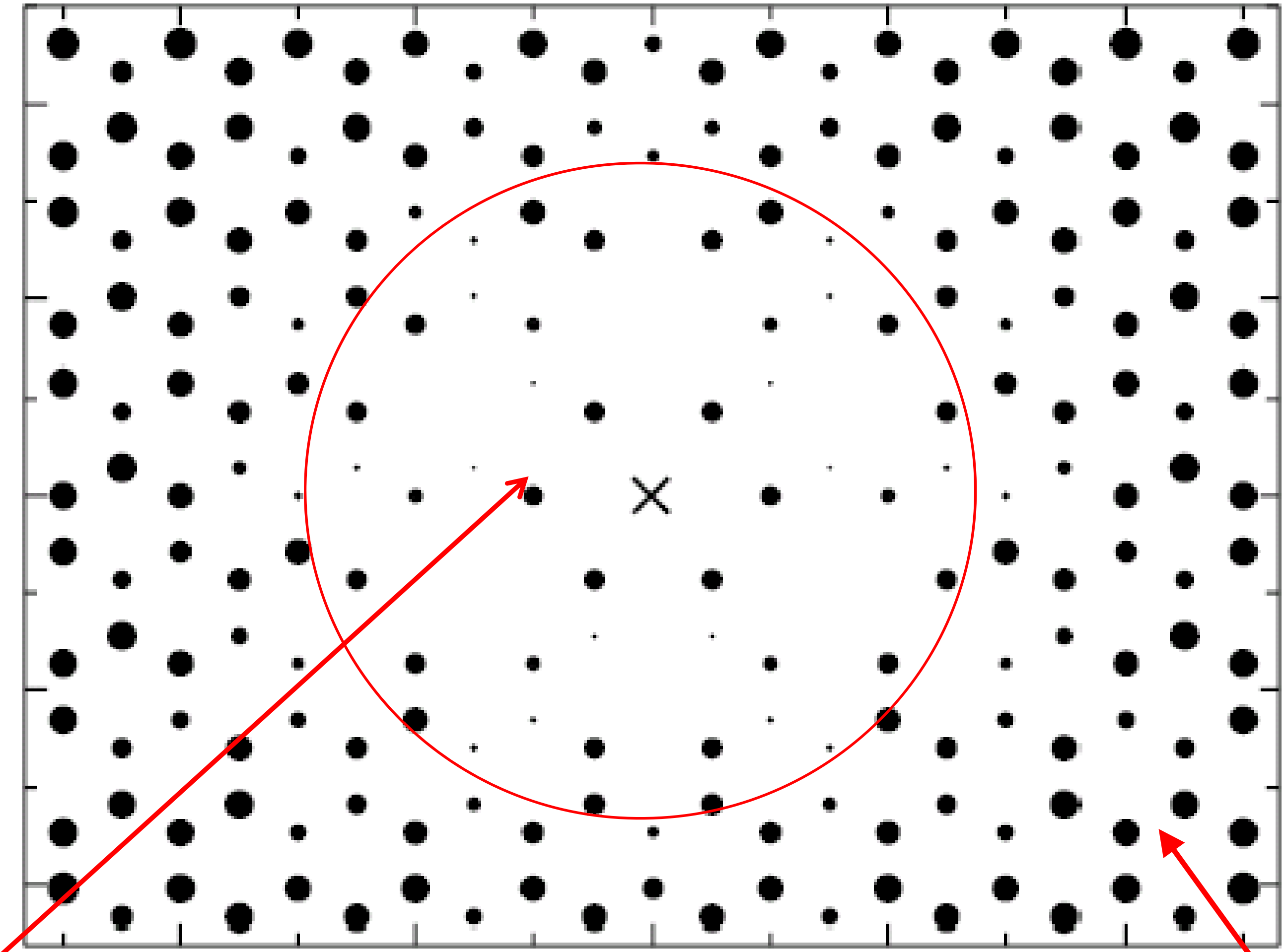


singlet

$$C_{ab} = e^2 \int dr_1 \int dr_2 \frac{|\Psi_a(r_1)|^2 |\Psi_b(r_2)|^2}{|r_1 - r_2|}, \quad J_{ab} = e^2 \int dr_1 \int dr_2 \frac{\Psi_a^*(r_1)\Psi_b(r_1)\Psi_b^*(r_2)\Psi_a(r_2)}{|r_1 - r_2|}$$

(expressible in q space)

Magnetism of two-defects

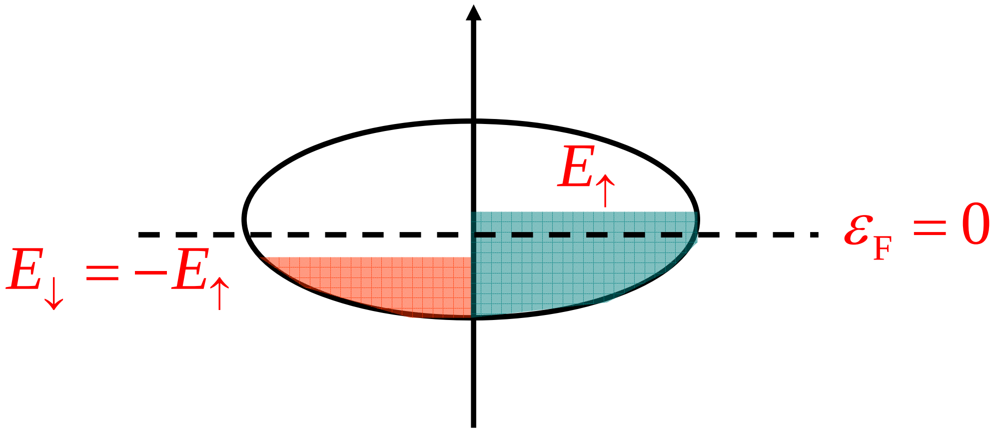


Singlet for AB
Triplet for AA
Agree with LDA

bandwidth ⟶ 0, triplet is favored

What about finite density of defects ?

Kinetic Energy (loss)



Exchange Energy (gain)

$$H = \frac{e^2}{8\pi\varepsilon_0} \sum_{ij\sigma_i} C^+_{i\sigma} C_{i\sigma} \frac{1}{|r_i - r_j|} C^+_{j\sigma'} C_{j\sigma'}$$

$$= \frac{e^2}{8\pi\varepsilon_0} \left[\sum_{ij} \frac{n_i n_j}{|r_i - r_j|} - \sum_{ij\sigma_i} C^+_{i\sigma} C_{j\sigma'} \frac{1}{|r_i - r_j|} C^+_{j\sigma'} C_{i\sigma} \right]$$

Flat-band ferromagnetism

Mielke-Tasaki mechanism:

Non-vanishing overlaps between Wannier orbitals

Prog. Theor. Phys. 99, 498, (1998) (for Hubbard model)

Essentially, exchange energy wins.

Example:

Arita et al, PRL88, 127202 (2002)

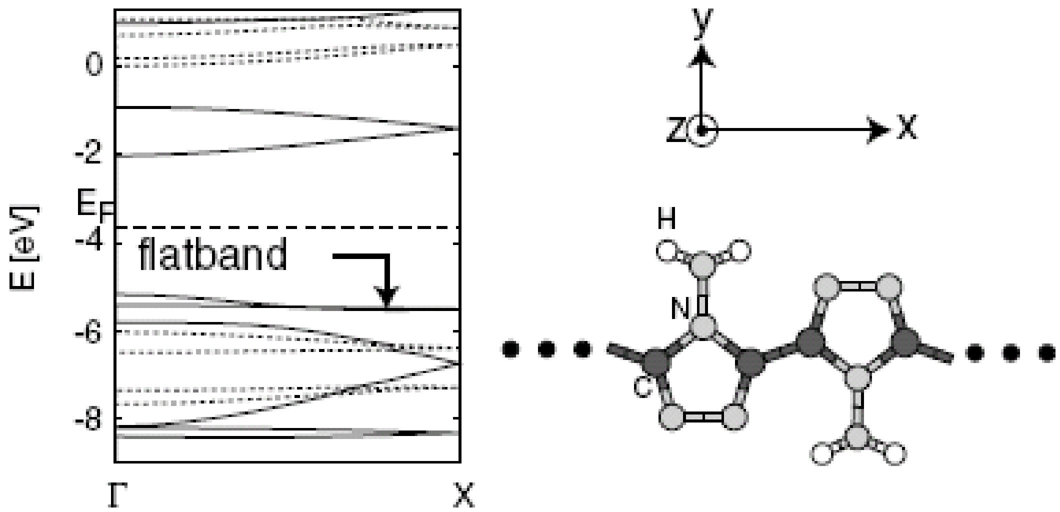
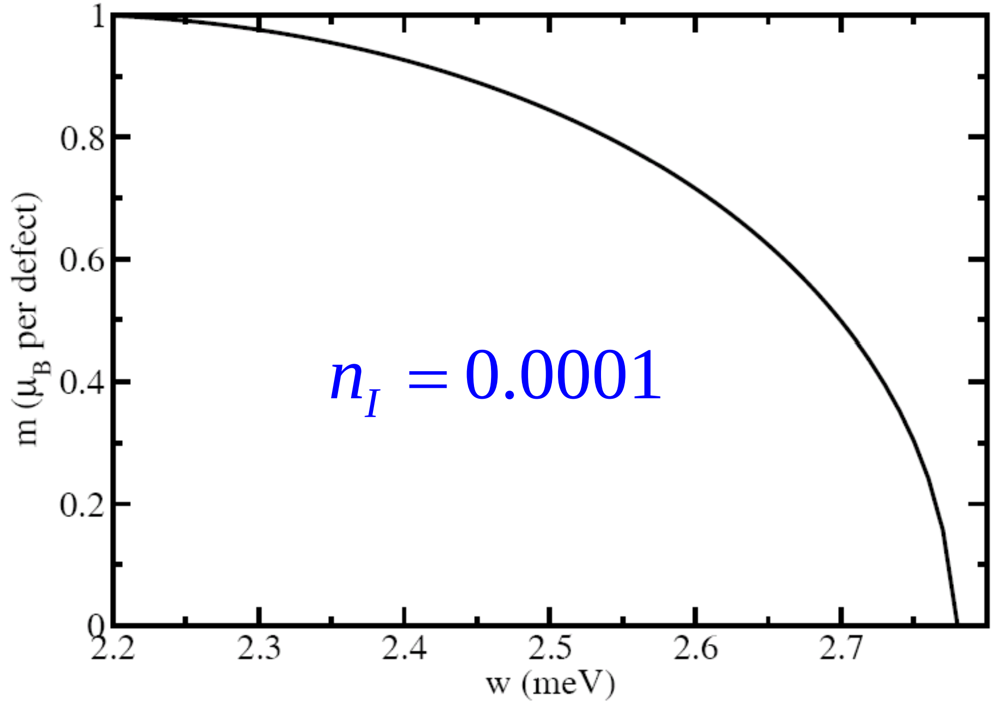
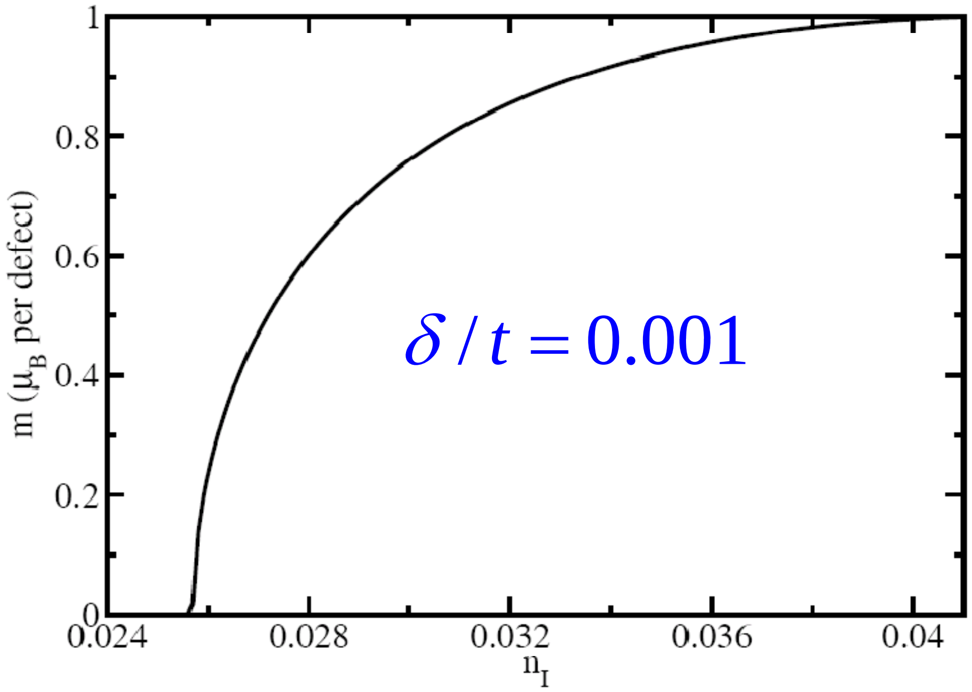


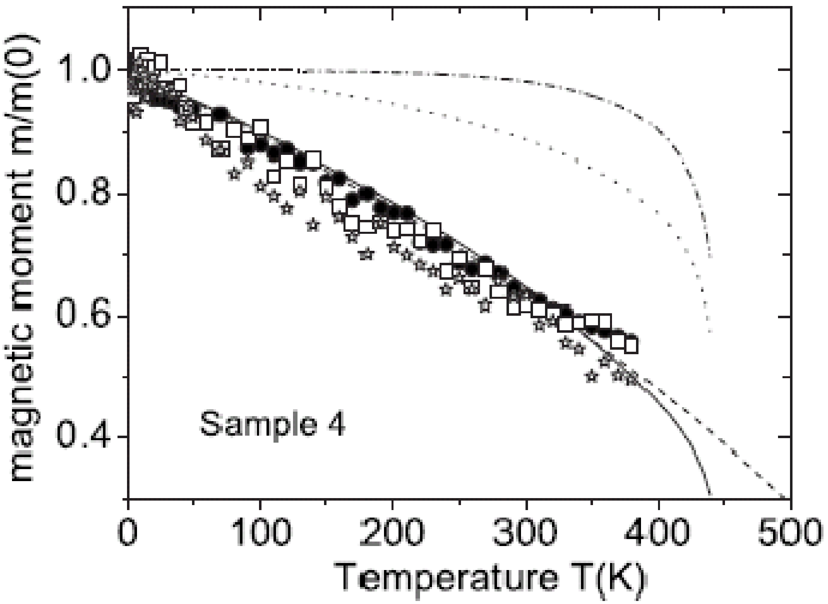
FIG. 2. The band structure (left panel) and the optimized atomic configuration (right) of the (undoped) polyaminotriazole obtained by the GGA-DFT. The solid (dotted) lines represent bands having π (σ) character.

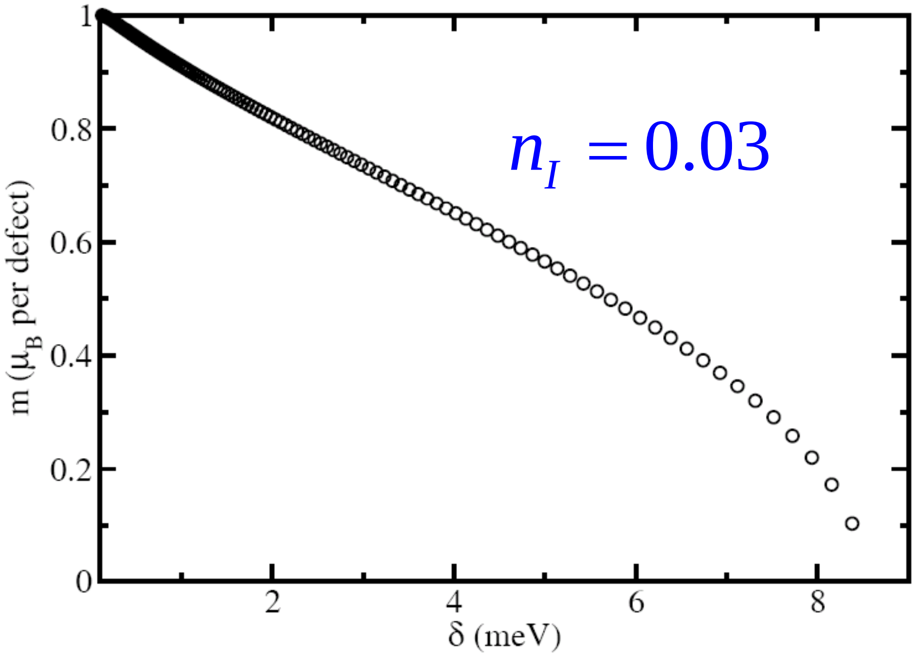




$\Leftarrow \mathrm{E}_{kinetic} \approx n_I^{3/2},\ E_{exchange} \approx n_I^2$

Linear-like temperature dependence of M





Barzola-Quiquia et al
Phys. Rev. B 76, 161403, 2007

$$\delta \propto T$$

Does the impurity band conduct?

Conductivity

$$\vec{J} = \frac{iet}{\hbar} \sum_{i\vec{\delta}} \vec{\delta} C^{+}_{i+\delta,\sigma} C_{i,\sigma}$$

$$\frac{\sigma}{4e^2/h} = \frac{w^2}{4\pi\delta^2}$$

Intrinsic λ = 30nm

[Martin et al., Nature Phys. 4, 144 (2008)]

$$n_I \approx 10^{-5} - 10^{-6}$$

$$\delta / t \approx 10^{-3}$$

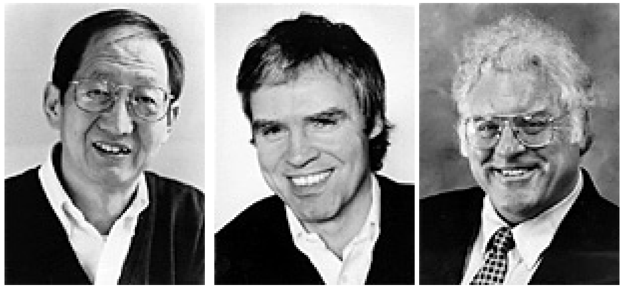
$$\frac{\sigma}{4e^2/h} = O(1)$$

Summary

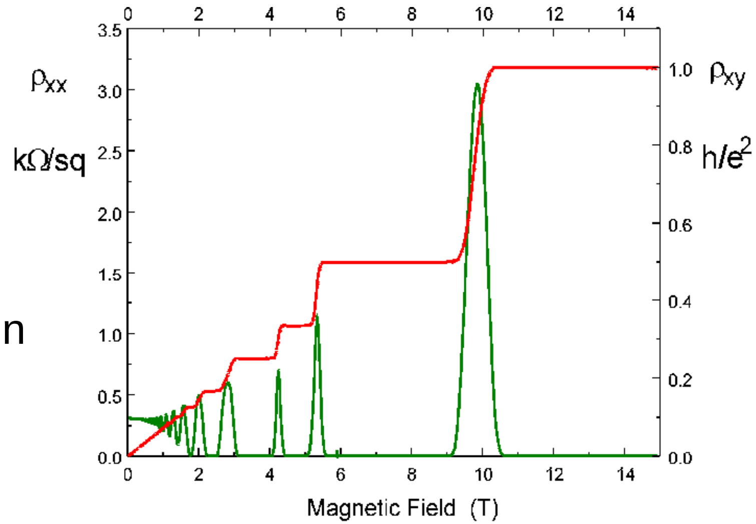
- Graphene is a promising material both for spintronics and electronics. It is relativistic!
- Defects and edges in graphene have anomalous contributions to transport and magnetism
- Edge states in zig-zag edges of nanoribbons or cracks are origin of paramagnetism in graphene ribbons
- Point defects on graphene are correlated and form an impurity band with a universal density of state characterized by Wigner's semi-circle law.

Integer and Fractional Quantum Hall effect

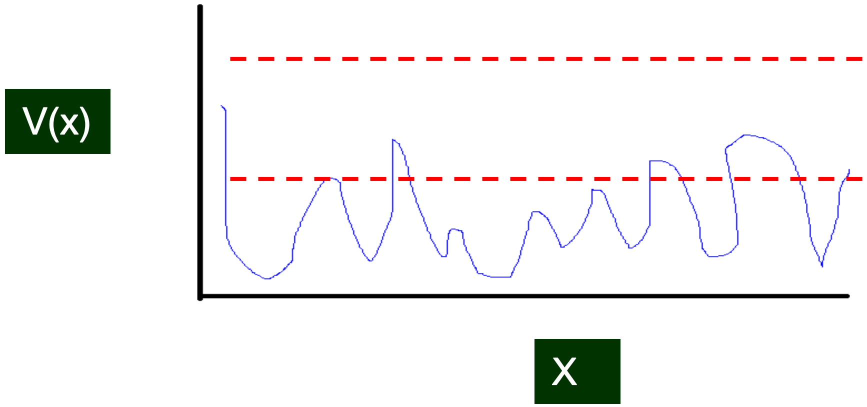
von Klitzing



D.C. Tsui, S.L. Stormer, R.B. Laughlin

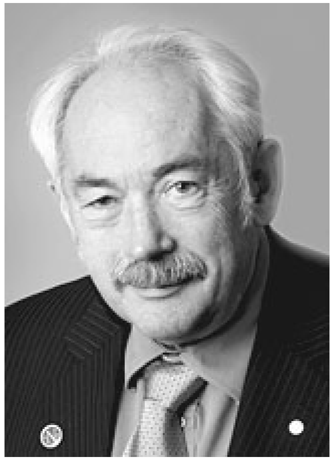


Anderson localization, metal-insulator transition, …



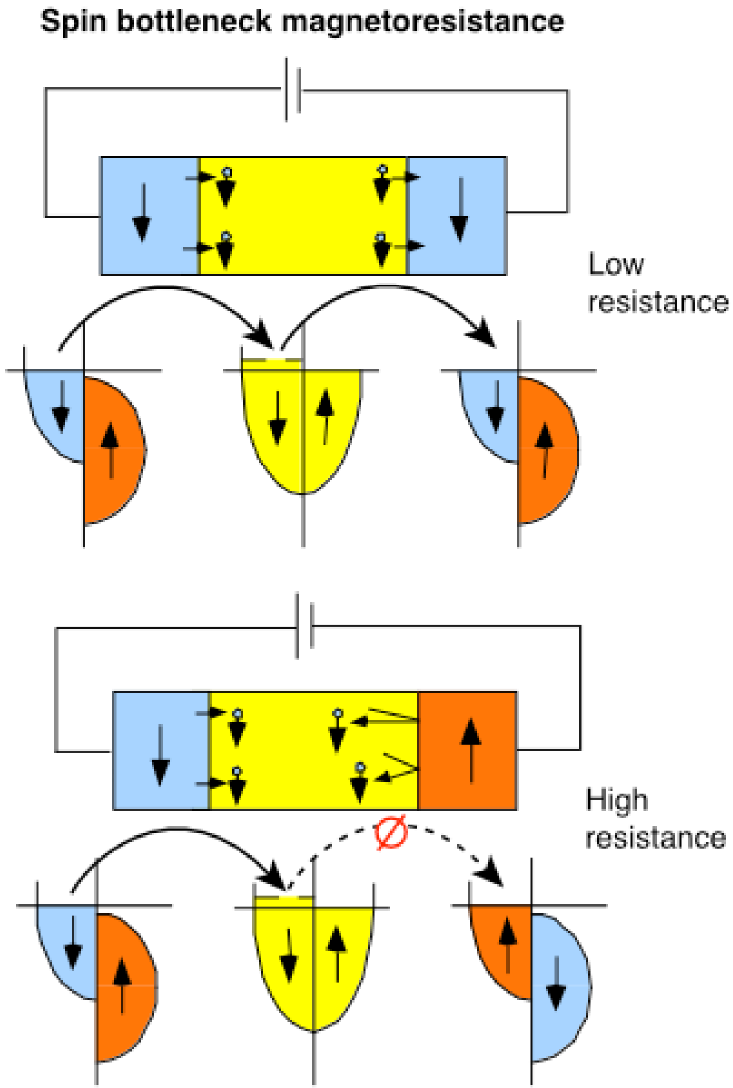
Giant Magnetoresistance

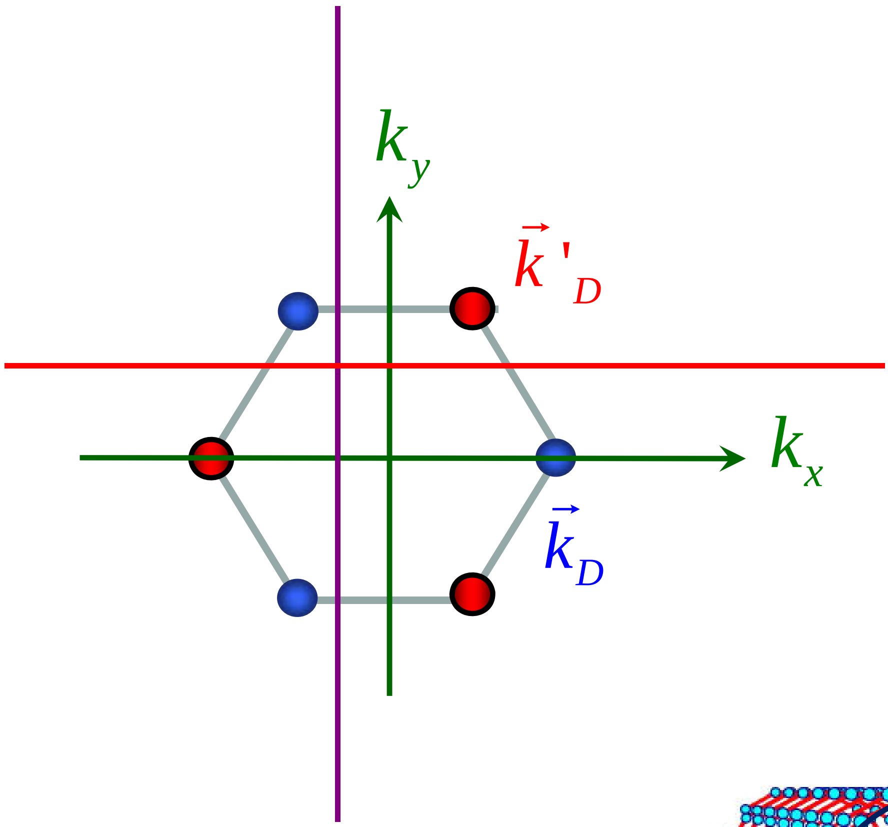
A. Fret



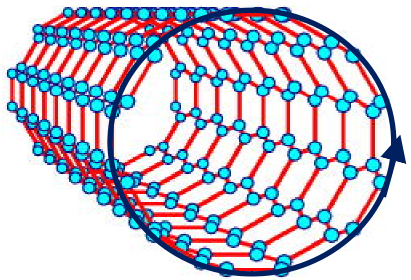
P.Grünberg

2007

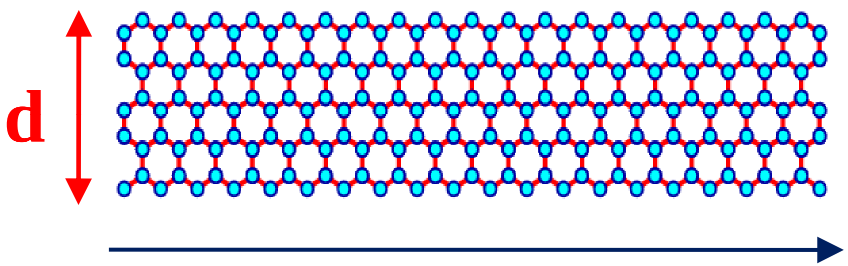




Carbon nanotube

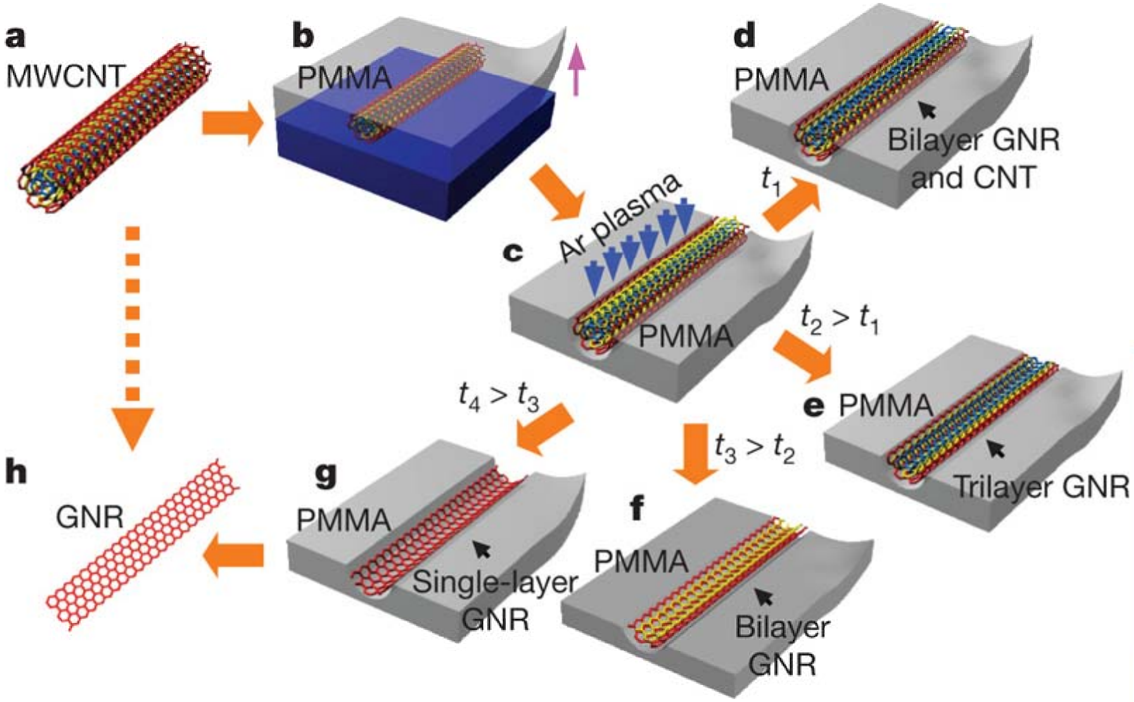


Carbon nanoribbon

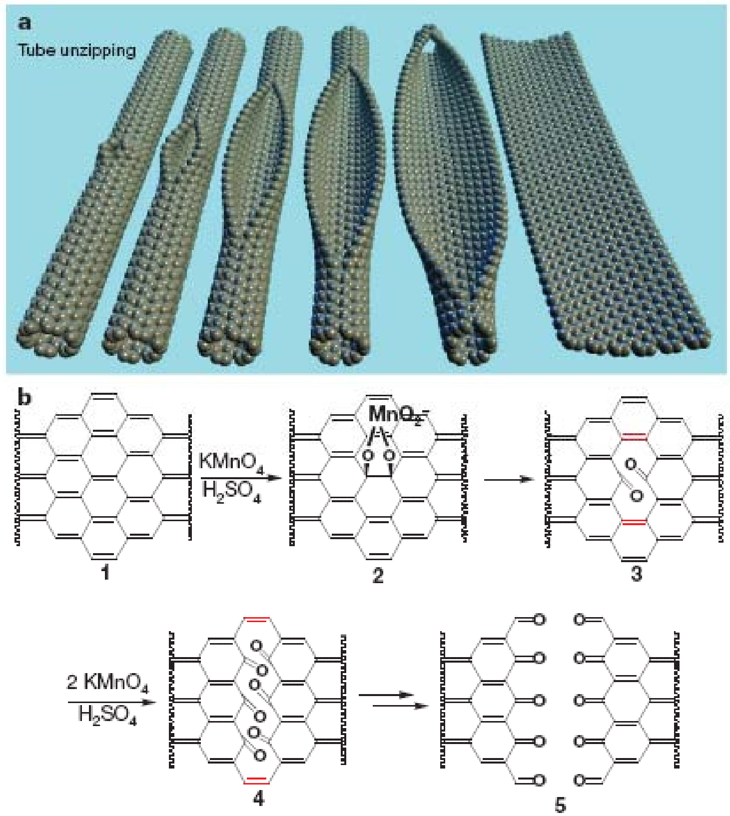


$d \sim 10nm$, gap ~ room tem.

Making Carbon Nanoribbon from Carbon Nanotube

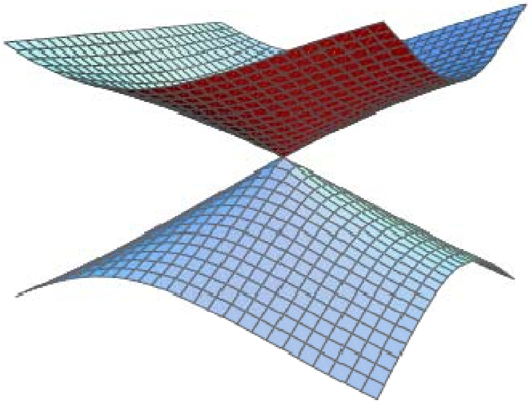


10-20nm, LY Jiao *et al. Nature* 458, 877 (2009)

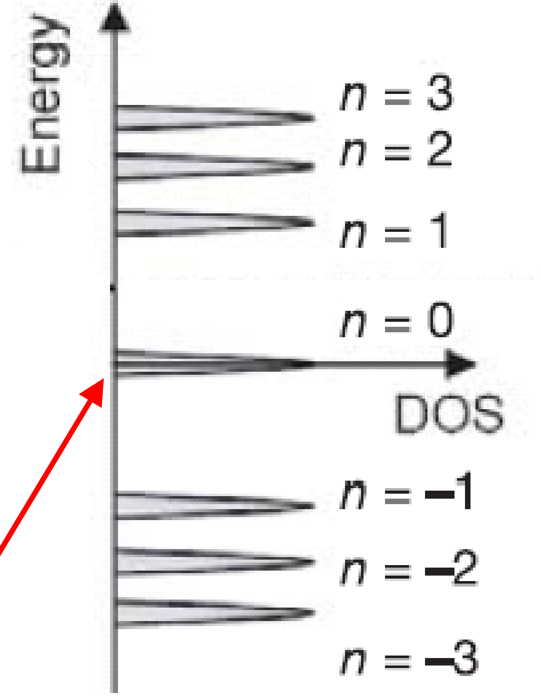


D. Y. Kosynkin *et al. Nature* 458, 872(2009)

Berry phase, diamagnetism, and Quantum Hall effect



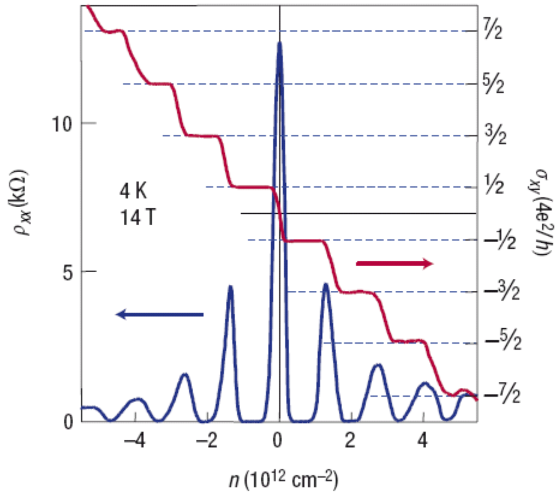
$B \neq 0$



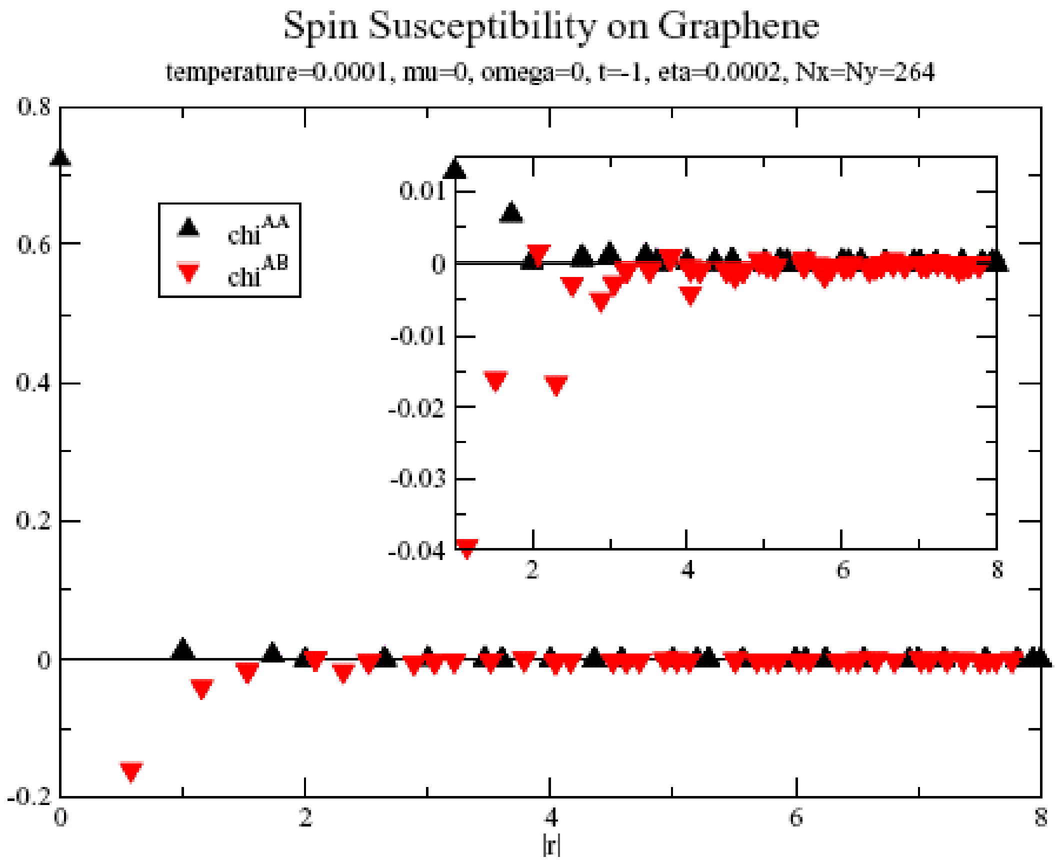
Berry phase:

$n + 1/2 \Rightarrow n$

$$\chi = -\frac{a^2 t^2 e^2}{2\pi\hbar^2}\delta(\varepsilon_F),$$ Quantum Hall effect



J.W. McClure, Phys. Rev. 104, 666 (1956); Geim and Novoselov, Nat. Mat. 6, 183 (2006)



$$J_{ij} = -j^2 \int d\tau \left\langle S_i^-(\tau) S_j^+(0) \right\rangle$$
$$= -j^2 \int d\tau \left\langle \overline{c}_{i\downarrow}(\tau) c_{j\downarrow}(0) \right\rangle \left\langle c_{i\uparrow}(\tau) \overline{c}_{j\uparrow}(0) \right\rangle$$

$$c_{i\sigma}(\tau) \to (-1)^i \overline{c}_{i\sigma}(\tau)$$
$$G(\tau) \text{ real } \Rightarrow J_{ij} \propto (-1)^{i+j+1}$$

Brey at al., Phys. Rev. Lett. 99, 116802 (2007); Saremi, Phys. Rev. B 76, 184430 (2007)