Isospin Symmetry Breaking in the Chiral Quark Model

Huiying Song, Xinyu Zhang, and Bo-Qiang Ma

Peking University, 2011.4.1

Outline

Chiral quark model

- Established by Weinberg, and developed by Manohar and Georgi, has been widely recognized as an effective theory of QCD at the low-energy scale.
- Applied to explain the violation of the Gottfried sum rule from the aspect of flavor asymmetry in the nucleon sea. E. J. Eichten, I. Hinchliffe, and C. Quigg, Phys. Rev. D **45**, 2269 (1992).
- Applied to explain the proton spin crisis. T. P. Cheng and L. F. Li, Phys. Rev. Lett. **74**, 2872 (1995).
- Applied to explain the NuTeV anomaly from the strange-antistrange asymmetry. Y. Ding, R.-G. Xu, and B.-Q. Ma, Phys. Lett. B **607**, 101 (2005); Phys. Rev. D **71**, 094014 (2005).

The Lagrangian

$$L = \bar{\psi}\left(iD_\mu + V_\mu\right)\gamma^\mu\psi + ig_{\mathrm{A}}\bar{\psi}A_\mu\gamma^\mu\gamma_5\psi + \cdots,$$

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

- ψ is quark field.
- $D_\mu = \partial_\mu + igG_\mu$ is the gauge-covariant derivative of QCD. G_μ stands for the gluon field, g stands for the strong coupling constant.
- g_{A} stands for the axial-vector coupling constant.

The Lagrangian

- V_μ and A_μ are the vector and the axial-vector currents.

$$\begin{pmatrix} V_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2}\left(\xi^+\partial_\mu\xi \pm \xi\partial_\mu\xi^+\right). \tag{1}$$

- Where $\xi = \exp(i\Pi/f)$

$$\Pi \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & \frac{-2\eta}{\sqrt{6}} \end{pmatrix}. \tag{2}$$

The interaction Lagrangian

- Expanding V_μ and A_μ in powers of Π/f,

$$V_\mu = 0 + O(\Pi/f)^2,$$

$$A_\mu = i\partial_\mu \Pi/f + O(\Pi/f)^2.$$

- $f \simeq 93$ MeV is the pseudoscalar decay constant.
- The effective interaction Lagrangian between GS bosons and quarks in the leading order:

$$L_{\Pi q} = -\frac{g_A}{f}\bar{\psi}\partial_\mu \Pi \gamma^\mu \gamma_5 \psi.$$

The light-front Fock decomposition

$$|U\rangle = \sqrt{Z_u}|u_0\rangle + a_{\pi^+}|d\pi^+\rangle + \frac{a_{u\pi^0}}{\sqrt{2}}|u\pi^0\rangle + a_{K^+}|sK^+\rangle, \quad (3)$$

$$|D\rangle = \sqrt{Z_d}|d_0\rangle + a_{\pi^-}|u\pi^-\rangle + \frac{a_{d\pi^0}}{\sqrt{2}}|d\pi^0\rangle + a_{K^0}|sK^0\rangle. \quad (4)$$

- Since the η is relatively heavy, we neglect the minor contribution from its suppressed fluctuation.
- Z_u and Z_d are the renormalization constants for the bare constituent u quark $|u_0\rangle$ and d quark $|d_0\rangle$, respectively, and $|a_\alpha|^2$ are the probabilities to find GS bosons in the dressed constituent-quark states $|U\rangle$ and $|D\rangle$.

Basic process

Figure: (a) A constituent quark q_i fluctuates into a Goldstone boson α plus a recoil constituent quark q_j. (b) A Goldstone boson emits a q_i and a $\bar{q}_j$.

$$q_j(x) = \int_x^1 \frac{dy}{y} P_{j\alpha/i}(y) q_i\left(\frac{x}{y}\right),$$

- Prob the internal quark structure of GS bosons

$$q_k(x) = \int \frac{dy_1}{y_1}\frac{dy_2}{y_2} V_{k/\alpha}\left(\frac{x}{y_1}\right) P_{\alpha j/i}\left(\frac{y_1}{y_2}\right) q_i\left(y_2\right).$$

Splitting function

- $P_{j\alpha/i}(y)$ is the splitting function, which gives the probability of finding a constituent quark j carrying the light-cone momentum fraction y together with a spectator GS boson α.
- $M_{j\alpha}^2 = \left(m_j^2 + k_T^2\right)/y + \left(m_\alpha^2 + k_T^2\right)/(1-y)$ is the square of the invariant mass of the final state.
- $V_{k/\alpha}(x)$ is the quark k distribution function in α and satisfies the normalization $\int_0^1 V_{k/\alpha}(x)dx = 1$.

The momentum cutoff function

- It is conventional to specify the momentum cutoff function at the quark-GS-boson vertex as:

$$g_{\mathrm{A}} \to g'_{\mathrm{A}} \exp\left[\frac{m_i^2 - M_{j\alpha}^2}{4\Lambda^2}\right], \qquad (5)$$

- $g'_{\mathrm{A}} = 1$, following the large N_c argument, and Λ is the cutoff parameter, determined by the experimental data of the Gottfried sum and the constituent-quark-mass inputs for the pion. Such a form factor has the correct t- and u-channel symmetry, $P_{j\alpha/i}(y) = P_{\alpha j/i}(1-y)$.

The PDFs of the proton

$$
\begin{aligned}
u(x) &= Z_u u_0(x) + P_{u\pi^-/d} \otimes d_0(x) + V_{u/\pi^+} \otimes P_{\pi^+ d/u} \otimes u_0(x) \\
&+ \frac{1}{2} P_{u\pi^0/u} \otimes u_0(x) + V_{u/K^+} \otimes P_{K^+ s/u} \otimes u_0(x) \\
&+ \frac{1}{2} V_{u/\pi^0} \otimes \left[P_{\pi^0 u/u} \otimes u_0(x) + P_{\pi^0 d/d} \otimes d_0(x) \right], \\
d(x) &= Z_d d_0(x) + P_{d\pi^+/u} \otimes u_0(x) + V_{d/\pi^-} \otimes P_{\pi^- u/d} \otimes d_0(x) \\
&+ \frac{1}{2} P_{d\pi^0/d} \otimes d_0(x) + V_{d/K^0} \otimes P_{K^0 s/d} \otimes d_0(x) \\
&+ \frac{1}{2} V_{d/\pi^0} \otimes \left[P_{\pi^0 u/u} \otimes u_0(x) + P_{\pi^0 d/d} \otimes d_0(x) \right],
\end{aligned}
$$

The PDFs of the proton

$$\begin{aligned}
\bar{u}(x) &= V_{\bar{u}/\pi^-} \otimes P_{\pi^- u/d} \otimes d_0(x) \\
&+ \frac{1}{2} V_{\bar{u}/\pi^0} \otimes \left[P_{\pi^0 u/u} \otimes u_0(x) + P_{\pi^0 d/d} \otimes d_0(x) \right], \\
\bar{d}(x) &= V_{\bar{d}/\pi^+} \otimes P_{\pi^+ d/u} \otimes u_0(x) \\
&+ \frac{1}{2} V_{\bar{d}/\pi^0} \otimes \left[P_{\pi^0 u/u} \otimes u_0(x) + P_{\pi^0 d/d} \otimes d_0(x) \right].
\end{aligned}$$

- Where the constituent quark-distributions u_0 and d_0 are normalized to two and one, respectively.

$$Z_u = 1 - \langle P_{\pi^+} \rangle - \tfrac{1}{2} \langle P_{u\pi^0} \rangle - \langle P_{K^+} \rangle,$$
$$Z_d = 1 - \langle P_{\pi^-} \rangle - \tfrac{1}{2} \langle P_{d\pi^0} \rangle - \langle P_{K^0} \rangle.$$

- $\langle P_\alpha \rangle \equiv \langle P_{j\alpha/i} \rangle = \langle P_{\alpha j/i} \rangle = \int_0^1 x P_{j\alpha/i}(x) \mathrm{d}x.$

The convolution integral and meson structure relation

$$P_{j\alpha/i} \otimes q_i = \int_x^1 \frac{\mathrm{d}y}{y} P_{j\alpha/i}(y)\, q_i\left(\frac{x}{y}\right),$$

$$V_{k/\alpha} \otimes P_{\alpha j/i} \otimes q_i = \int_x^1 \frac{\mathrm{d}y_1}{y_1} \int_{y_1}^1 \frac{\mathrm{d}y_2}{y_2} V_{k/\alpha}\left(\frac{x}{y_1}\right) P_{\alpha j/i}\left(\frac{y_1}{y_2}\right) q_i(y_2)\,. \quad (6)$$

- In addition, $V_{k/\alpha}(x)$ follows the relationship

$$V_{u/\pi^+} = V_{\bar{d}/\pi^+} = V_{d/\pi^-} = V_{\bar{u}/\pi^-} = 2V_{u/\pi^0} = 2V_{\bar{u}/\pi^0} = 2V_{d/\pi^0} = 2V_{\bar{d}/\pi^0} = \frac{1}{2}V_\pi,$$

$$V_{u/K^+} = V_{d/K^0}. \quad (7)$$

The inputs

- The parton distributions of mesons are the parametrization GRS98, since the parametrization is more approximate to the actual value, M. Gluck, E. Reya, and M. Stratmann, Eur. Phys. J. C **2**, 159 (1998).

$$V_\pi(x) = 0.942x^{-0.501}(1 + 0.632\sqrt{x})(1 - x)^{0.367},$$
$$V_{u/K^+}(x) = V_{d/K^0}(x) = 0.541(1 - x)^{0.17} V_\pi(x). \qquad (8)$$

- Constituent-quark distributions u_0 and d_0, but there is no proper parametrization of them because they are not directly related to observable quantities in experiments. We adopt the constituent-quark-model distributions as inputs for constituent-quark distributions. For the proton, we have:

The inputs

$$u_0(x) = \frac{2x^{c_1}(1-x)^{c_1+c_2+1}}{\mathrm{B}[c_1+1, c_1+c_2+2]},$$
$$d_0(x) = \frac{x^{c_2}(1-x)^{2c_1+1}}{\mathrm{B}[c_2+1, 2c_1+2]}.$$

- Where $\mathrm{B}[i,j]$ is the Euler beta function. Such distributions satisfy the number and the momentum sum rules
$$\int_0^1 u_0(x)\mathrm{d}x = 2, \quad \int_0^1 d_0(x)\mathrm{d}x = 1,$$
$$\int_0^1 x u_0(x)\mathrm{d}x + \int_0^1 x d_0(x)\mathrm{d}x = 1.$$
- $c_1 = 0.65$ and $c_2 = 0.35$ are adopted in the calculation, following the original choice.
R. C. Hwa and M. S. Zahir, Phys. Rev. D **23**, 2539 (1981).

The parameters

- We assume that the ISB is entirely from the mass difference between isospin multiplets.
- $(m_u + m_d)/2 = 330$ MeV, $m_{\pi^\pm} = 139.6$ MeV, $m_{\pi^0} = 135$ MeV, $m_{K^\pm} = 493.7$ MeV, and $m_{K^0} = 497.6$ MeV.
- $\delta m = m_d - m_u$, $\delta m = 4$ MeV and $\delta m = 8$ MeV.

$$S_{\mathrm{G}} = \int_0^1 \frac{\mathrm{d}x}{x} \left[F_2^{\mathrm{p}}(x) - F_2^{\mathrm{n}}(x)\right]$$
$$= \frac{1}{3} - \frac{8}{9}\langle P_{\pi^-}\rangle + \frac{2}{9}\langle P_{\pi^+}\rangle + \frac{5}{18}\left(\langle P_{u\pi^0}\rangle - \langle P_{d\pi^0}\rangle\right).$$

The parameters

- $S_{\mathrm{G}} = 0.235 \pm 0.026$, M. Arneodo *et al.* [New Muon Collaboration], Phys. Rev. D **50**, R1 (1994).
- $\Lambda_\pi \sim 1500$ MeV.
- Λ_K cannot be determined in the same way. Usually, it is assumed that $\Lambda_K = \Lambda_\pi$, A. Szczurek, A. J. Buchmann, and A. Faessler, J. Phys. G **22**, 1741 (1996),
  K. Suzuki and W. Weise, Nucl. Phys. A **634**, 141 (1998).
- $SU(3)_f$ symmetry breaking implies that $\langle P_K \rangle < \langle P_\pi \rangle$. So we adopt a wide range of Λ_K from 900 to 1500 MeV.

Isospin symmetry breaking

- The isospin symmetry breaking (ISB) at parton level:

$$
\begin{aligned}
\delta u_{\mathrm{V}}(x) &= u_{\mathrm{V}}^{\mathrm{p}}(x) - d_{\mathrm{V}}^{\mathrm{n}}(x), \\
\delta d_{\mathrm{V}}(x) &= d_{\mathrm{V}}^{\mathrm{p}}(x) - u_{\mathrm{V}}^{\mathrm{n}}(x), \\
\delta \bar{u}(x) &= \bar{u}^{\mathrm{p}}(x) - \bar{d}^{\mathrm{n}}(x), \\
\delta \bar{d}(x) &= \bar{d}^{\mathrm{p}}(x) - \bar{u}^{\mathrm{n}}(x).
\end{aligned}
$$

- Where $q_{\mathrm{V}}^{\mathrm{N}}(x) = q^{\mathrm{N}}(x) - \bar{q}^{\mathrm{N}}(x)$ $(q = u,\ d,\ \mathrm{N} = \mathrm{p},\ \mathrm{n})$.

u_V ISB

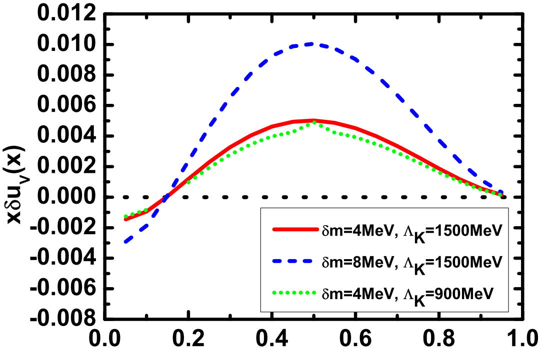


Figure: The ISB of the u_V-quark distribution $x\delta u_V(x)$ versus x in the chiral quark model with different inputs.

d_{V} ISB

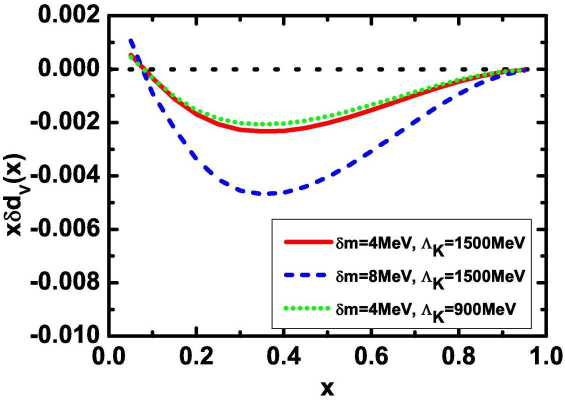


Figure: The ISB of the d_{V}-quark distribution $x\delta d_{\mathrm{V}}(x)$ versus x in the chiral quark model with different inputs.

Sea quark ISB

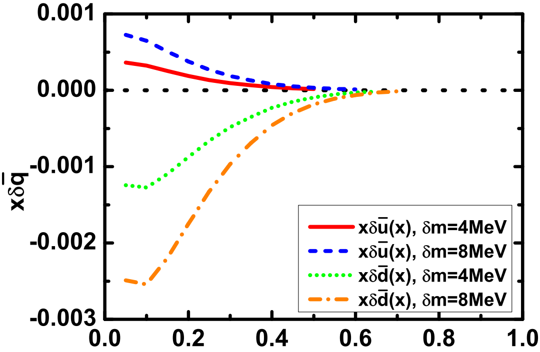


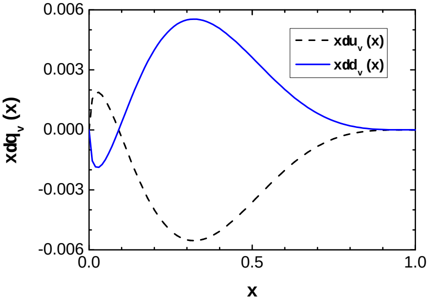
Figure: The ISB of the sea quark distribution $x\delta\bar{q}(x)$ versus x in the chiral quark model with different inputs.

MRST parametrization

- Valance quark ISB:

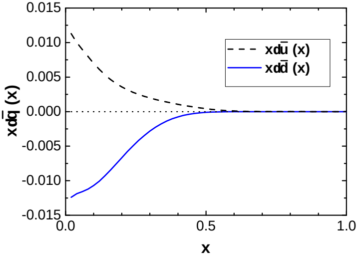
$$\delta u_{\mathrm{V}} = -\delta d_{\mathrm{V}} = \kappa(1-x)^4 x^{-0.5}(x-0.0909).$$

- $-0.8 \leq \kappa \leq +0.65$ with a 90% confidence level, and the best fit value is $\kappa = -0.2$.



MRST parametrization

- Sea quark ISB: $\delta\bar{u}(x) = k\bar{u}^{\mathrm{p}}(x), \quad \delta\bar{d}(x) = -k\bar{d}^{\mathrm{p}}(x)$, with the best fit value $k = 0.08$.
- $\bar{u}(x)$ and $\bar{d}(x)$ are from CTEQ6 parametrization.



A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne, Eur. Phys. J. C **35**, 325 (2004).

A short summary

- In most regions, $x\delta u_{\rm V}(x) > 0$ and $x\delta\bar{u}(x) > 0$. On the contrary, $x\delta d_{\rm V}(x) < 0$ and $x\delta\bar{d}(x) < 0$.
- Our predictions that $x\delta\bar{u}(x) > 0$ and $x\delta\bar{d}(x) < 0$ are consistent with the MRST parametrization. and, moreover, the shapes of $x\delta\bar{u}(x)$ and $x\delta\bar{d}(x)$ are similar to the best phenomenological fitting results given by the MRST group.
- It can also be found that the difference between various choices of Λ_K is minor, but the different choices of δm can have remarkable influence on the distributions. Especially, larger δm can lead to larger ISB, and this is concordant with our principle that ISB results from the mass difference between isospin multiplets at both hadron and parton levels.
- It should also be noted that $\delta q_{\rm V}(x)$ $(q = u, d)$ must have at least one zero point due to the valance-quark-normalization conditions.

NuTeV anomaly

- NuTeV Collaboration reported
  $\sin^2\theta_{\rm W} = 0.2277 \pm 0.0013\,({\rm stat}) \pm 0.0009\,({\rm syst})$.
  G. P. Zeller *et al.* [NuTeV Collaboration], Phys. Rev. Lett. **88**, 091802 (2002).
- Other electroweak processes :
  $\sin^2\theta_{\rm W} = 0.2227 \pm 0.0004$
- The 3 standard deviations is called the NuTeV anomaly.
- Discussions of NuTeV anomaly:
  1. New physics beyond the standard model.
  2. The nuclear effect.
  3. Nonisoscalar targets.
  4. Strange-antistrange asymmetry.
  5. ISB and so on.

PW ratio

- The Paschos-Wolfenstein (PW) ratio:
  E. A. Paschos and L. Wolfenstein, Phys. Rev. D **7**, 91 (1973).

$$R^{-}=\frac{\langle\sigma_{\rm NC}^{\nu \rm N}\rangle-\langle\sigma_{\rm NC}^{\overline{\nu}\rm N}\rangle}{\langle\sigma_{\rm CC}^{\nu \rm N}\rangle-\langle\sigma_{\rm CC}^{\overline{\nu}\rm N}\rangle}=\frac{1}{2}-\sin^2\theta_{\rm W}, \tag{9}$$

- The assumptions for the PW ratio:
  1. Isoscalar target.
  2. Strange-antistrange symmetry $s(x)=\bar{s}(x)$.
  3. Isospin symmetry between p and n.

The modified PW ratio

- Take the ISB into account:

$$R_{\mathrm{N}}^{-} = \frac{\langle \sigma_{\mathrm{NC}}^{\nu \mathrm{N}} \rangle - \langle \sigma_{\mathrm{NC}}^{\bar{\nu} \mathrm{N}} \rangle}{\langle \sigma_{\mathrm{CC}}^{\nu \mathrm{N}} \rangle - \langle \sigma_{\mathrm{CC}}^{\bar{\nu} \mathrm{N}} \rangle} = R^{-} + \delta R_{\mathrm{PW}}^{\mathrm{ISB}}, \quad (10)$$

$$\delta R_{\mathrm{PW}}^{\mathrm{ISB}} = \left(\frac{1}{2} - \frac{7}{6} \sin^2 \theta_{\mathrm{W}} \right) \frac{\int_0^1 x \left[\delta u_{\mathrm{V}}(x) - \delta d_{\mathrm{V}}(x) \right] \mathrm{d}x}{\int_0^1 x \left[u_{\mathrm{V}}(x) + d_{\mathrm{V}}(x) \right] \mathrm{d}x}, \quad (11)$$

- $\delta R_{\mathrm{PW}}^{\mathrm{ISB}}$ is the correction from the ISB to the PW ratio. $u_{\mathrm{V}}(x)$ and $d_{\mathrm{V}}(x)$ standing for valance-quark distributions of the proton.
- $Q_{\mathrm{V}} = \int_0^1 x \left[u_{\mathrm{V}}(x) + d_{\mathrm{V}}(x) \right] \mathrm{d}x$

The correction to NuTeV anomaly of ISB

Table: The renormalization constant, the total momentum fraction of valance quarks, and the correction of the ISB to the NuTeV anomaly in the chiral quark model.

δm (MeV)	Λ_K (MeV)	Z_u	Z_d	$Q_{\rm V}$	$\delta R_{\rm PW}^{\rm ISB}$
4	900	0.7497	0.7463	0.8451	0.0008
4	1200	0.7220	0.7185	0.8222	0.0008
4	1500	0.6932	0.6896	0.7985	0.0009
8	900	0.7515	0.7444	0.8455	0.0016
8	1200	0.7239	0.7165	0.8227	0.0017
8	1500	0.6953	0.6874	0.7990	0.0019

A short summary

- The ISB correction is of the order of magnitude of 10^{-3} and is more significant with a larger δm or Λ_K.
- MRST parametrization result: $-0.009 \leq \Delta R_{\mathrm{PW}}^{\mathrm{ISB}} \leq +0.007$.
- Our result is consistent with the MRST parametrization result.
- The NuTeV anomaly can be totally removed if $\Delta R_{\mathrm{PW}} = -0.005$, so the ISB correction is remarkable.
- The ISB correction is in an opposite direction to remove the NuTeV anomaly in the chiral quark model.

Summary

- We assume that isospin symmetry breaking is the result of mass differences between isospin multiplets and discuss the ISB of the valance-quark and the sea-quark distributions between the proton and the neutron in the framework of the chiral quark model.
- It is remarkable that our results of ISB for both the valence-quark and sea-quark distributions are consistent with the MRST parametrization of the ISB of valance- and sea-quark distributions.

Summary

- We analyze the effects of isospin symmetry breaking to the NuTeV anomaly.
- We find that the correction to the NuTeV anomaly is in an opposite direction, so the NuTeV anomaly cannot be removed by isospin symmetry breaking in the chiral quark model. However, its influence is remarkable and should be taken into careful consideration.
- The correcion to the NuTeV anomaly from isospin is not conclusive. Therefore, it is important to do more precision experiments and careful theoretical studies on isospin symmetry breaking.
  Huiying Song, Xinyu Zhang, and Bo-Qiang Ma,
  Phys. Rev. D **82**, 113011 (2010);
  Eur. Phys. J. C **71**, 1542 (2011).


Thank you!