Effective Chiral Lagrangian from Strong Dynamical Models

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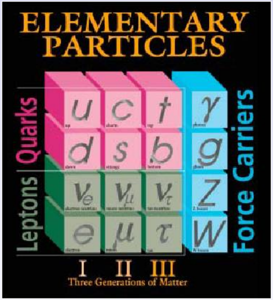
Outline

Standard Model and its Difficulties

The standard model:

- Fermions: physical world
  Bosons: force carriers
- fits experiments
- Problems:
  - triviality, unnatrualness
  - Higgs boson



$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

$$\Downarrow \qquad v = 246GeV$$

$$SU(3)_C \otimes U(1)_{em}$$

New Physics

New physics beyond the standard model(BSM):

- include Higgs:
    - Supper Symmetry
    - Little Higgs
    - L-R Models
    - ···
- exclude Higgs:
    - Higgsless Model
    - Strong Dynamics
    - ···

Solve the problems:

- asymptotically free→triviality
- Fermion condensation → unnaturalness

Technicolor & Extended TC

Technicolor: QCD-like

$$G_{TC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

Introduce techni-Fermion condensation: $\langle\bar{\psi}\psi\rangle \neq 0$
Ordinary quarks: merely dynamical masses from QCD
ordinary leptons: massless
require: ETC gives particles "hard masses"
Problems:

- large top mass
- wrong FCNC
- large S

Topcolor

Top quark mass: 173.1GeV
near EWSB scale
top condensation leads to EWSB?

$$SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2 \otimes SU(2)_L$$

Pagels-Stokar formula:

$$f_\pi^2 = \frac{N_c}{16\pi^2} m_t^2 (ln\frac{\Lambda^2}{m_t^2} + k)$$

$\Lambda \sim 10^{15}$GeV,composite Higgs boson mass:

$$m_H^2(\mu) = \frac{\Lambda^2}{g^2} - {\frac{2N_c}{4\pi}}^2 (\Lambda^2 - \mu^2)$$

fine-tuning: 1:10^{-30}, unnatural

topcolor assisted technicolor(TC2 Model)

Typical Gauge group:

$$G_{TC}\otimes SU(1)_1\otimes SU(2)_2\otimes U(1)_1\otimes U(1)_2\otimes SU(2)_L$$

- Top condensation gives top quark mass
- ETC gives other quarks masses
- Technicolor interaction breaks EW symmetry

TC2 Models

C.T.Hill's schematic TC2 model:(PLB,1995,345:483-489)

- schematic, no detail
- $\rho \sim 1$ needs fine-tuning

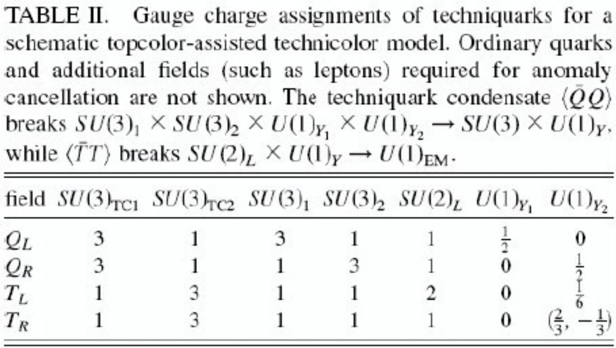


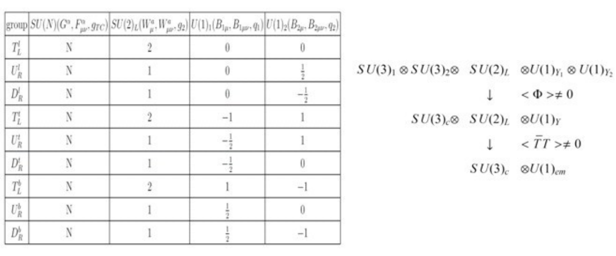
TABLE II. Gauge charge assignments of techniquarks for a schematic topcolor-assisted technicolor model. Ordinary quarks and additional fields (such as leptons) required for anomaly cancellation are not shown. The techniquark condensate $\langle \bar{Q}Q \rangle$ breaks $SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2} \to SU(3) \times U(1)_Y$, while $\langle \bar{T}T \rangle$ breaks $SU(2)_L \times U(1)_Y \to U(1)_{EM}$.

field	$SU(3)_{TC1}$	$SU(3)_{TC2}$	$SU(3)_1$	$SU(3)_2$	$SU(2)_L$	$U(1)_{Y_1}$	$U(1)_{Y_2}$
Q_L	3	1	3	1	1	$\frac{1}{2}$	0
Q_R	3	1	1	3	1	0	$\frac{1}{2}$
T_L	1	3	1	1	2	0	$\frac{1}{6}$
T_R	1	3	1	1	1	0	$(\frac{2}{3}, -\frac{1}{3})$

TC2 Models

K.Lane's natural TC2 model:(PLB,1995,352:382-387)

$$G_{TC} \otimes SU(3)_1 \otimes SU(3)_2 \otimes SU(2)_L \otimes U(1)_1 \otimes U(1)_2$$

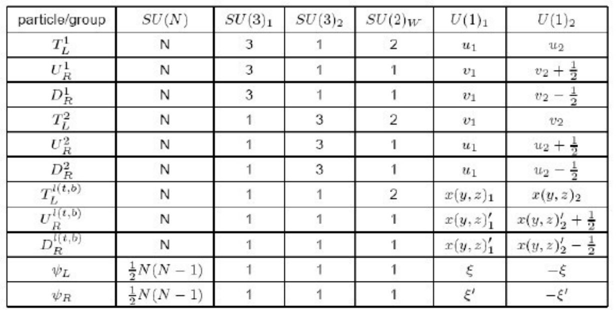


group	$SU(N)(G^a, F^a_{\mu\nu}, g_{TC})$	$SU(2)_L(W^a_\mu, W^a_{\mu\nu}, g_2)$	$U(1)_1(B_{1\mu}, B_{1\mu\nu}, q_1)$	$U(1)_2(B_{2\mu}, B_{2\mu\nu}, q_2)$
T^l_L	N	2	0	0
U^l_R	N	1	0	$\frac{1}{2}$
D^l_R	N	1	0	$-\frac{1}{2}$
T^t_L	N	2	-1	1
U^t_R	N	1	$-\frac{1}{2}$	1
D^t_R	N	1	$-\frac{1}{2}$	0
T^b_L	N	2	1	-1
U^b_R	N	1	$\frac{1}{2}$	0
D^b_R	N	1	$\frac{1}{2}$	-1

$$\begin{array}{rcl} SU(3)_1 \otimes SU(3)_2 \otimes & SU(2)_L & \otimes U(1)_{Y_1} \otimes U(1)_{Y_2} \\ & \downarrow & <\Phi> \neq 0 \\ SU(3)_c \otimes & SU(2)_L & \otimes U(1)_Y \\ & \downarrow & <\bar{T}T> \neq 0 \\ & SU(3)_c & \otimes U(1)_{em} \end{array}$$

TC2 Models

K.Lane's generation mixing TC2 model:(PRD,1996,54:2204-2212)



particle/group	$SU(N)$	$SU(3)_1$	$SU(3)_2$	$SU(2)_W$	$U(1)_1$	$U(1)_2$
T_L^1	N	3	1	2	u_1	u_2
U_R^1	N	3	1	1	v_1	$v_2+\frac{1}{2}$
D_R^1	N	3	1	1	v_1	$v_2-\frac{1}{2}$
T_L^2	N	1	3	2	v_1	v_2
U_R^2	N	1	3	1	u_1	$u_2+\frac{1}{2}$
D_R^2	N	1	3	1	u_1	$u_2-\frac{1}{2}$
$T_L^{l(t,b)}$	N	1	1	2	$x(y,z)_1$	$x(y,z)_2$
$U_R^{l(t,b)}$	N	1	1	1	$x(y,z)'_1$	$x(y,z)'_2+\frac{1}{2}$
$D_R^{l(t,b)}$	N	1	1	1	$x(y,z)'_1$	$x(y,z)'_2-\frac{1}{2}$
ψ_L	$\frac{1}{2}N(N-1)$	1	1	1	ξ	$-\xi$
ψ_R	$\frac{1}{2}N(N-1)$	1	1	1	ξ'	$-\xi'$

$\langle\bar{T}^iT^j\rangle$ non-diagonal(i,j=1,2)
ψ guarantees the theory walking

EWCL

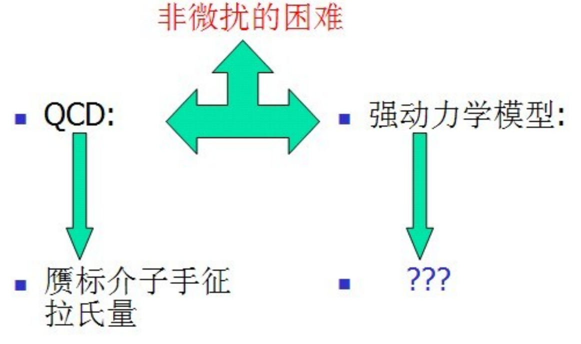
Low energy effective theory for EW interaction:

- universal: contain all possible terms allowed by the symmetry
- economic: only contain the particles below the EW scale
- contributions from heavy particles hidden in the coefficients

$$
\begin{aligned}
& S_{EM} \\
= & \int d^4x\Big(-\frac{1}{4}f^2 tr[X^\mu X_\mu] + \frac{1}{4}\beta_1 f^2 (tr[\tau^3 X^\mu])^2 \\
+ & \frac{1}{2}\alpha_1 g_1 B_{\mu\nu} tr[\tau^3 \overline{W}^{\mu\nu}] + i\alpha_2 g_1 B_{\mu\nu} tr[\tau^3 X^\mu X^\nu] + 2i\alpha_3 tr[\overline{W}^{\mu\nu} X_\mu X_\nu] \\
+ & \alpha_4 (tr[X^\mu X_\nu])^2 + \alpha_5 (tr[X^\mu X_\mu])^2 + \alpha_6 tr[X^\mu X^\nu] tr[\tau^3 X_\mu] tr[\tau^3 X_\nu] \\
+ & \alpha_7 tr[X^\mu X_\mu] tr[\tau^3 X^\nu] tr[\tau^3 X_\nu] + \frac{1}{4}\alpha_8 (tr[\tau^3]\overline{W}_{\mu\nu})^2 \\
+ & i\alpha_9 tr[\tau^3 \overline{W}_{\mu\nu}] tr[\tau^3 X^\mu X^\nu] + \frac{1}{2}\alpha_{10} (tr[\tau^3 X^\mu] tr[\tau^3 X^\nu])^2 \\
+ & \alpha_{11}\epsilon^{\mu\nu\rho\lambda} tr[\tau^3 X_\mu] tr[X_\nu \overline{W}_{\rho\lambda}] + \alpha_{12} tr[\tau^3 X_\mu] tr[X_\nu \overline{W}^{\mu\nu}] \\
+ & \alpha_{13}\epsilon^{\mu\nu\rho\lambda} g_1 B_{\mu\nu} tr[\tau^3 \overline{W}_{\rho\lambda}] + \alpha_{14}\epsilon^{\mu\nu\rho\lambda} tr[\tau^3 \overline{W}_{\mu\nu}] tr[\tau^3 \overline{W}_{\rho\lambda}]
\end{aligned}
$$

$$X_\mu = U^\dagger (D_\mu U) \qquad \overline{W}_{\mu\nu} = U^\dagger \frac{\tau^a}{2} W^a_{\mu\nu} U$$

EWCL from Dynamical EWSB models



非微扰的困难

- QCD:
- 赝标介子手征拉氏量

- 强动力学模型:
- ???

EWCL & LECs

e.g.: Oblique correction parameters: S, T, U:

$$\begin{aligned} S &= -16\pi\alpha_1 \\ \alpha T &= 2\beta_1 \\ U &= -16\pi\alpha_8 \end{aligned}$$

expremental:

$$\begin{aligned} S &= 0.01 \pm 0.10 \\ T &= 0.03 \pm 0.11 \\ U &= 0.06 \pm 0.10 \end{aligned}$$

Electroweak Chiral Lagrangian

QCD →Pseudo-scalar meson effective Lagrangian

- Wang Q, Kuang Y P, Xiao M, et al (PRD,2000,61:054011)
- Yang H, Wang Q, Kuang Y P, et al (PRD,2002,66:014019)
- Jiang S Z, Wang Q, (PRD 2010, 81:094037)
- Jiang S Z, Zhang Y, et al (PRD, 2010, 81:014001)

Strong dynamical models→EWCL

- C.T.Hill's schematic TC2 model and one-dobulet model: HH Zhang, et.al, Phys.Rev.D,2008,77:055003
- K.Lane's natural TC2 model: JY Lang,et.al, Phys.Rev.D,2009,79:015002
- R.S.Chivukula's hypercharge-universal topcolor model: JY Lang,et.al, Phys.Lett.B,2009,673: 63-67

EWCL from C.T. Hill's schematic TC2 Model

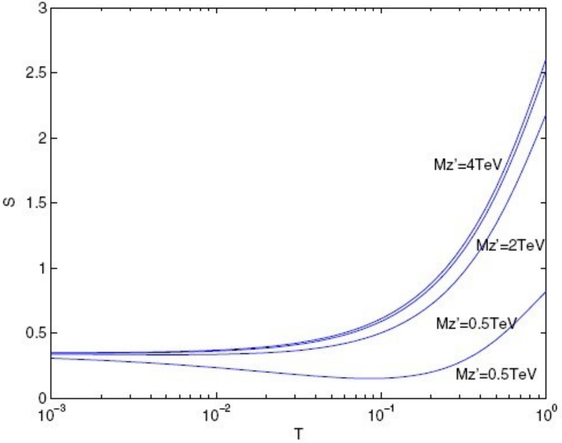


FIG. 4 (color online). The S parameter for the topcolor-assisted technicolor model. $F_0^{\mathrm{TC2}} = 250$ GeV, the T parameter and $M_{Z'} = \{0.5, 1, 2, 4\}$ TeV are as input parameters of the model.

EWCL from K.Lane's natural TC2 Model

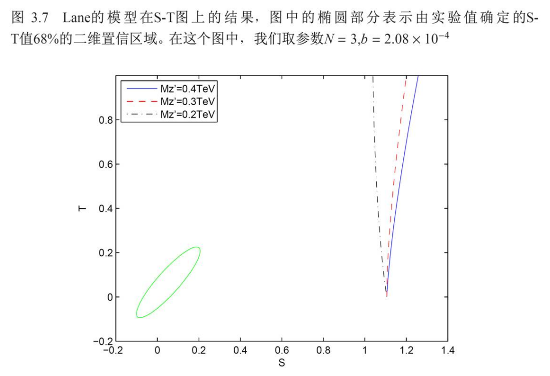


图 3.7 Lane的模型在S-T图上的结果，图中的椭圆部分表示由实验值确定的S-T值68%的二维置信区域。在这个图中，我们取参数$N = 3, b = 2.08 \times 10^{-4}$

- Trln expansion of non-diagonal condensation
- walking effects
- more models
- $\cdots$

Calculating Method

Goal:

$$\begin{aligned} & \int \mathcal{D}Z'_\mu \mathcal{D}G^\alpha_\mu \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS[Z'_\mu, G^\alpha_\mu, \bar{\psi}, \psi, W^\alpha_\mu, B_\mu]} \\ = & \int \mathcal{D}_\mu(U) e^{iS_{EW}[U, W^\alpha_\mu, B_\mu]} \end{aligned}$$

key steps:

- Integrate in Goldstone boson fields
- Integrate out BSM gauge bosons
- Integrate out BSM fermions
- Integrate out Z'

Integrate in U Field

Introduce operator

$$O(x) \equiv tr_{lc}\psi_L^i(x)\bar{\psi}_R^i(x)$$

decomposed as

$$O(x) = \zeta_L^\dagger(x)\sigma(x)\zeta_R(x)$$

Hermite matrix $\sigma(x)$: modular degree of freedom;
Unitary matrix $\zeta_L^\dagger(x), \zeta_R(x)$: $SU(2)_L$和$U(1)_Y$ phase degree of freedom.

$$U(x) \equiv \zeta_L^\dagger(x)\zeta_R(x)$$

Insert a constant

$$\int \mathcal{D}_\mu(U)\mathcal{F}[O]\delta(\zeta_L O\zeta_R^\dagger - \zeta_R O^\dagger \zeta_L^\dagger) = const.$$

Integrate in U Field

Do special chiral rotation $SU(2)_L \times U(1)$:
$V_L = \zeta_L,\ V_R = \zeta_R$

$$\begin{aligned} & S_{\rm eff}[U,W,B,Z'] \\ = \quad & S_{\rm Gauge}[W,B,Z'] - i\mathrm{Tr}\ln(\mathrm{i}\not{\partial} + \not{V} + \not{A}\gamma^5) \\ - \quad & i\ln\frac{\int \mathcal{D}G\mathcal{D}\bar\psi_\zeta\mathcal{D}\psi_\zeta\mathcal{F}[O_\zeta]\delta(O_\zeta - O_\zeta^\dagger)e^{iS[G,W_\zeta,B_\zeta,\bar\psi_\zeta,\psi_\zeta]}}{\int\mathcal{D}\bar\psi_\zeta\mathcal{D}\psi_\zeta e^{i\int d^4x\bar\psi_\zeta(i\not{\partial}+\not{V}_\zeta+\not{A}_\zeta\gamma^5)\psi_\zeta}} \end{aligned}$$

Finally

$$\begin{aligned} & S_{\rm eff}[U,W,B,Z'] \\ = & S_{\rm Gauge}[W,B,Z'] + S_{\rm norm}[U,W,B,Z'] + S_{\rm anom}[U,W,B,Z'] \end{aligned}$$

where

$$\begin{aligned} & S_{\rm norm}[U,W,B,Z'] \\ = \quad & -i\ln\int\mathcal{D}G\mathcal{D}\bar\psi_\zeta\mathcal{D}\psi_\zeta\mathcal{F}[O_\zeta]\delta(O_\zeta - O_\zeta^\dagger)e^{iS[G,W_\zeta,B_\zeta,\bar\psi_\zeta,\psi_\zeta]} \\ & S_{\rm anom}[U,W,B,Z'] \\ = \quad & \mathrm{Tr}\ln(\mathrm{i}\not{\partial} + \not{V} + \not{A}\gamma^5) - \mathrm{Tr}\ln(\mathrm{i}\not{\partial}_\zeta + \not{V}_\zeta + \not{A}_\zeta\gamma^5) \\ & S_{\rm Gauge}[W,B,Z'] \\ = \quad & \int d^4x\Big(-\frac{1}{4}W^\alpha_{\mu\nu}F^{\alpha,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{1}{2}M_0^2Z'_\mu Z'^\mu\Big) \end{aligned}$$

Integrate out BSM gauge bosons

Use formula

$$\begin{aligned}
&\int \mathcal{D}G^{\alpha}_{\mu}\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\{i\int d^4x[-\frac{1}{4}F^{\alpha}_{\mu\nu}F^{\alpha,\mu\nu}+\bar{\psi}(i\not{\partial}-g_{TC}t^{\alpha}\not{G}^{\alpha}+\not{V}+\not{A}\gamma^5)\psi]\}\\
=&\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\{i\int d^4x[\bar{\psi}(i\not{\partial}+\not{V}+\not{A}\gamma^5)\psi\\
&+\sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x_n\frac{(-ig_{TC})^n}{n!}G^{\alpha_1\cdots\alpha_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n)J^{\mu_1}_{\alpha_1}(x_1)\cdots J^{\mu_n}_{\alpha_n}(x_n)]\}
\end{aligned}$$

where

$$J^{\alpha\mu}\equiv\bar{\psi}t^{\alpha}\gamma^{\mu}\psi$$

We can integrate out technigluons.

Integrate out BSM fermions

Using some approximations, we can obtain

$$S[U,W,B] = -i\mathrm{Tr}\ln[\mathrm{i}\partial\!\!\!/ + v\!\!\!/ + a\!\!\!/\gamma^5 - \Sigma(\bar{\nabla}^2)]$$

If $\Sigma(\bar{\nabla}^2)$ is diagonal , we can use

$$\begin{aligned}
& -i\mathrm{Tr}\ln[\mathrm{i}\partial\!\!\!/ + v\!\!\!/ + a\!\!\!/\gamma^5 - \Sigma(\bar{\nabla}^2)] \\
= & \int d^4x tr_f[(F_0)^2a^2 - K_1(d_\mu a^\mu)^2 - K_2(d_\mu a_\nu - d_\nu a_\mu)^2 + K_3(a^2)^2 + K_4(a_\mu a_\nu)^2 \\
& - K_{13}V_{\mu\nu}V^{\mu\nu} + iK_{14}a_\mu a_\nu V^{\mu\nu}] + \mathcal{O}(p^6)
\end{aligned}$$

where K's are function of self-energy $\Sigma(p^2)$. (PRD 2002,66:014019).
a contains no colron field⇒ massless coloron !

Introduce non-diagonal Σ_5

New self energy form:

$$\Sigma + i\tau^2\gamma^5\Sigma_5$$

- non-diagonal condensation
- massive coloron

SD equations:

$$i\Sigma(-p^2) = \int \frac{d^4q}{(2\pi)^4}\frac{12\pi C_2(N)\alpha_{TC}(-(p-q)^2)}{(p-q)^2}\frac{\Sigma(-q^2)}{q^2+\Sigma_5^2(-q^2)+\Sigma^2(-q^2)}$$

$$i\Sigma_5(-p^2) = \int \frac{d^4q}{(2\pi)^4}\frac{12\pi C_2(N)\alpha_{TC}(-(p-q)^2)}{(p-q)^2}\frac{\Sigma_5(-q^2)}{q^2+\Sigma_5^2(-q^2)+\Sigma^2(-q^2)}$$

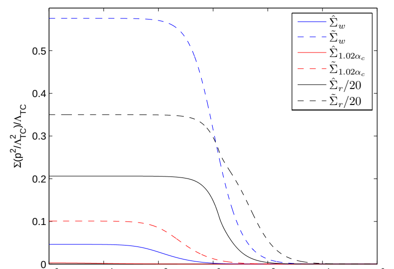
Some results about Self-energy

After complex computation, we get:

- $\Sigma = \hat{\Sigma}\cos\Theta$, $\Sigma_5 = \hat{\Sigma}\sin\Theta$
- vacuum energy only depends on $\hat{\Sigma}$
- new expansion formula: $-i\mathrm{Tr}\ln[i\not{\partial} + \not{v} + \not{a}\gamma^5 - \Sigma(\hat{\nabla}^2) - i\tau^2\gamma^5\Sigma_5(\hat{\nabla}^2)]$

  Θ-dependent parts: keep the same， only replace Σ by $\hat{\Sigma}$;

  Θ-independent parts : EWCL coefficients change little， coloron masses change largely



Integrate out Colorons and Z'

colorons only interact with ordinary quarks
Only kinetic terms and mass terms:

$$M^2_{\text{coloron}} = \frac{C}{\hat{E} + 2(\mathcal{K} + \hat{\mathcal{K}}_{13}^{\Sigma\neq 0}) + (2/g_3^2 - 8\hat{\mathcal{K}}_{13}^{\Sigma\neq 0})/(\cot\theta' + \tan\theta')^2}$$

Jackiw's classic field method，intergrate out Z':

$$\int \mathcal{D}Z' e^{iS[U,W,B,Z']} = e^{iS[U,W,B,Z_c]}$$

We can get the relations between K's and the effective Lagrangian coefficients.

Analytic Results

$$f^2 = 5\hat{F}_0^2 \qquad\qquad \beta_1 = \frac{10a_3^2\hat{F}_0^2}{\bar{M}_{Z'}^2}$$

$$\alpha_1 = \frac{5}{2}(1-2\beta_1)(\hat{\mathcal{K}}_2^{\Sigma\neq0} - \hat{\mathcal{K}}_{13}^{\Sigma\neq0}) + \frac{\beta_1 f^2}{2M_{Z'}^2} - \frac{\gamma\beta_1}{2a_3}$$

$$\alpha_2 = (\beta_1 - \frac{1}{2})(\frac{5}{2}\hat{\mathcal{K}}_{13}^{\Sigma\neq0} - \frac{5}{8}\hat{\mathcal{K}}_{14}^{\Sigma\neq0}) + \frac{\beta_1 f^2}{2M_{Z'}^2} - \frac{\gamma\beta_1}{2a_3}$$

$$\alpha_3 = (\beta_1 - \frac{1}{2})(\frac{5}{2}\hat{\mathcal{K}}_{13}^{\Sigma\neq0} - \frac{5}{8}\hat{\mathcal{K}}_{14}^{\Sigma\neq0})$$

$$\alpha_4 = (2\beta_1 + \frac{1}{4})(\frac{5}{2}\hat{\mathcal{K}}_{13}^{\Sigma\neq0} - \frac{5}{8}\hat{\mathcal{K}}_{14}^{\Sigma\neq0}) + (\frac{5}{16}\hat{\mathcal{K}}_{4}^{\Sigma\neq0} - \frac{5}{32}\hat{\mathcal{K}}_{14}^{\Sigma\neq0}) + \frac{\beta_1 f^2}{2M_{Z'}^2}$$

$$\alpha_5 = -\frac{5}{2}(4\beta_1 + \frac{1}{4})\hat{\mathcal{K}}_{13}^{\Sigma\neq0} + \frac{5}{4}(3\beta_1 + \frac{1}{4})\hat{\mathcal{K}}_{14}^{\Sigma\neq0} + \frac{5}{32}(\hat{\mathcal{K}}_{3}^{\Sigma\neq0} - \hat{\mathcal{K}}_{4}^{\Sigma\neq0}) - \frac{\beta_1 f^2}{2M_{Z'}^2}$$

$$\begin{aligned}\alpha_6 = &-\frac{\beta_1 f^2}{2M_{Z'}^2} + \frac{\beta_1^2}{4a_3^2}[(2a_0^2 + \hat{a}_0^2)\hat{\mathcal{K}}_{3}^{\Sigma\neq0} + (2a_0^2 + \hat{a}_0^2 + 5a_3^2)\hat{\mathcal{K}}_{4}^{\Sigma\neq0}\\ &+10a_3^2\hat{\mathcal{K}}_{13}^{\Sigma\neq0} - 5a_3^2\hat{\mathcal{K}}_{14}^{\Sigma\neq0} - 2a_0^2\hat{D}_4] - \frac{\beta_1}{2}(\frac{5}{2}\hat{\mathcal{K}}_{4}^{\Sigma\neq0} + 15\hat{\mathcal{K}}_{13}^{\Sigma\neq0} - 5\hat{\mathcal{K}}_{14}^{\Sigma\neq0})\end{aligned}$$

Analytic Results

$$
\begin{aligned}
\alpha_7 &= \frac{\beta_1 f^2}{2M_{Z'}^2} - \frac{\beta_1^2}{4a_3^2}[(\frac{5}{2}a_3^2 + a_0^2 + \frac{1}{2}\hat{a}_0^2)\hat{\mathcal{K}}_3^{\Sigma\neq0} + (a_0^2 + \frac{1}{2}\hat{a}_0^2 - \frac{5}{2}a_3^2)\hat{\mathcal{K}}_4^{\Sigma\neq0} \\
&\quad -10a_3^2\hat{\mathcal{K}}_{13}^{\Sigma\neq0} + 5a_3^2\hat{\mathcal{K}}_{14}^{\Sigma\neq0} + a_0^2\hat{D}_3] - \frac{\beta_1}{2}(\frac{5}{4}\hat{\mathcal{K}}_3^{\Sigma\neq0} - \frac{5}{4}\hat{\mathcal{K}}_4^{\Sigma\neq0} - 15\hat{\mathcal{K}}_{13}^{\Sigma\neq0} + 5\hat{\mathcal{K}}_{14}^{\Sigma\neq0}) \\
\alpha_8 &= -\frac{\beta_1 f^2}{2M_{Z'}^2} + 10\beta_1(\hat{\mathcal{K}}_2^{\Sigma\neq0} - \hat{\mathcal{K}}_{13}^{\Sigma\neq0}) \\
\alpha_9 &= -\frac{\beta_1 f^2}{2M_{Z'}^2} + \beta_1(5\hat{\mathcal{K}}_2^{\Sigma\neq0} - 10\hat{\mathcal{K}}_{13}^{\Sigma\neq0} + \frac{5}{4}\hat{\mathcal{K}}_{14}^{\Sigma\neq0}) \\
\alpha_{10} &= \frac{5\beta_1^2}{4}(\hat{\mathcal{K}}_3^{\Sigma\neq0} + \hat{\mathcal{K}}_4^{\Sigma\neq0}) + \frac{\beta_1^4}{8a_3^4}g_{4Z} - \frac{\beta_1^3}{2a_3^3}[(2a_3^3 + 6a_0^2a_3 + 3\hat{a}_0^2a_3)(\hat{\mathcal{K}}_3^{\Sigma\neq0} + \hat{\mathcal{K}}_4^{\Sigma\neq0}) \\
&\quad +2a_0^2a_3\hat{D}_2] \\
\alpha_{11} &= \alpha_{12} = \alpha_{13} = \alpha_{14} = 0
\end{aligned}
$$

Inputs: $M_{Z'}, T$

γ, D_inon-leading order (β_1 expansion)

Walking Technicolor

increase techni-quark condensation:

- large techni-quark mass
- large techni-NGB mass
- correct FCNC

In the region $\Lambda_{TC} < \mu < \Lambda_{ETC}$: near constant α^*(ultraviolet fixed point)

In infrared region:near constant α^*(UVFP)

2-loop diagram:

$$\beta(\alpha) \;=\; -\beta_0 \frac{g_{\rm TC}^3}{(4\pi)^2} - \beta_1 \frac{g_{\rm TC}^5}{(4\pi)^4}$$

Walking Technicolor

$$\beta_0 > 0 \qquad \beta_1 < 0$$

Banks-Zaks IRFP

$$\alpha_* = -\frac{4\pi\beta_0}{\beta_1}$$

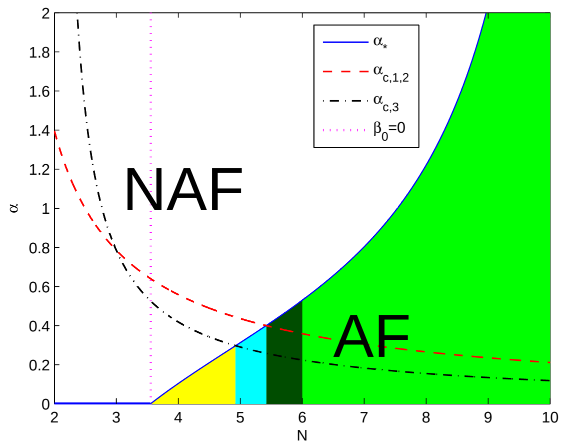
$$\begin{aligned} 2N\beta_0 &= \frac{11}{3}C_2(SU(N)_{\rm TC}) - \frac{4}{3}[T(R_1)+T(R_2)+T(R_3)] \\ (2N)^2\beta_1 &= \frac{34}{3}C_2^2(SU(N)_{\rm TC}) - \sum_{i=1}^{3}[\frac{20}{3}C_2(SU(N)_{\rm TC})T(R_i) + 4C_2(R_i)T(R_i)]\ . \end{aligned}$$

critical coupling to form condensation:

$$\alpha_c = \frac{2\pi N}{C_2(R)}$$

Phase Diagram

TechniColor gauge group SU(N)， N≥6



Coupling Constant

- running

$$\alpha(x)=\frac{4\pi}{\beta_0}\times\begin{cases}7 & \ln x\leq -2\\ 7-\frac{4}{5}(2+\ln x)^2 & -2\leq\ln x\leq 0.5\\ \frac{1}{\ln x} & \ln x\geq 0.5\end{cases}$$

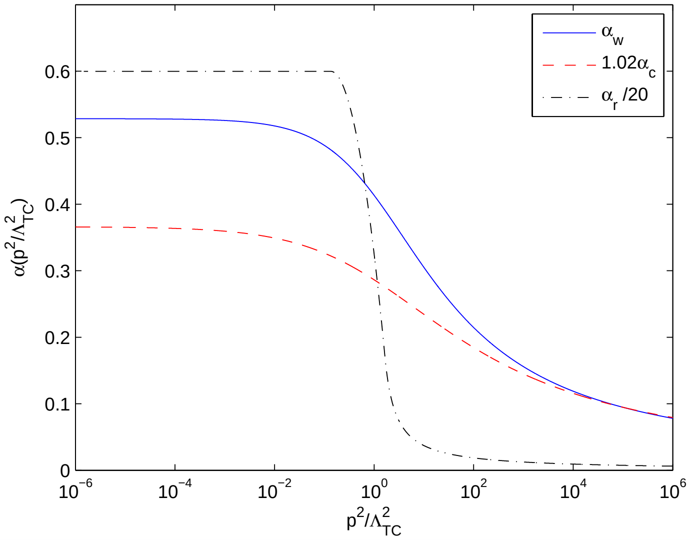
- walking

$$\frac{1}{\alpha(x)}=\frac{\beta_0}{2\pi}\ln x+\frac{1}{\alpha_*}\ln\frac{\alpha(x)}{\alpha_*-\alpha(x)}$$

- ideal walking

$$\alpha_*=1.02\alpha_c$$

Coupling Constant



Hypercharges

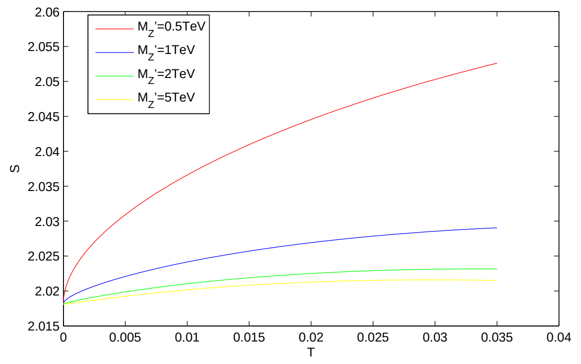
26(+ordinary fermions 8)hypercharges，23 restraints

Inputs:$x_1, y_1, y_1 + y_2$

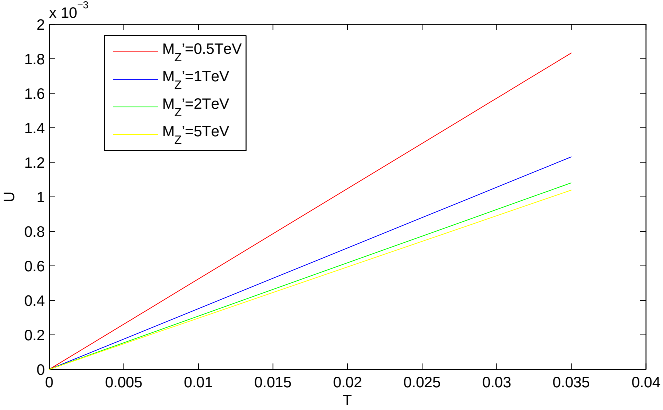
particle	$SU(N)$	$SU(3)_1$	$SU(3)_2$	$SU(2)_W$	$U(1)_1$	$U(1)_2$
T_L^1	N	3	1	2	u_1	u_2
U_R^1	N	3	1	1	v_1	$v_2 + \frac{1}{2}$
D_R^1	N	3	1	1	v_1	$v_2 - \frac{1}{2}$
T_L^2	N	1	3	2	v_1	v_2
U_R^2	N	1	3	1	u_1	$u_2 + \frac{1}{2}$
D_R^2	N	1	3	1	u_1	$u_2 - \frac{1}{2}$
$T_L^{l(t,b)}$	N	1	1	2	$x(y,z)_1$	$x(y,z)_2$
$U_R^{l(t,b)}$	N	1	1	1	$x(y,z)'_1$	$x(y,z)'_2 + \frac{1}{2}$
$D_R^{l(t,b)}$	N	1	1	1	$x(y,z)'_1$	$x(y,z)'_2 - \frac{1}{2}$
ψ_L	$\frac{1}{2}N(N-1)$	1	1	1	ξ	$-\xi$
ψ_R	$\frac{1}{2}N(N-1)$	1	1	1	ξ'	$-\xi'$

Results

$a = -39.4015,\ a' = -45.5381,\ b = 14.3188,\ b' = 8.1822,\ c = -39.4015,\ c' = -45.5381,\ d = -12.2733,\ d' = -14.3188,\ \xi = 4.6025,\ \xi' = -4.6025,\ x_1 = 25,\ x_1' = 18.8634,\ x_2 = -25.5528,\ x_2' = -19.4161,\ y_1 = 25,\ y_1' = 22.9545,\ y_2 = -25,\ y_2' = -22.9545,\ z_1 = -7.7287,\ z_1' = 18.8634,\ z_2 = 7.7287,\ z_2' = -18.8634,\ u_1 = -4.1027,\ v_1 = -6.1482,\ u_2 = 4.1948,\ v_2 = 6.2404.$

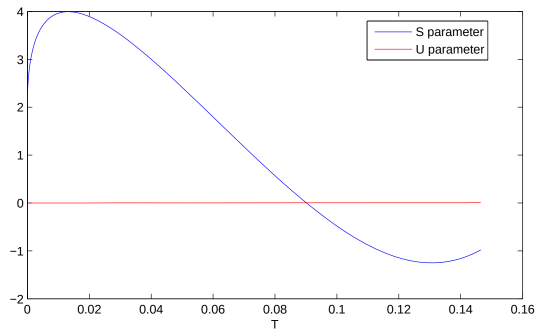


Results


x 10^-3
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
U
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
T
M_Z'=0.5TeV
M_Z'=1TeV
M_Z'=2TeV
M_Z'=5TeV

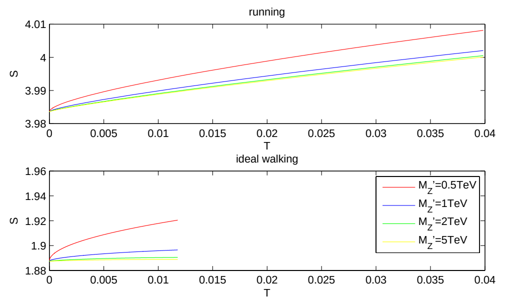
results

$a = -19.1970$, $a' = -22.1970$, $b = 7$, $b' = 4$, $c = -19.1970$, $c' = -22.1970$, $d = -6$, $d' = -7$, $\xi = 2.25$, $\xi' = -2.25$, $x_1 = -50$, $x_1' = -53$, $x_2 = 2.7393$, $x_2' = 5.7393$,$y_1 = 36.0643$,$y_1' = 35.0643$,$y_2 = -12.4356$, $y_2' = -11.4356$, $z_1 = 20.0643$, $z_1' = 33.0643$, $z_2 = 3.5644$, $z_2' = -9.4356$, $u_1 = 0.4116$, $v_1 = -0.5884$, $u_2 = -0.4111$, $v_2 = 0.5889$



Results

Compare: running and walking

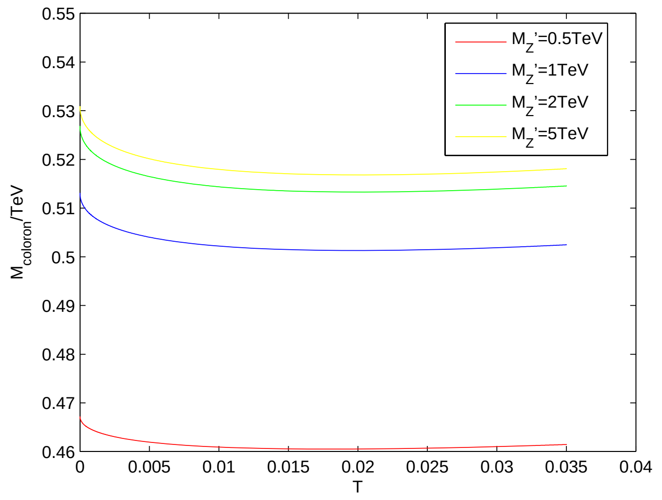


TC scale

$$\Lambda_{\rm TC} = \begin{cases} 0.21\text{TeV} & \text{running} \\ 4.5\text{TeV} & \text{walking} \\ 73\text{TeV} & \text{ideal walking} \end{cases}$$

Results

Coloron mass is not sensitive to walking effect:



Conclusion

Conclusion:

- walking effect can reduce S to half of running;
- In typical parameter space of this TC2 model: S∼2;
- there exits the parameter space: T a little bigger, $S < 0$;
- $\alpha_2, \alpha_3, \alpha_4, \alpha_5$ order 10^{-2}
  $\alpha_6, \alpha_7, \alpha_9$ order 10^{-5}
  α_{10} order 10^{-10};
  $\alpha_6, \alpha_7, \alpha_9$, especially α_{10}, sensitive to walking effect.
- cover all the typical TC2 models。

Thank you!