Quantum Modifications of Gravity Waves in de Sitter Spacetime

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Historical Developments

1. Spacetime is governed by $G_{\mu\nu} + (-\Lambda\, g_{\mu\nu}) = 8\pi G\, T_{\mu\nu}$ and $T^{\mu}{}_{\mu} = 0$, de Sitter spacetime is therefore classically stable (Lifshitz 46).
2. 1975, Grishchuk; Gravitational waves regarded as small corrections oscillating in spacetime and are completely analogous to EM waves.
3. 1984, Ford; Possibility of breaking de Sitter (time translation) invariant to get time depend cosmological constant. Observed that massless, minimally coupled scalar field in de Sitter spacetime suffers from IR divergence.
   $\Box\phi = 0,\ ds^2 = dt^2 - a^2(t)dx^2$
   with $a(t) = \exp^{Ht}$, then $< \phi^2 >= \frac{H^3}{4\pi^3} t$
   First piece of work to discuss the possibility that pure quantum gravity may lead to instability of de Sitter spacetime, however, with a negative result up to second order.

4. Gravitons in de Sitter spacetime; the metric perturbations in TT gauge satisfy minimally coupled scalar wave equation,
$\Box h_{\mu}^{\nu} = 0$
Gravitons are therefore, equivalent to a pair of minimally coupled scalar fields.
As a result, de Sitter invariant vacuum(Bunch-Davies)are not applicable, one may obtain time dependence in
$< h^{\mu\nu} h_{\mu\nu} >= \frac{H^3}{2\pi^3} t$.
And this will certainly back react to induce a time dependence in cosmological constant.
After all, gauge invariant quantities like; $<^{(2)} R_{\mu\nu} >$ doesn't vary with time in the correct vacuum!

5. Recent, disputes between groups about time dependence in de Sitter spacetime:
   - Instability; Yes, Woodard et al Annals of Phys. 321(2006)875, PRD 78(2008)
     Dimensional Regularization obtained a finite constant 1PI 1-point function $\Rightarrow$ small positive renormalization (screening) of the cosmological constant at two loop level
   - Instability; No, J. Garriga and T. Tanaka, PRD 77(2008) 024021
     Tadpole correction is not gauge invariant, need to construct gauge invariant composite operators which measure the renormalized curvature scalar and find no secular(linear in time) screening.

6. E. Witten, Three D Gravity Reconsidered (arXiv: 0706.3359)
   Only negative cosmological constant with black hole solutions are self-consistent in 3d, rely on AdS/CFT, and a positive cosmological constant is inconsistent at ALL dimensions.
7. A. Polyakov, de Sitter Space and Eternity (arXiv: 0709.2899)
   Infrared effects (fluctuations of metric $g_{\mu}\nu$ near zero not near a classical background)are likely to screen away the cosmological constant. In time-dependent picture, the screening is equivalent to instability of spacetime with constant curvature. Initially present curvature is therefore gradually decaying. "de Sitter spacetime carries the IR seeds of it own destruction.

Important Results:

1. The inflationary scenario is remarkably successful, and most of the models are built on the de Sitter spacetime.
2. Recent observations support the accelerating universe, probably driven by a small nonvanishing cosmological constant.
3. The de Sitter spacetime is a key role in understanding both the early universe and its fate.

Important Questions:

1. Effects of the quantum backreaction from the matter field on the de Sitter geometry?
2. the mechanism of the possible screening of the cosmological constant due to the quantum effects? That is, any hint to the fine-tuning problem?
3. Any instability of the de Sitter spacetime due to quantum effects?
4. Duration of the de Sitter phase and how it exits?

Main Theme of this work:

Quantum backreaction from the matter field on the de Sitter geometry

1. When the quantum matter field is involved,the expectation value of the renormalized stress tensor thus distorts the spacetime.
2. Its value contains both geometric and state-dependent contributions.
3. We focus on the tensor perturbations caused by the conformal vacuum fluctuations of a conformally invariant matter field.
4. We compute the one-loop correction on the propagation of finite wavelength gravity waves, and found a correction depends on the interval over which the interaction with the quantum matter field is switch on, or equivalently, the duration of inflation.

Semiclassical Einstein's equation

When the quantum matter field is involved,

1. What does the corresponding Einstein's equation look like?

$$G_{\mu\nu} \stackrel{?}{=} 8\pi G \left\langle T_{\mu\nu} \right\rangle .$$

2. It can not be true because $\left\langle T_{\mu\nu} \right\rangle$ formally diverges.
3. Regularization & renormalization at one loop introduce new terms

$$G_{\mu\nu} + \lambda_1 \mathcal{K}_{\mu\nu} + \lambda_2 \mathcal{A}_{\mu\nu} = 8\pi G \left\langle T_{\mu\nu} \right\rangle_{ren} ,$$

Where

$$
\begin{aligned}
\mathcal{K}_{\mu\nu} &\equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\int dx^4\,\sqrt{-g}\,R^2 \\
&= -2R_{\mu\nu}R + \frac{1}{2}g_{\mu\nu}R^2 + 2R_{;\,\mu\nu} - 2g_{\mu\nu}\Box R\,, \\
\mathcal{A}_{\mu\nu} &\equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\int dx^4\,\sqrt{-g}\,C_{\rho\sigma\alpha\beta}C^{\rho\sigma\alpha\beta} \\
&= 4\nabla_\alpha\nabla_\nu R^\alpha_\mu - 2\nabla_\rho\nabla^\rho R_{\mu\nu} - g_{\mu\nu}\nabla_\rho\nabla^\rho R - g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} \\
&\qquad + 4R^\rho_\mu R_{\rho\nu} - \frac{2}{3}\mathcal{K}_{\mu\nu}\,.
\end{aligned}
$$

When the quantum matter field is involved,

1. The expectation value of the renormalized stress tensor thus distorts the spacetime,
2. Its value contains both geometric and state-dependent contributions.
3. Extra terms are hints that perturbative gravity is unrenormalizable.
4. They contains 4th-order derivatives of the metrics; thus possible inducing runaway.
5. Since it is not seen in observations, usually their coefficients are set zero.

Trace anomaly

At the classical level, $T^{\mu}_{\mu} = 0$ for a conformally invariant field, but no longer holds for its quantum counterpart:

1. The anomalous trace for a free field is a state-independent, local geometric quantity.
2. For our case, the unambiguous part of the anomalous trace arises from a geometrical term of the form,

$$\mathcal{B}_{\mu\nu} = \frac{1}{2}\, g_{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} + \frac{2}{3}\, R_{\mu\nu} R - R_{\mu}{}^{\alpha} R_{\nu\alpha} - \frac{1}{4}\, g_{\mu\nu} R^2 \,.$$

and $T_{\mu\nu} = C\, \mathcal{B}_{\mu\nu}$.

Backreactions of quantum matter fields

In general, the quantum fluctuations of the matter field will perturb the background geometry, and this metric perturbation may in turn modify the stress tensor.

0th order effect

We first take a look at the 0th order effect:

1. since $\langle T^{(0)}_{\mu\nu}\rangle = C\mathcal{B}^{(0)}_{\mu\nu}$, the Einstein's equation becomes

$$R^{(0)}_{\mu\nu} - \Lambda_0\gamma_{\mu\nu} = 8\pi GC\left[\mathcal{B}^{(0)}_{\mu\nu} - \frac{1}{2}\gamma_{\mu\nu}\mathcal{B}^{(0)}\right].$$

2. since $R^{(0)}_{\mu\nu}$ and $\mathcal{B}^{(0)}_{\mu\nu}$ are all proportional to $\Lambda\gamma_{\mu\nu}$, it amounts to a shift of the cosmological constant,

$$\Lambda_0 \to \Lambda_0 + \frac{8\pi}{3}\ell_p^2\Lambda_0^2 C\,.$$

1st order effect

To first order in the perturbation $h_{\mu\nu}$, Horowitz and Wald show

$$\langle T^{(1)}_{\mu\nu} \rangle = \beta \mathcal{K}^{(1)}_{\mu\nu} + C\mathcal{B}^{(1)}_{\mu\nu} + P_{\mu\nu} + Q_{\mu\nu}\,,$$

where

$$P_{\mu\nu} = -16\pi\alpha\, a^{-2}\, \partial^{\rho}\partial^{\sigma} \left[\ln(a)\, \tilde{C}_{\mu\rho\nu\sigma} \right]\,,$$

with $\tilde{C}_{\mu\rho\nu\sigma}$ being the Weyl tensor. For perturbed Minkowski spacetime with the perturbation $\tilde{h}_{\mu\nu} = a^{-2} h_{\mu\nu}$, the non-local term $Q_{\mu\nu}$ given by

$$Q_{\mu\nu} = \alpha\, a^{-2} \int d^4x'\, H_{\lambda}(x - x')\, \tilde{\mathcal{A}}^{(1)}_{\mu\nu}\,.$$

The values of the two constants, C and α can be determined explicitly, and they are all positive.

field	C	α
conformal scalar	$1/(2880\,\pi^2)$	$1/(3840\,\pi^3)$
spin $\frac{1}{2}$	$11/(5760\,\pi^2)$	$1/(1280\,\pi^3)$
photon	$31/(1440\,\pi^2)$	$1/(320\,\pi^3)$

The remaining two constants, β and λ, are undetermined. A shift in either of these constants adds additional terms proportional to $\mathcal{K}_{\mu\nu}$ and $\mathcal{A}_{\mu\nu}$, respectively.

1. The local geometric tensors $\mathcal{K}_{\mu\nu}$ and $\mathcal{B}_{\mu\nu}$ produce no effects on the tensor perturbations other than finite shifts of the cosmological and Newton's constants,

$$\Lambda = \Lambda_0 + \frac{8\pi}{3} G_0 \, C \, \Lambda^2 \,,$$
$$G = \ell_p^2 = G_0 \, \left[1 + 64 G_0 \, \beta \, \Lambda - \frac{8\pi}{3} G_0 \, C \, \Lambda \right]^{-1} \,.$$

2. We may consider only the effects of the $P_{\mu\nu}$ and $Q_{\mu\nu}$ terms on the tensor perturbations,

$$\Box_s h_i^j = -16\pi \ell_p^2 \, (P_i^j + Q_i^j) \,.$$

Spatially homogeneous solutions

We look for a growing, spatially homogeneous solution, as these will be the most rapidly growing modes if there is an instability,

$$\tilde{h}_{ij} = a^{-2}\, h_{ij} = h_i^j = e_i^j\, \eta^{-b}\,,$$

where e_i^j is the fixed polarization tensor. The exponent b satisfies

$$b(b+3) = -\xi \Big\{ b(b+1)(b+2)(b+3) \Big[\gamma_\epsilon + \Psi(b) + \ln \frac{H\lambda}{2} \Big] + (b+2)(b+3)(2b+1) \Big\}\,,$$

with $\xi = 64\pi^2 \ell_p^2 H^2 \alpha$.

Forms of spatially homogeneous solutions

1. The homogeneous solutions in the absence of the quantum stress tensor ($\xi = 0$) are $b = 0$ and $b = -3$, which are both stable to the first order metric perturbation.
2. The only possibility for an unstable solution which is within the domain of validity of the semiclassical theory is one with a small positive value of b.
3. If we expand the previous eq. for $|b| \ll 1$, we find that no such solutions exist.
4. Hence, de Sitter spacetime is stable against tensor perturbations in the presence of the quantum stress tensor.

Effects on gravity waves

Study the effect of the quantum stress tensor on gravity waves in de Sitter spacetime by iteration.

Plane wave solutions of gravity waves $\Box_s h^j_i = 0$ are of the form,

$$h^{\nu}_{\mu} = c_0\, e^{\nu}_{\mu}\,(1 + ik\eta)\,\mathrm{e}^{i(\mathbf{k}\cdot\mathbf{x} - k\eta)}\,,$$

in the presence of the quantum stress tensor, the modified gravity wave may be expressed as $h^{\nu}_{\mu} + h'^{\nu}_{\mu}$, where

$$h'^{\nu}_{\mu}(x) = 16\pi\ell_p^2 \int d^4x'\,\sqrt{-g(x')}\,G_R(x,x')\,P^{\nu}_{\mu}\,.$$

Initial time, η_0 dependence

Let η_0 be the time when the interaction is turned one. In the limit $|\eta_0| \gg |\eta|$, the dominant contribution is proportional to $|\eta_0|$,

$$h'^{\nu}_{\mu}(x) \sim -64\pi^2\, i\, e^{\nu}_{\mu}\, c_0\, H^2 k\, \ell_p^2\, |\eta_0|\, (1 + ik\eta)\, \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{x} - k\eta)}\,,$$

which has the same functional form as does h^{ν}_{μ}, but is out of phase by $-\pi/2$ due to the factor of $-i$.

1. The correction term is larger the earlier the coupling between the quantum stress tensor and the metric perturbation is switched on.
2. But how large is it?
3. Check the ratio of the magnitude of the correction to that of the original wave,

Ratio of corrections

We find

$$\Gamma = \left| \frac{h'^{\nu}_{\mu}}{h^{\nu}_{\mu}} \right| = 64\pi^2 H^2 k \ell_p^2 |\eta_0| = 64\pi^2 H\, k_P\, \ell_p^2 \,,$$

here $k_P = k/a(\eta_0) = kH|\eta_0|$ is the physical wavenumber of the mode k as measured by a comoving observer at time $\eta = \eta_0$. If we require that

- The curvature of the de Sitter spacetime be well below the Planck scale, $H\,\ell_p \ll 1$, and
- The mode in question is always below the Planck scale, $k_P\,\ell_p \ll 1$.

Then the quantum correction to the gravity wave is smaller than the original wave.

Numerical estimation of tensor mode modification I

The modification of tensor perturbation should leave an imprint on the CMB power spectrum. The spectrum will by multiplied by a factor $1+\Gamma^2$. Under assumptions:

1. most of the vacuum energy that drives inflation is converted into radiation at reheating, $\Rightarrow\ H^2 = \frac{8\pi}{3}\ell_p^2 E_R^4$.
2. expansion by a factor of about $E_R/(1\,\mathrm{eV})$ between the end of inflation and last scattering.
3. a further expansion by a factor of 10^3 to the present.
4. $a_R = 1$ at reheating.
5. S is the factor by which the universe expands from the initial conformal time $\eta = \eta_0$ to the end of inflation.

Numerical estimation of tensor mode modification II

We end up with

$$\Gamma^2 = 1.34 \times 10^{-76} \left(\frac{10^{25}\,\mathrm{cm}}{\ell_0}\right)^2 \left(\frac{E_R}{10^{15}\,\mathrm{GeV}}\right)^6 S^2 \,,$$

where ℓ_0 is the proper length of the scale we are interested in, and $10^{25}\,\mathrm{cm} \simeq 1^\circ$ angular size today.

6. The minimal inflation needed to solve the horizon and flatness problems requires $S \approx 10^{23}$. then the one-loop correction is negligible.
7. but if $E_R \approx 10^{15}\mathrm{GeV}$ and $S \approx 10^{38}$ leads to an effect of order unity at 1° scales.
8. In contrast to the nearly flat spectrum due to free graviton fluctuations, the one-loop effect is highly tilted toward the blue end of the spectrum.

Summary

1. We found no growing, spatially homogeneous solutions in a spatially, flat universe, which implies that de Sitter spacetime is stable at the one-loop level in the presence of conformal matter.
2. We compute the one-loop correction on the propagation of finite wavelength gravity waves, and found a correction depends on the interval over which the interaction with the quantum matter field is switch on, or equivalently, the duration of inflation.

3. So long as the curvature of de Sitter spacetime and the initial proper frequency of the mode are below the Planck scale, the fractional correction is small.
4. The effect take the form of both a phase shift and an amplitude change, and can be absorbed in a complex amplitude shift.
5. The initial time (during of inflation) effect is in principle observable in that gravity wave modes are no longer exactly solutions of the Lifshitz equation $\Box_s h_i^j = 0$.
6. If inflation lasts for a sufficiently long time, then modes which are cosmological interest today appear to have been above the Planck scale at the onset of inflation. This is the cosmological version of the transplanckian problem, which also arises in black hole physics.