$\exists$ AdS/CFT

Anti-de Sitter space (QGR)
CFT conformal field theory
QFT with conformal symm.

$\}$ Correspondence.
↑
Evidence only
No proof (?)

Could be a revolutionary new principle.

What could be the new principle that explain this correspondence?

Holographic principle

$\exists$ two approaches to motivate the holographic principle

Historically, motivated by BH study

BH is a solution to GR

e.g. Schwarzschild soln:

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

Gravity

$\phi \rightarrow A_\mu \rightarrow g_{\mu\nu}$

$\times$ $\quad$ $\times$ EM $\quad$ equivalence principle

RG flow of QFT

metric

M = mass of BH

Vac. soln. to Einstein eqn. $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \leftarrow \frac{8\pi G T_{\mu\nu}}{c^4}$

$\cancel{T_{\mu\nu} = M \delta_{\mu 0} \delta_{\nu 0} \delta^{(3)}(\vec{x})}$ 　　Vac. $T_{\mu\nu} = 0$

M = ADM mass. ← ?

Normally, Energy = KE + P.E.

$$= \int (\partial_\mu \phi)^2 + V(\phi) \; d^3x$$

derivatives of field → KE

potential V → P.E.

In GR, Energy density $\stackrel{?}{=} (\partial g_{\mu\nu})^2 + V(g_{\mu\nu})$

Wrong!

$x^\mu \to x'^\mu$

Equivalence principle : $g_{\mu\nu} \to g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$

Riemann Normal Coord. $\begin{cases} g'_{\mu\nu}(P) = \eta_{\mu\nu} \\ \partial g'_{\mu\nu}(P) = 0 \end{cases}$

In any diffeomorphic inv. theory, there is no notion of local energy density.

ADM = Arnowitz-Deser-Misner

$$= \int_{\partial M} \text{asymptotic behaviour of metric.}$$

$\exists$ other definition of energy in GR

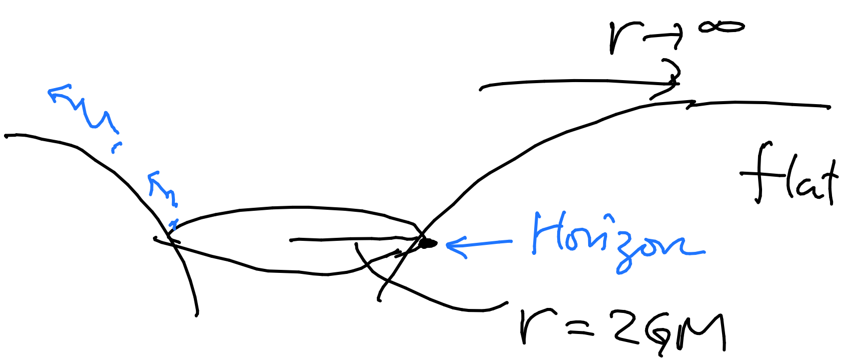
Bondi Mass for lightlike object.

BH solution has a horizon:

$g_{00} = 0$ when $r = 2GM/c^2$ $\quad c=1$

red shift is infinite.

$(ds^2)_{BH} \xrightarrow{r\to\infty}$ Minkowski metric. $\Rightarrow$ r = radial distance as measured by the asymptotic observer.



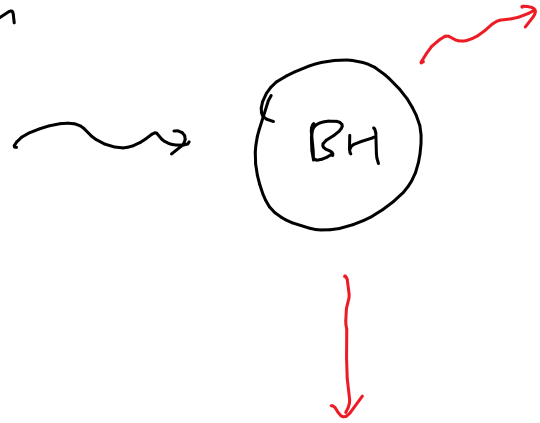
Horizon radius $R = 2GM$

∃ Hawking radiation

$$T = \frac{\hbar}{8\pi GM}$$

wave function

$\psi(x)$



BH Disappear + "blackbody radiation".

thermal.

Does not contain any quantum correlation

$$\rho = \text{density matrix}$$
$$= e^{-\beta E}$$

nonunitary transformation!

Something is wrong?

Information Loss Paradox.

Another problem is the BH entropy.

$$E = M = R/2G$$

$$A = 4\pi R^2$$

$$dA = 8\pi R\, dR$$

$$dE = \frac{dR}{2G} = \frac{1}{16\pi G R}\, dA$$

$$dE = \frac{1}{16\pi G R} dA = \frac{1}{32\pi G^2 M} dA = \underbrace{\frac{\hbar}{8\pi G M}}_{T} \cdot \frac{dA}{\hbar 4G}$$

1st law of thermodynamics

$$dE = T dS$$

$\Rightarrow S = \frac{A}{4G}$, $\exists$ entropy of BH $\propto A$

(70's) puzzle i. Area dependence

ii. what are we counting ?

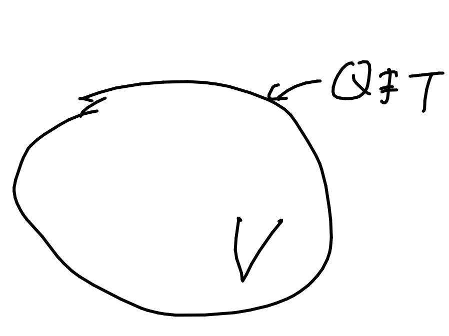
↑ Quantum theory of gravity ?

90's : t'Hooft & Susskind.

Any Quantum theory of gravity will satisfy a holographic principle:

QG in a Space V

$\equiv$ QFT lives on ∂V



97: Maldacena proposes AdS/CFT duality.

Second motivation to holography (from QFT):

One important property of QFT: Second quantization

Coupling get modified by Q. Corrections

In renormalizable QFT, Renormalizability

Anomalies.

$$\beta(g) \triangleq \mu \frac{\partial g}{\partial \mu}$$ ← RG flow equation

g = renomalized Coupling

μ = energy scale. ← origin of μ: $\exists$ UV div.

⇒ introduce UV cutoff Λ

$\Rightarrow g = g(\Lambda, p)$

$p^2 = -\mu^2$

$\tilde{g} = g(\Lambda, \mu^2)$

eliminate Λ,

$g = g(\mu, \tilde{g})$

eg $\beta = -c g^3$

$$-c(g^2)^2 = \frac{\partial g^2}{\partial \ln \mu} \frac{1}{2}$$

$$\frac{dg^2}{(g^2)^2} = -2c\, d(\ln\mu)$$

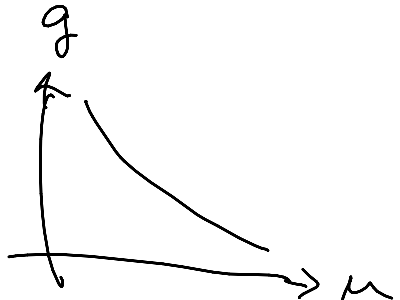
$$\parallel$$

$$-d\left(\frac{1}{g^2}\right)$$

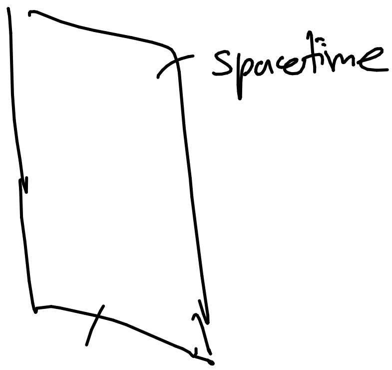
$$\frac{1}{g^2} - A = 2c \ln\mu$$

$$\mu = \mu_0, \quad \frac{1}{g^2(\mu_0)} = A - 2c \ln\mu_0$$

$$\frac{1}{g^2} - \frac{1}{g^2(\mu_0)} = 2c \ln\left(\frac{\mu}{\mu_0}\right)$$



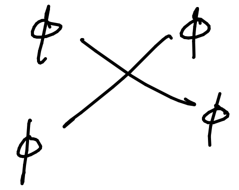
Classical FT



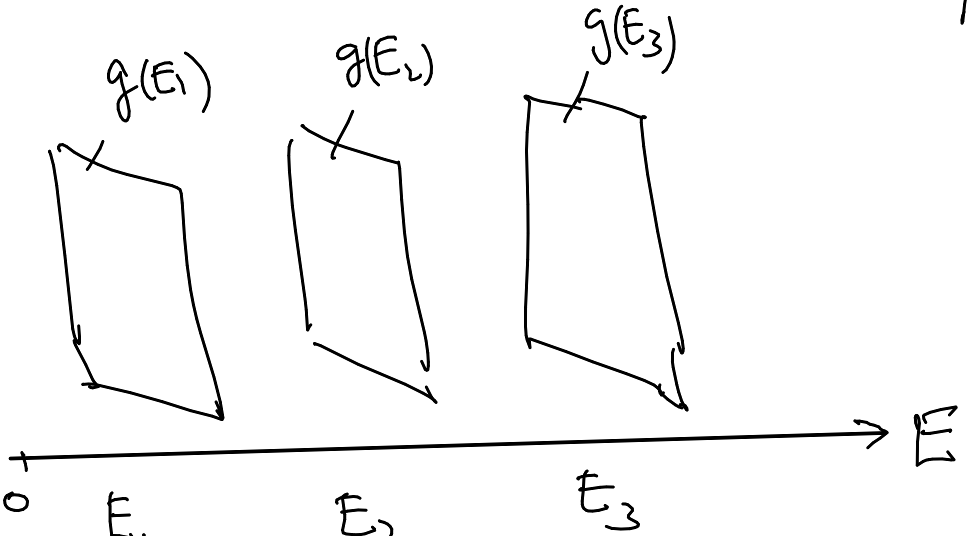
$g = \text{constant}$ for all E

work for all E

eg $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{g}{4!}\phi^4$



QFT



∞ Collection of theories related by RG flow

Each member in the Collection is specified by

$$(\vec{x}, t, E) \leftarrow 5d$$

Suggested a description in terms of a 5d manifold.

A certain theory of the 5d describes the Complete RG properties of the QFT.

Lets restrict to CFT.

Ask: What is the 5d. description?

What is a CFT?

What is a QFT that ∃ Lorentz inv.?

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

QFT is inv. under Lorentz transf.

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

$\uparrow$

$$S = \int d^4x \, \mathcal{L}$$

st $ds^2 = \eta_{\mu\nu} dx'^\mu dx'^\nu$ inv.

$\mathcal{L}$ is invariant.

$\mathcal{L}$ = fields — scalar field $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$

— ψ

— A_μ

CFT is a QFT that is inv. under Conformal transf.

$$ds^2 \to ds'^2 = f(x)\, ds^2$$

$f(x)$ = indep. of x (global)

A scaling of coord $x^\mu \to \lambda x^\mu$ λ = const.

is a conformal transf.

(Dilatation)

Conformal transformations form a group, Conformal group.

In 2d, Conformal group is ∞ dim. (Virasoro group)

In general dim d,

$$\text{Lorentz group} = SO(1, d-1)$$

$$\text{Conformal group} = SO(2, d)$$

$SO(n,m)$ = orthonormal transf. that keep

$$ds^2 = -dx_1^2 - \cdots - dx_n^2 + dy_1^2 + \cdots + dy_m^2$$

Conjecture for QFT 4d:

A QFT that is scale inv. + unitary

is also Conformal inv.