2018/9/17

Last time: $\mu \frac{\partial g}{\partial \mu} = \beta(g)$ g = Coupling constant

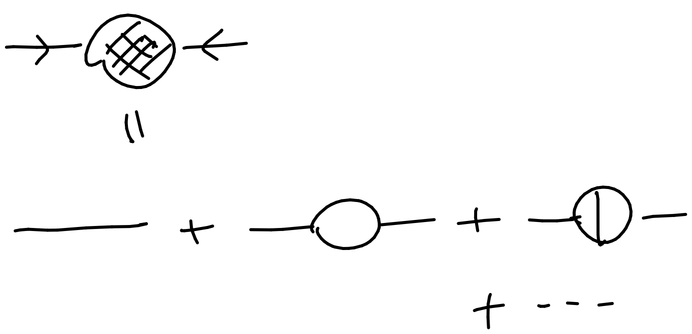
RG flow eqn.

Rmk: Why does Coupling run in QFT? change with energy scale

QFT is complicated.

pertubation theory

Feynman diagrams.



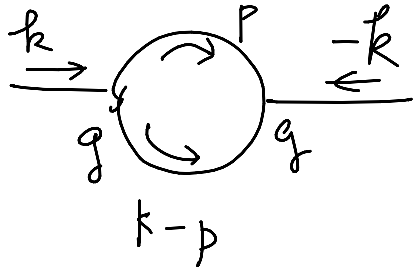
Loops expansions.

$\exists$ infinities asscocided with the loops.

loop expansion = expansion in $\int d^d p$

each internal propagator $\sim \frac{1}{p^2+m^2}$

e.g.



$d=4.$

$$\int d^4p \, \frac{1}{p^2+m^2} \, \frac{1}{(k-p)^2+m^2}$$

$|p| = \infty \to \Lambda$

$$= \int d^4p \, \frac{1}{p^4} + \cdots$$

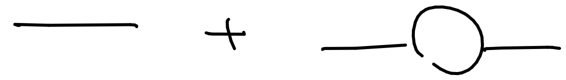
$$= \underbrace{\text{log. div.}}_{\log \Lambda} \times g_b^2$$

$V_{int} = \dfrac{g_b}{3!}\phi^3$

g_b = bare coupling = parameter in the $\mathcal{L}$

physical observable coupling = $g_{phys.}$

Full contribution up to 1 loop.

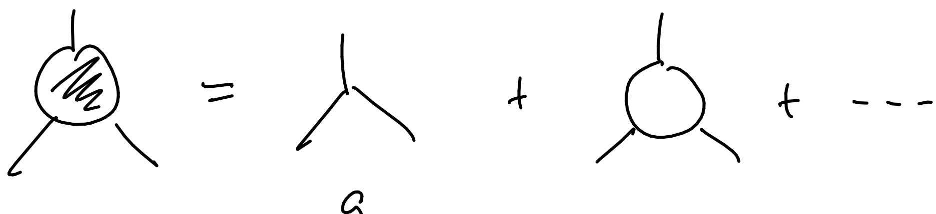
= 

tree level 1-loop.

Λ = cut off

$= A_0(k) + g_b^2 \log\Lambda \times A_1(k) + \cdots \qquad \int^\Lambda dp$

another div. is the coupling



g_b

In general, we can absorb these div.

with the help of renormalization :

$$\phi = Z_2 \phi_{phys}$$

$$g_. = Z_3 \, g_{phys}$$

A theory is renormalizable if one can absorb all divergence by redefinition of fields, couplings, masses. etc

As a result,

$$g = g(p, \Lambda)$$

or we can eliminate Λ;

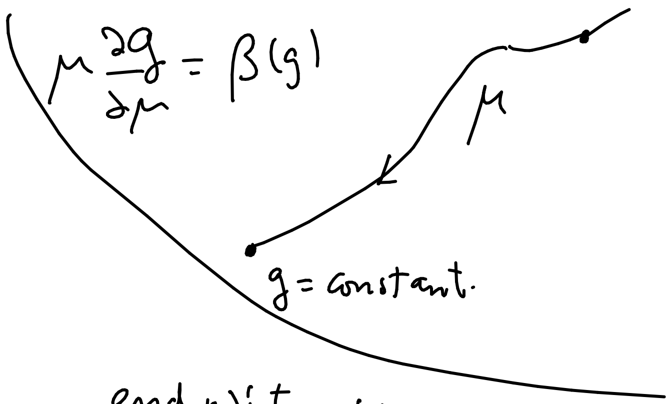
$$g_0 = g_0(p^2 = \mu^2, \Lambda) \Rightarrow \Lambda = \Lambda(g_0)$$

$$g = g(p, \Lambda) = g(p, \mu)$$

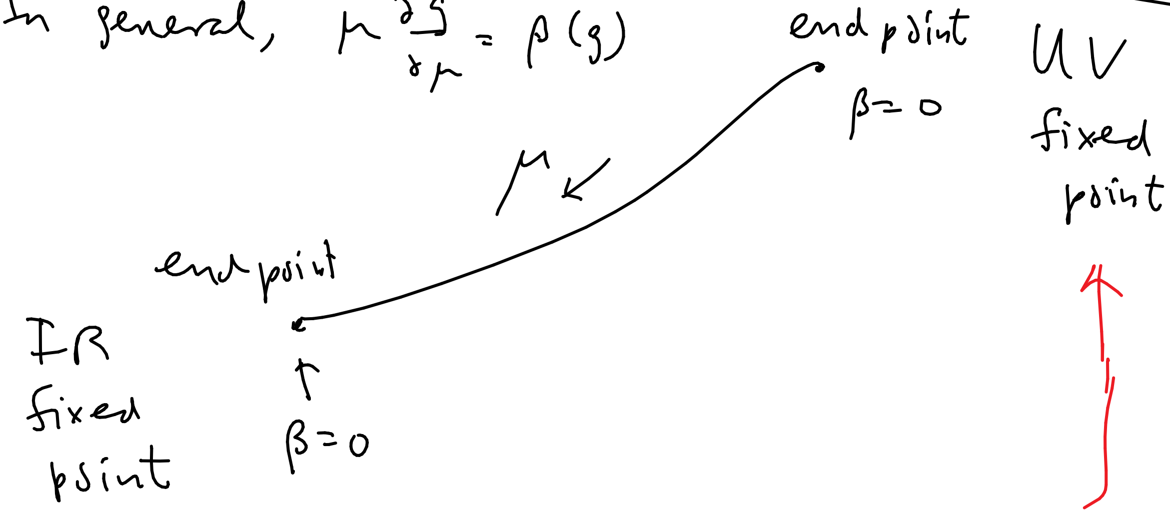
$$\boxed{\mu \frac{\partial}{\partial \mu} g = \beta(g) \qquad \text{RG flow eqn.}}$$

Interesting limit: $\beta = 0$

$\underline{g}$ = Constant.



In general, $\mu \frac{\partial g}{\partial \mu} = \beta(g)$



Scaling Symmetry $\Leftarrow \beta = 0$

$$\begin{cases} x \to \lambda x \\ \mu \to \frac{1}{\lambda}\mu \end{cases}$$

$\mu \frac{\partial}{\partial \mu} g(\mu) = \beta(g(\mu))$

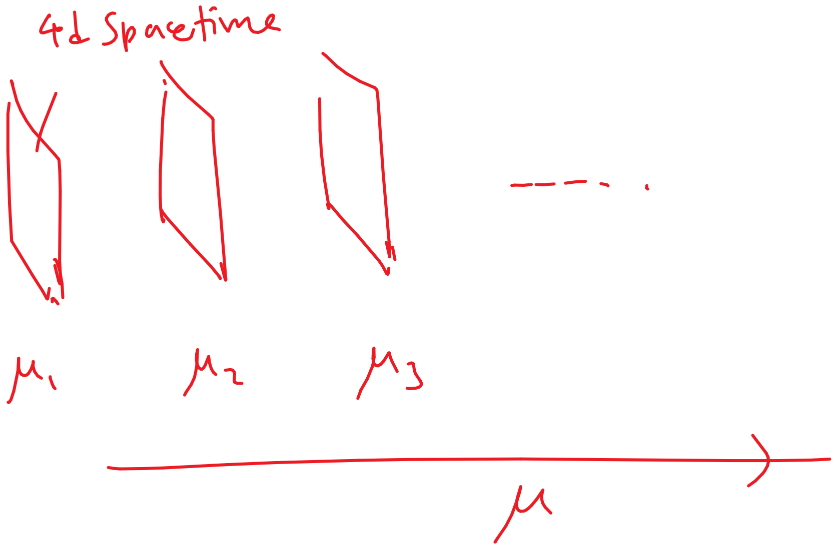
inv. unless $\beta = 0$

d = 4
Unitary

Conformal
Symm.

$x \to \tilde{x}$

$d\tilde{s}^2 = \lambda^2 ds^2$



family of QFT at different energies related by RG eqn.

Effectively, we have a 5d spacetime.

$$\shortparallel$$

(4d physical spacetime $\oplus$ μ)

Based on our previous experience with holographic principle (from BH), this suggests that

<u>RG flow of QFT_{4d}</u> may be described by a 5d gravitational metric.

(RG = GR)

Take this conjecture, we can ask:

what properties of the metric do we expect?

$$ds^2 = g_{MN}(x)\, dx^M dx^N \qquad x^M = (x^\mu, r)$$

x^μ: 4d, r: energy

$$= A(r)\eta_{\mu\nu}\, dx^\mu dx^\nu + B(r)\, dr^2$$

Consider the case $\beta = 0$:

We have scaling symm: $x \to \lambda x$, $r \to \frac{1}{\lambda} r$ in QFT

requiring same symm. in the metric:

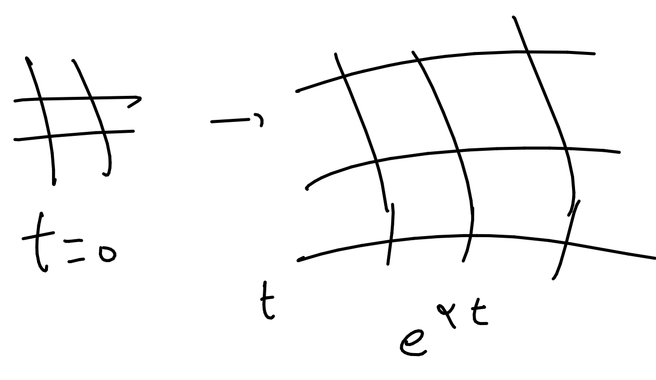
$$\Rightarrow \begin{cases} A = r^2 & : \quad r^2 dx^2 \to \frac{r^2}{\lambda^2} \cdot \lambda^2 dx^2 = r^2 dx^2 \\ B = \frac{1}{r^2} \end{cases}$$

$$ds^2 = L^2\left(r^2\, dx^2 + \frac{1}{r^2}\, dr^2\right)$$

AdS_5 metric   Anti-de Sitter metric.

de-Sitter

$$ds^2 = -dt^2 + e^{2\alpha t}\, d\vec{x}^2$$



or in terms of $z = \frac{1}{r}$

$$ds^2 = \frac{L^2}{z^2}\left(dx_\mu^2 + dz^2\right)$$

L= AdS length

$r = \infty$
$z = 0$
bdy of AdS

$z = \infty$
horizon

$$d\eta = e^{-\alpha t} dt$$

$$\eta = -\frac{1}{\alpha} e^{-\alpha t}$$

$$ds^2 = e^{2\alpha t}\left(-d\eta^2 + d\vec{x}^2\right)$$

$$= \frac{1}{\alpha^2 \eta^2}\left(-d\eta^2 + d\vec{x}^2\right)$$

AdS space is a soln to the Einstein eqn. with cosmological const. Λ

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 0$$

with $-2\Lambda = \frac{d(d+1)}{L^2}$

d = space dim.
$d+1$ = spacetime dim $\Big\}$ of QFT

$= \frac{12}{L^2}$ for QFT_{3+1} $\quad d+2$ = Gravity

Einstein eqn comes from the Hilbert Einstein action

$$S = \frac{1}{2G}\int d^{d+2}x \sqrt{-g}\,(R - 2\Lambda) + \cdots$$

matters, or higher derivative corrections to GR.

Action is necessary(?) for construction of the theory of quantum gravity.

From the newton constant, define Planck length:

$$\frac{\hbar G}{c^3} = \ell_p^{\,d}$$

Now, we have two length scales: ℓ_p + L

$$\frac{\ell_p}{L} = \text{dim. less} = \text{Control quantum gravity corrections}$$

$$\text{Quantum gravity} = \text{Classical Einstein gravity} + \text{Quantum Corrections} \sum_n \left(\frac{\ell_p}{L}\right)^n$$

Since we don't know how to compute these quantum corrections, it is better to consider the case where ℓ_p/L is small.

Currently, we study AdS/CFT with $L/\ell_p \gg 1$

Turns out L/ℓ_p measures the d.o.f in the QFT:

$$\boxed{\text{Small quantum gravity corrections} \Longleftrightarrow \text{Large no. of d.o.f in QFT.}}$$

AdS/CFT = Correspondence b/w Quantum gravity on AdS_5

and

gauge theory in 4d.

The gauge theory is $\mathcal{N}=4$ SYM with gauge group $\leftarrow \beta = 0$

$G = U(N)$ g_{YM} Constant.

$A_\mu = A_\mu^a T^a$

$X^I = X^{Ia} T^a$ $I = 1, \cdots, 6$

$\psi = \psi^a T^a$ 4 Weyl fermions

T^a = Lie alg. generator in adj. reps.

$a = 1, \cdots, N^2$

<u>gauge theory:</u>

ϕ complex scalar

$$\mathcal{L} = -\frac{1}{2}|(\partial\phi)|^2 + m^2|\phi|^2$$

$$\phi \to e^{i\alpha}\phi$$ α = const. global symm.

$$D_\mu = \partial_\mu + A_\mu$$

$$\mathcal{L} = -\frac{1}{2}|D_\mu\phi|^2 + m^2\phi^2$$

$$\partial_\mu(e^{i\alpha}\phi) = e^{i\alpha}\partial_\mu\phi + \underline{(\partial_\mu e^{i\alpha})\phi}.$$

$$A_\mu \to A_\mu - (\partial_\mu e^{i\alpha})e^{-i\alpha}$$

$$D_\mu\phi \to e^{i\alpha}(D_\mu\phi)$$

To have small QG corrections, need large N.

$$\boxed{\frac{L^4}{l_p^4} = 4\pi g_{YM}^2 N} \quad — \circledast$$

$\lambda = g_{YM}^2 N$ = large.

't Hooft coupling.

used for expansion in terms of planarity of Feynman diagrams:

need A_μ to be dynamical

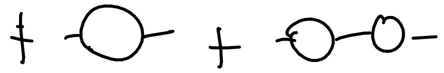
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$U(1)$ gauge theory = EM

gauge principle : promote global rotation to local symm.

has been applied to construction of $SU(2)$, $SU(3)$ YM theory

Feynman diagrams / loop expansions ← + 

Feynman diagrams \ large N expansion ← planarity of diagrams.

$\circledast$ means: strongly coupled gauge theory

is described by (dual)

classical gravity (with small QG corrections)

strong - weak duality.

- physical questions:

Suppose this is all true, how can we see it in expt.?

What is the observational statement?

Start with an observable in QFT side, how is it described in the gravity side?

Map / Dictionary?

CFT: Conformal operators / scalar operators.

$$\mathcal{O}(x) \to \mathcal{O}(x) \qquad \text{Lorentz transf.}$$

$$\mathcal{O}(x) \to \lambda^{-\Delta} \mathcal{O}(x) \qquad \text{scaling } x \to \lambda x$$

dim. Δ

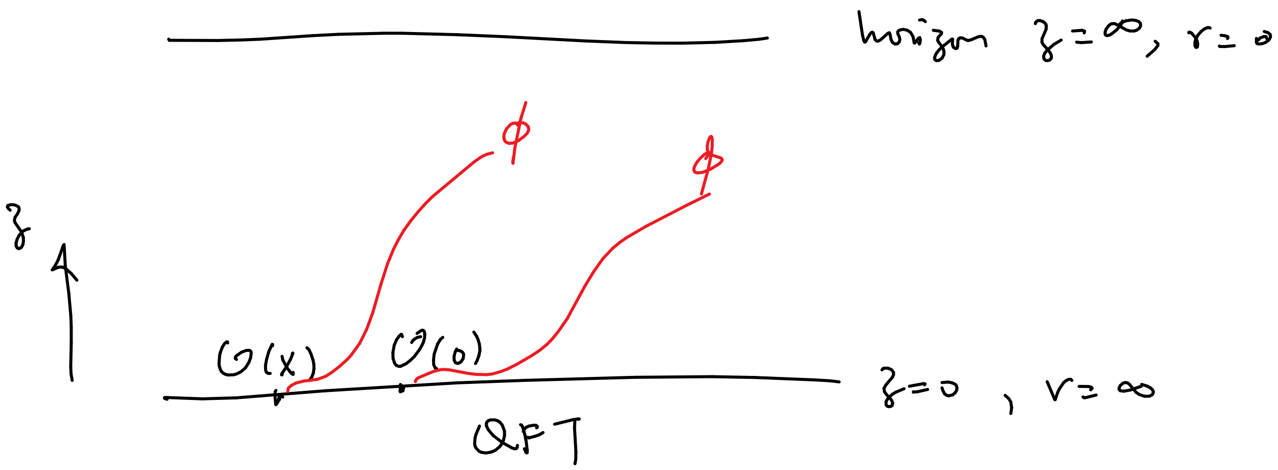
Conformal symm $\Rightarrow \langle \mathcal{O}(x)\, \mathcal{O}(0) \rangle = \dfrac{c}{|x|^{2\Delta}}$

In mom. space this becomes

$$\langle \mathcal{O}(\vec{k},\omega), \mathcal{O}(0) \rangle_R \sim k^{2\Delta - d - 1}$$

$$k^2 = \vec{k}^2 - \omega^2/c^2$$

Q. How is this encoded in gravity?



Naturally, operator $\mathcal{O}$ couples to scalar field ϕ in AdS_5

$$S = \int d^{d+2}x \sqrt{-g} \left(-\frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - m^2 \phi^2 \right)$$

EOM: $$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g}\, g^{MN} \partial_N \phi) - m^2 \phi = 0$$

metric is AdS_5 metric.

$$\phi(x^\mu, r) = \int \frac{d\omega\, d^d k}{(2\pi)^{d+1}} f_k(r)\, e^{ik_\mu x^\mu}$$

get $$\left[\frac{\partial^2}{\partial r^2} + \frac{d+2}{r} \frac{\partial}{\partial r} - \left(\frac{k^2 L^2}{r^4} + \frac{m^2 L^2}{r^2} \right) \right] f_k(r) = 0$$

Consider large r regime, we can try soln. of the form

$$f_k(r) = r^\alpha$$

$$\alpha^2 + (d+1)\alpha - m^2 L^2 = 0$$

$$\Rightarrow \quad \alpha = -\Delta \,, \quad \Delta - (d+1)$$

Where $\Delta = \frac{d+1}{2} + \sqrt{\left(\frac{d+1}{2}\right)^2 + m^2 L^2}$

$$f_k(r) = r^{\Delta-(d+1)}\left(A(k) + O\left(\tfrac{1}{r}\right)\right) + r^{-\Delta}\left[B(k) + O\left(\tfrac{1}{r}\right)\right]$$

Comment: 1° Δ real

$$m^2 L^2 \geq -\frac{(d+1)^2}{4}$$

m^2 can be negative & still make sense.

This lower bound is known as

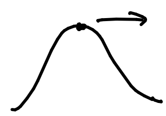
Breitenlohner-Freedman bound.

SUGRA: this bound gurantee stable vac.

If violated, Δ is imaginary, no good for CFT

it corresponds to tachyonic mode

in gravity.



2.° $f_k(r) \sim$ fun. of $k/r \triangleq \xi$.

$$f_k(r) = r^{\Delta-(d+1)}\left(A(k) + O(\tfrac{1}{r})\right) + r^{-\Delta}\left[B(k) + O(\tfrac{1}{r})\right]$$

$$A(k) = \frac{A_0 \leftarrow \text{indep. of } k}{k^{\Delta-d-1}} \quad , \quad B(k) = \frac{B_0}{k^{-\Delta}}$$

$$\underbrace{\frac{B(k)}{A(k)} = \frac{B_0}{A_0} \cdot k^{2\Delta-d-1}}_{\text{Gravity}} = \underbrace{\langle \mathcal{O}(\vec{k},\omega)\, \mathcal{O}(0) \rangle_R}_{\text{CFT}}$$

This is one way how CFT is mapped to gravity!

Q. What about higher point correlation functions?

Q What about non-scalar type of operators?

Q. How are gravity observables mapped to CFT?

⋮