0. Overview

Theory of gravity is complicated!

Let's recall what we know about EM:

EM, static: Coulomb force

$$\vec{F} = -\nabla\phi$$

$Q_1 \qquad Q_2$

$$\phi_{EM} \sim \frac{1}{r}$$

time dependent: Maxwell eqn.

$$\phi \rightarrow \underbrace{(\phi, \vec{A})}_{} = A_\mu$$

($\vec{A}$ ← new physics)

Gravity static: Newton

$$\vec{F} = -\nabla\phi$$

$m_1 \qquad m_2$

$$\phi_{grav.} \sim \frac{1}{r}$$

time dep.

? $A_\mu^{(grav)}$? wrong!

right theory is GR:

$$g_{\mu\nu} = (\underset{\substack{\downarrow \\ \phi}}{g_{00}},\ \underbrace{g_{0i},\ g_{ij}}_{\text{new physics}})$$

$$\mu = 0, \underbrace{1, 2, 3}_{i}$$

$g_{\mu\nu}(\vec{x}, t)$ is called the metric field

- Mathematically, $g_{\mu\nu}$ describes the <u>geometry</u> of <u>spacetime</u>. How?

↑ differential geometry
Mathematics

↑ will do this in this course

- Physically, we like to know how does physics determines $g_{\mu\nu}$?

This is through the Einstein eqn.

will learn about why this is true.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} \qquad \text{vs.} \qquad \partial^\mu F_{\mu\nu} = J_\nu$$

$R_{\mu\nu}$ = some nonlinear functions of $\partial^2 g_{\mu\nu}$

Crucial difference

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

2nd order, linear PDE

Next term, applications of GA

Given a theory we would like to know what does it describe i.e. look for soln. (new) and to study their properties

	GR.		EM
Soln:	1° point mass	Newton ↓ modified (Schwarzschild) vs.	point charge.
	2° wave (gravitational)		EM wave

1° • point charge is described by $\phi \sim \frac{1}{r}$ which has a singularity. But the singularity is not so bad as we could replace the point charge by a smooth distribution and the potential becomes regular. i.e. singularity is due to our idealization using a point description.

This is not the same in GR. Powerful mathematical theorem due to Penrose + Hawking (Singularity theorem) states that

as long as some energy condition for matters are satisfied, one cannot avoid the formation of singularity.

reasonable

Therefore, GR predicts its own faliure.

GR is not a complete theory as GR cannot tell as what happens at a singularity!

$\Rightarrow$ ~~GR~~ Einstein eqn. need to be modified! How?

Key research problem
needs Quantum gravity?

- Blackhole :

The metric of a point mass is described by

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)dr^2 + r^2 d\Omega^2 \qquad (c=1)$$

Apart from the singularity at $r=0$, the metric describes a spacetime where no even light can escape once it reaches the region

$$r \leq r_s \qquad r_s = 2GM$$

Therefore, it appears to the outside a region of radius r_s completely black. This is called a blackhole.

Thermodynamics : $M = \frac{1}{2G} r_s$

$$\Rightarrow A = 16\pi G^2 M^2$$

$$dA = 32\pi G^2 M\, dM$$

$$dM = \frac{dA}{32\pi G^2 M} = \frac{\kappa}{8\pi G^2 M} d\left(\frac{A}{4\kappa}\right)$$

κ = arb constant of area dimension

If compared to thermodynamics, $dE = TdS$, this suggests $\begin{cases} T = \frac{\kappa}{8\pi G^2 M} \\ S = A/4\kappa \end{cases}$

classically, there is no reason to expect a non-zero temperature for the BH as a nonzero temperature would mean that there is a thermal radiation. But classically ~~the~~ nothing can escape from the BH !

Hawking found that (QFT in curved spacetime) pair creation process around the BH could lead to a nonzero temperature (black body radiation)

Hawking found : $T_{BH} \propto \frac{1}{M}$, proportional constant $\sim \hbar$!

More precisely, he found $\kappa \propto l_p^2$ $\qquad l_p = \sqrt{\frac{G\hbar}{c^3}}$ = Planck's length .

The relation $S = \frac{A}{4}$ is strange as for $\qquad$ ($l_p = 1$ unit)

usual thermodynamical system, entropy is extensive

and so $S \propto V$.

The area nature of blackhole entropy has a profound implication!

't Hooft suggested that Quantum gravity is holographic in nature:

The gravity physics in a region V is described by the degrees of freedom residing on the boundary ∂V of V.

Thus it is natural that BH has an entropy proportional to the area.

2° Gravitional wave is predicted by GR but has not been observed since it is very weak. LIGO