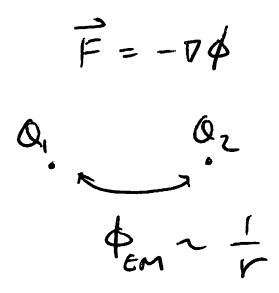


0. Overview

Theory of gravity is complicated!

Let's recall what we know about EM:

EM, static: Coulomb force

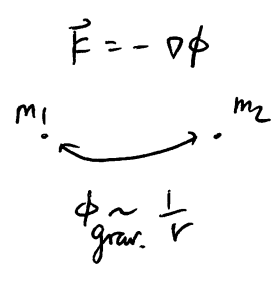


time dependent: Maxwell eqn.

$\phi \rightarrow (\phi, \vec{A}) = A_\mu$

↑ new physics

Gravity static: Newton



time dep.

? $A_\mu^{(grav)}$? Wrong!

right theory is GR:

$g_{\mu\nu} = (g_{00}, \underbrace{g_{0i}, g_{ij}}_{\text{new physics}})$

↓
 ϕ

$\mu = 0, 1, 2, 3$
i

$g_{\mu\nu}(\vec{x}, t)$ is called the metric field

Mathematically, $g_{\mu\nu}$ describes the geometry of spacetime. How?

↑
differential geometry
Mathematics

↑ will do this in this
course

• Physically, we like to know how does physics determines $\{g_{\mu\nu}\}$?

This is through the Einstein eqn.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

will learn about why this is true.

vs. $\partial^\mu F_{\mu\nu} = J_\nu$

$R_{\mu\nu}$ = some nonlinear functions of $\partial^2 g_{\mu\nu}$

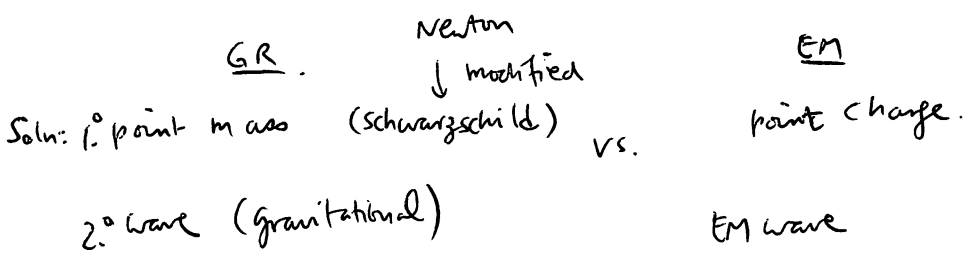
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

2nd order, (linear) PDE

→
Crucial difference

Next term, Applications of GR

Given a theory we would like to know what does it describe i.e. look for soln. (new) and to study their properties



1^o • point charge is described by $\phi \sim \frac{1}{r}$ which has a singularity. But the singularity is not so bad as we could replace the point charge by a smooth distribution and the potential becomes regular. i.e. singularity is due to our idealization using a point description.

This is not the same in GR. Powerful mathematical theorem due to (Singularity theorem)

Penrose + Hawking states that

as long as some energy condition for matters are satisfied, one cannot avoid the formation of singularity. ↖ reasonable

Therefore, GR predicts its own failure.

GR is not a complete theory as GR cannot tell us what happens at a singularity!

⇒ ~~GR~~ need to be modified! How?
Einstein eq. ← key research problem
needs quantum gravity?

• Blackhole:

The metric of a point mass is described by

$$ds^2 = - \left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right) dr^2 + r^2 d\Omega^2 \quad (c=1)$$

Apart from the singularity at $r=0$, the metric describes a spacetime where no even light can escape once it reaches the region

$$r \leq r_s \quad r_s = 2GM$$

Therefore, it appears to the outside a region of radius r_s . This is completely black

called a blackhole.

Thermodynamics: $M = \frac{1}{2G} r_s$

$$\Rightarrow A = 16\pi G^2 M^2$$

$$dA = 32\pi G^2 M dM$$

$$dM = \frac{dA}{32\pi G^2 M} = \frac{\kappa}{8\pi G^2 M} d\left(\frac{A}{4\kappa}\right)$$

$\kappa = \text{arb constant of area dimensions}$

If compared to thermodynamics, this suggests

$$\begin{cases} T = \frac{\kappa}{8\pi G^2 M} \\ dE = T dS \\ S = \frac{A}{4\kappa} \end{cases}$$

0.4.
classically, there is no reason to expect a non-zero temperature for the BH as a non-zero temperature would mean that there is a thermal radiation. But classically ~~the~~ nothing can escape from the BH!

Hawking found that (QFT in curved spacetime) pair creation process around the BH could lead to a non-zero temperature (black body radiation)

Hawking found $T_{BH} \propto \frac{1}{M}$, proportional constant $\sim \hbar$!

More precisely, he found $k \propto l_p^2$ $l_p = \sqrt{\frac{\hbar G}{c^3}}$ = Planck's length.

The relation $S = \frac{A}{4}$ is strange as for $(l_p = 1 \text{ unit})$

usual thermodynamical system, entropy is extensive

and so $S \propto V$.

The area nature of blackhole entropy has a profound implication!

't Hooft suggested that Quantum gravity is holographic in nature:

The gravity physics in a region V is described by the degrees of freedom residing on the boundary ∂V of V .

Thus it is natural that BH has an entropy proportional to the area.

2° Gravitational wave is predicted by GR but has not been observed since it is very weak. LIGO