A_5 as the symmetry of four generation leptons

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arXiv:1011.3199

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2010/11/23

Outline

- Introduction
- Non-abelian discrete symmetries A_4 and A_5
- The model
- Results
- Conclusion

Introduction

- It is now generally accepted that the SM consists of three fermion families, however, the number of generations is not fixed by the theory.

- Revival of interest in 4th generation:

  - electroweak precision fit does not exclude 4th generation

  - one of the simplest kinds of new physics is a sequential replication of the three generations of chiral matter

  - LHC has the potential to discover or fully exclude SM4

Current bounds

$m_{t'} > 256$ GeV; $m_{b'} > 128$ GeV (CC decay; 199 GeV for 100% NC decay);

$m_{\tau'} > 100.8$ GeV; $m_{\nu'_\tau} > 90.3$ GeV (Dirac coupling; 80.5 GeV for Majorana coupling)

- CKM unitarity : current measurement errors of the CKM quark mixing matrix elements leave room for the possible extension from 3×3 to 4×4

- The first row : $|V_{ub'}|^2 = 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 \simeq 0.0008 \pm 0.0011$

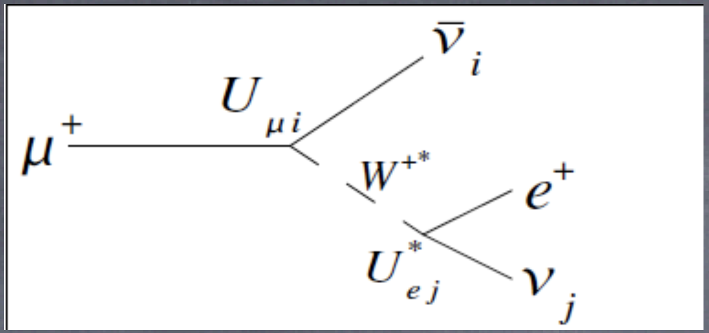
- The second row : $|V_{cb'}|^2 = 1 - |V_{cd}|^2 - |V_{cs}|^2 - |V_{cb}|^2 \simeq -0.003 \pm 0.027$

- The first column : $|V_{t'd}|^2 = 1 - |V_{ud}|^2 - |V_{cd}|^2 - |V_{td}|^2 \simeq -0.001 \pm 0.005$

overall $|V_{ub'}|,\ |V_{t'd}|,\ |V_{cb'}| \lesssim 0.04$ $1\ \sigma$ T. Tait et al. 2007

For comparison $|V_{ub}| \sim 0.004$ and $|V_{cb}| \simeq 0.04$

- PMNS mixings :



H.Lacker and A.Menzel 2010

$$\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu(\gamma)) \propto G_F^2 \sum_{i=1,2,3} |U_{\mu i}|^2 \sum_{k=1,2,3} |U_{ek}|^2 = G_F^2(1-|U_{\mu 4}|^2)(1-|U_{e4}|^2)$$

$$\Gamma(\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau(\gamma)) \propto G_F^2 \sum_{i=1,2,3} |U_{\tau i}|^2 \sum_{k=1,2,3} |U_{lk}|^2 = G_F^2(1-|U_{\tau 4}|^2)(1-|U_{l4}|^2) \quad l=e/\mu$$

- Unitarity violation possible, but only observable if G_F can be extracted independently $\rightarrow$ test universality violation

- Perfect universality means

$$\frac{1-|U_{\mu 4}|^2}{1-|U_{e4}|^2}=1 \longrightarrow |U_{e4}|=|U_{\mu 4}|=|U_{\tau 4}| \longrightarrow |U_{e4}|^2+|U_{\mu 4}|^2+|U_{\tau 4}|^2=3|U_{e4}|^2\le 1 \longrightarrow |U_{e4}|\le\frac{1}{\sqrt{3}}$$

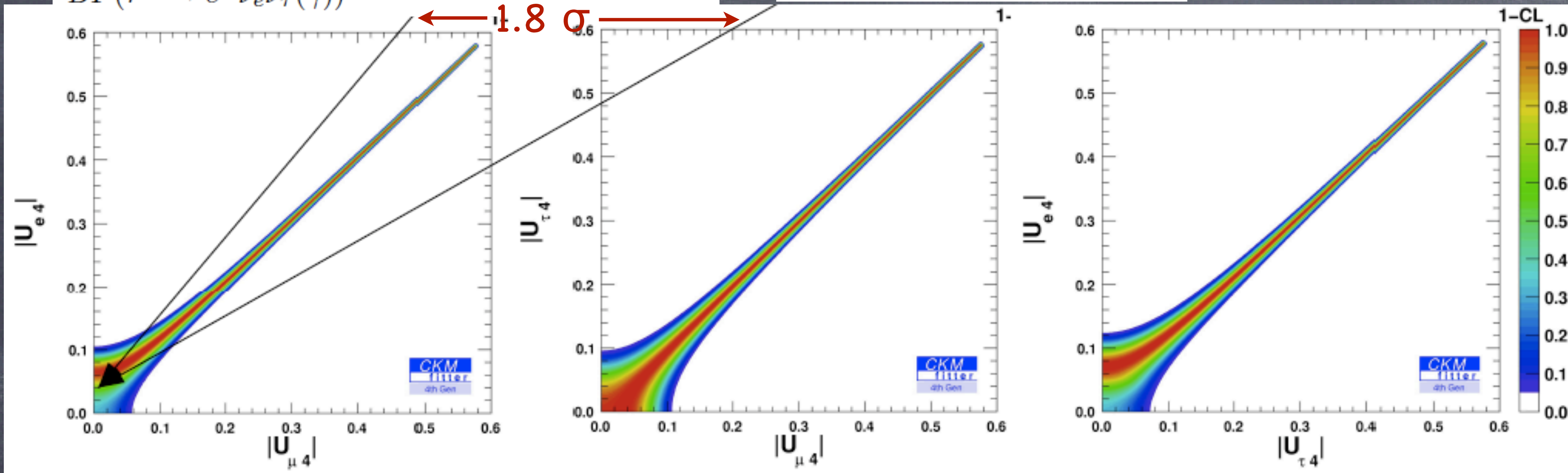
$\longrightarrow$ G_F could be up to 50% larger than 3SM value

H.Lacker and A.Menzel 2010

$$\frac{BF(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{BF(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))} = 0.9796 \pm 0.0016 \pm 0.0036$$

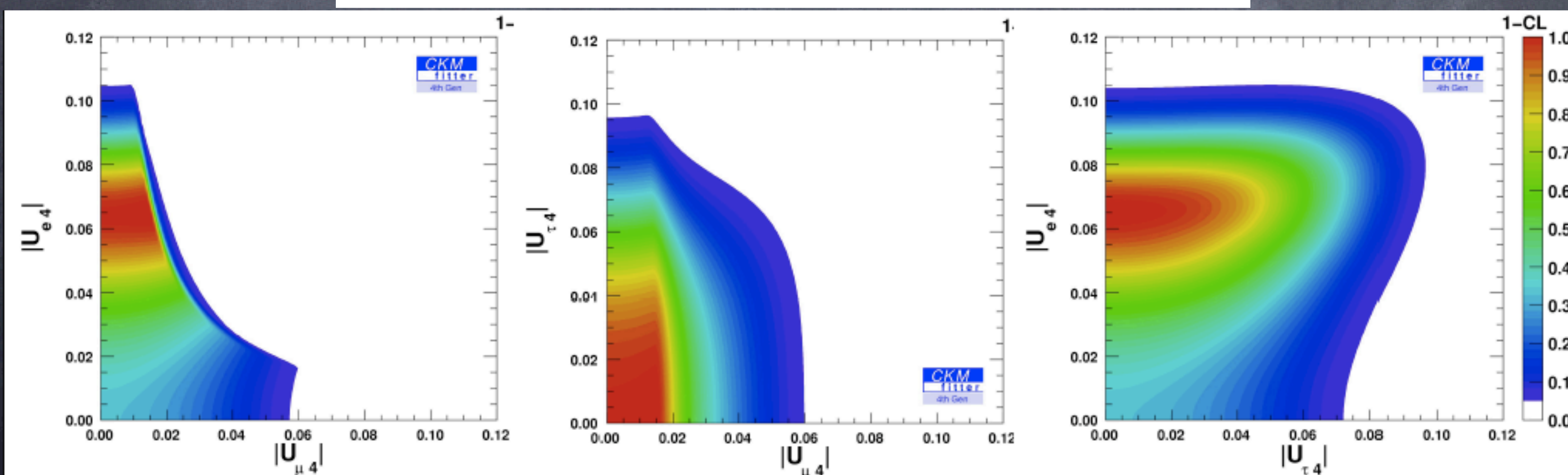
$$\frac{BF(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{BF(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))}_{pred.} = 0.9726$$

1.8 σ



Radiative decays

$$\Gamma(\mu^- \to e^- \gamma) = \frac{3G_F^2 m_\mu^5}{192\pi^3}\frac{\alpha(m_\mu)m_{\nu_4}^4}{32\pi m_W^4}|U_{\mu 4}U_{e4}^*|^2\{I^{(2)}(\frac{m_{\nu_4}^2}{m_W^2}) - I^{(3)}(\frac{m_{\nu_4}^2}{m_W^2})\}$$



$BF(\mu^- \to e^- \gamma) < 1.2\cdot 10^{-11}$ $BF(\tau^- \to e^- \gamma) < 3.3\cdot 10^{-8}$ $BF(\tau^- \to \mu^- \gamma) < 4.4\cdot 10^{-8}$

- Plus semileptonic K decays and charged pion decays

$$R_K = BF(K^+ \to e^+\nu_e)/BF(K^+ \to \mu^+\nu_\mu)$$

$$R_\pi = \frac{BF(\pi^+ \to e^+\nu_e)}{BF(\pi^+ \to \mu^+\nu_e)}$$

to constrain $(1 - |U_{e4}|^2)/(1 - |U_{\mu4}|^2)$

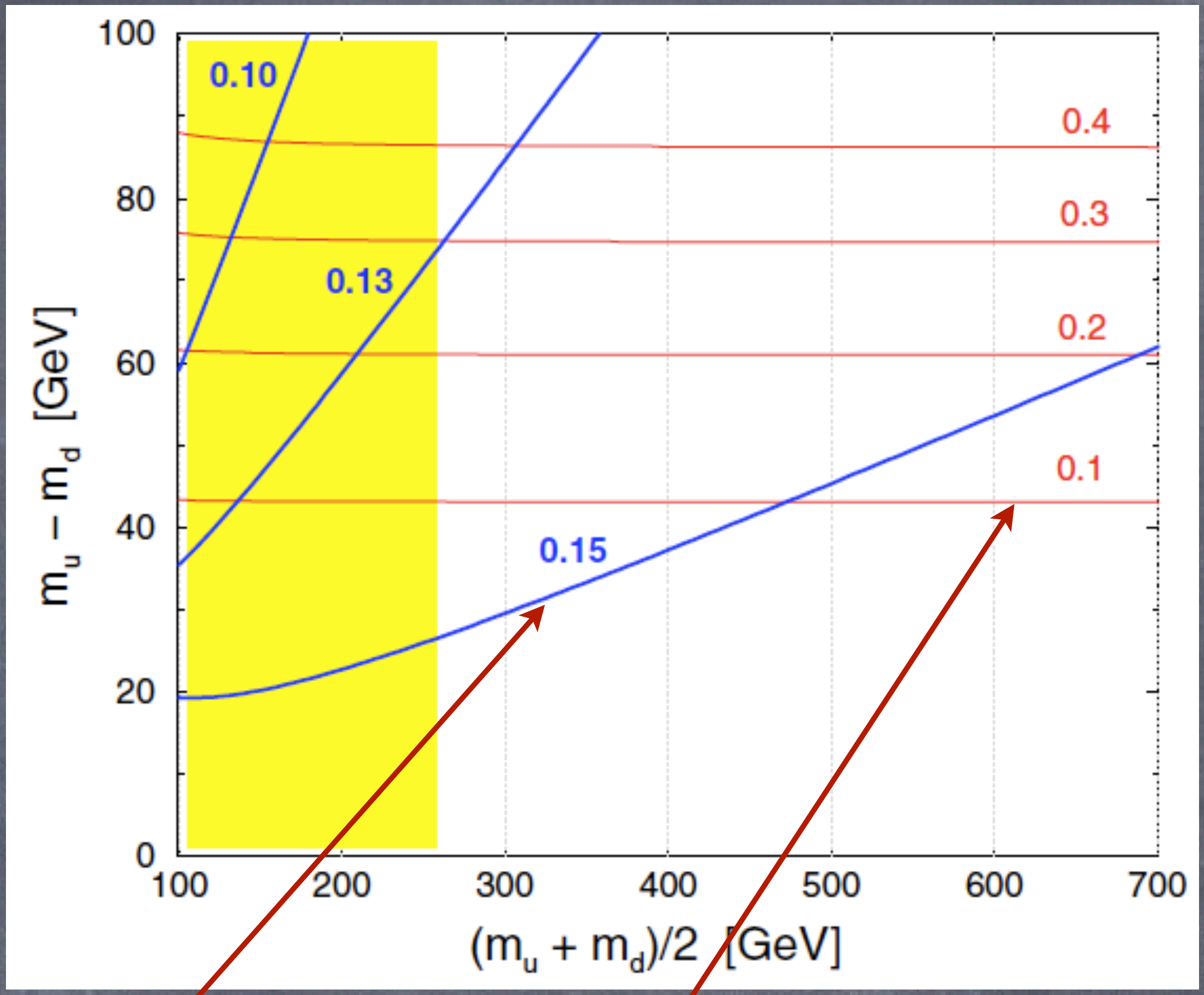
finally

$$\left|U^{4\times4}\right| = \begin{pmatrix} * & * & * & \begin{matrix} <0.089 \\ >0.021 \end{matrix} \\ * & * & * & <0.029 \\ * & * & * & <0.085 \\ <0.115 & <0.115 & <0.115 & \begin{matrix} <0.9998 \\ >0.9934 \end{matrix} \end{pmatrix}$$

@ 2σ

One should also consider the electroweak constraints

$$\Delta S = \frac{N_c}{6\pi}\left(1 - 2Y\ln\frac{m_u^2}{m_d^2}\right)$$



ΔS

ΔT

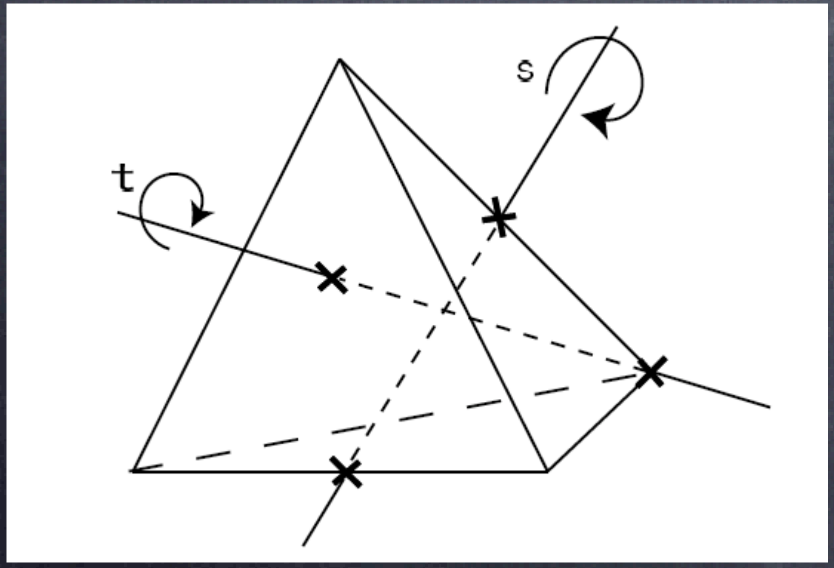
Flavor symmetry of leptons

A_4 symmetry :

- It was observed that A_4 symmetry can describe the neutrino mixing pattern

$A_4 \to Z_3$: charged leptons and $A_4 \to Z_2$ in neutrino sector

- A_4 is an even permutations of 4 objects (S_4) with order is equal to (4!)/2 = 12. A_4 group is isomorphic to the group of three dimensional rotations of a regular tetrahedron(T).



$$s^2 = t^3 = (st)^3 = e$$

A_4 continues...

- Three one-dimensional unitary representations are given by

$$
\begin{array}{lll}
1 & s=1, & t=1, \\
1' & s=1, & t=e^{i4\pi/3}=\omega^2, \\
1'' & s=1, & t=e^{i2\pi/3}=\omega.
\end{array}
$$

and one three-dimensional unitary representation

$$
t=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \quad \text{and} \quad s=\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}
$$

- The 12 elements of A_4 can be represented by the 3×3 matrices given by : $e,s,t,st,ts,t^2,st^2,sts,tst,t^2s,tst^2,t^2st$.

- Or equivalently we can construct the 12 representation matrices by taking 3×3 identity matrix e, the reflection matrices = diag(1,−1,−1), r_2 = diag(−1,1,−1), and r_3 = diag(−1,−1,1), the cyclic and anticyclic matrices

$$c = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad c^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

as well as $r_i c r_i$ and $r_i c^{-1} r_i$

A_4	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{3}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{3}$
$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{1}$	$\mathbf{3}$
$\mathbf{1}''$	$\mathbf{1}''$	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{3}$
$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3} \oplus \mathbf{3}$

- The A_4 has two subgroups

  G_s : a reflection subgroup generated by s

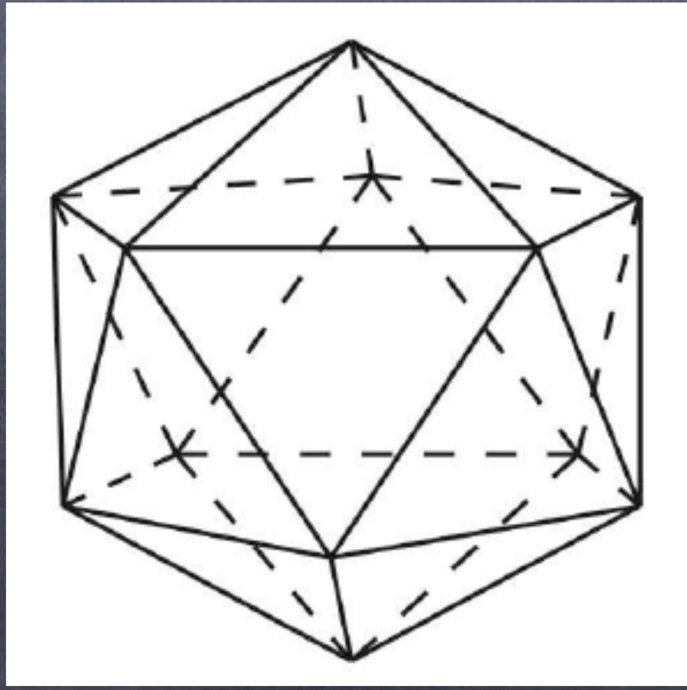
  G_t : the group generated by t, isomorphic to Z_3

A_5 symmetry

- If we assume A_4 is the flavor symmetry of three generation leptons and adopt the hypothesis of the existence of fourth generation -- the simplest approach is to demand the A_4 model can be minimally embedded

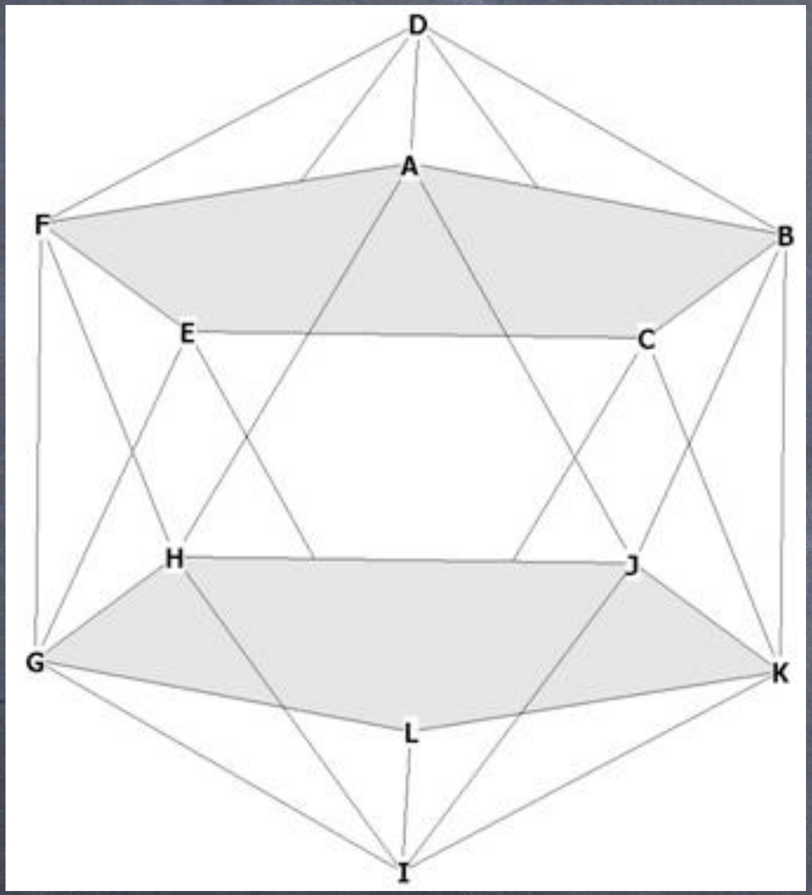
$$\Rightarrow A_5 \text{ group}$$

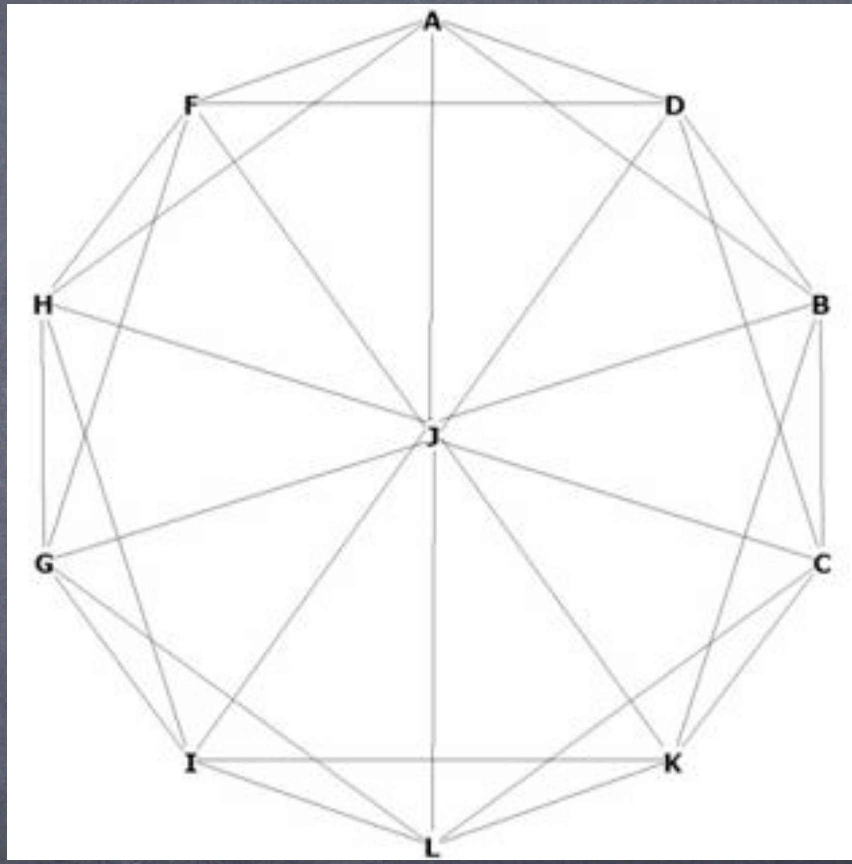
- A_5 is a symmetry of even permutation of five objects with order 60. A_5 is equivalent to the symmetry of a regular icosahedron (I).



Some interesting features of icosahedron :

- The icosahedron is built around the pentagon and the golden section. It seems absurd, since every face of the icosahedron is an equilateral triangle.





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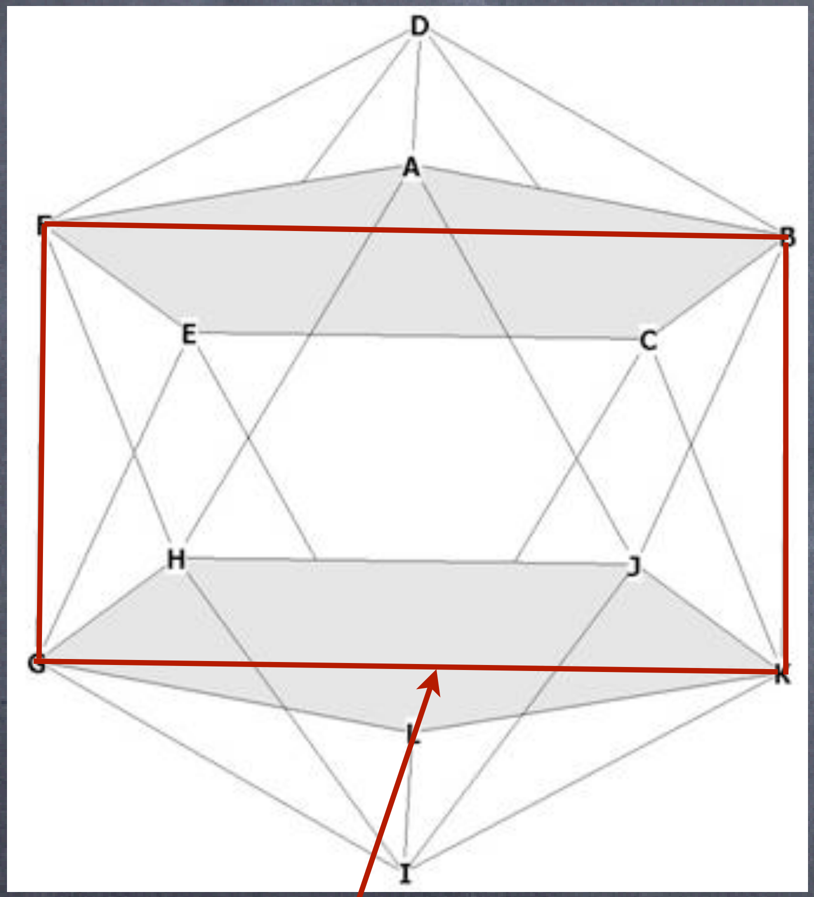

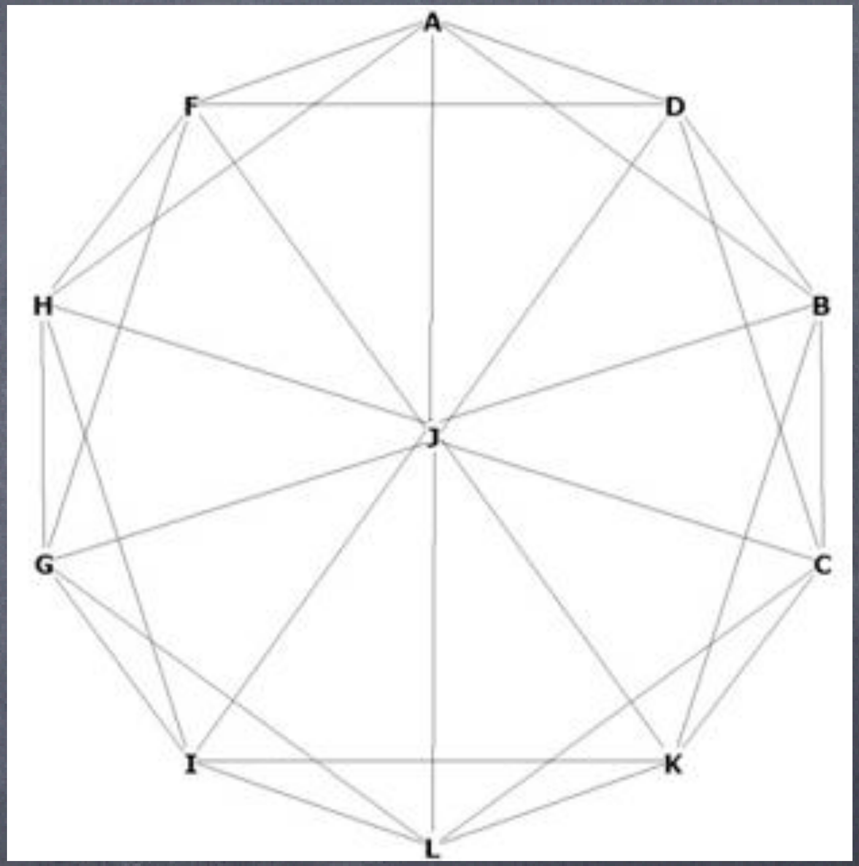

Golden ratio Φ = FB/FG

$$\frac{1}{\phi} = \phi - 1$$

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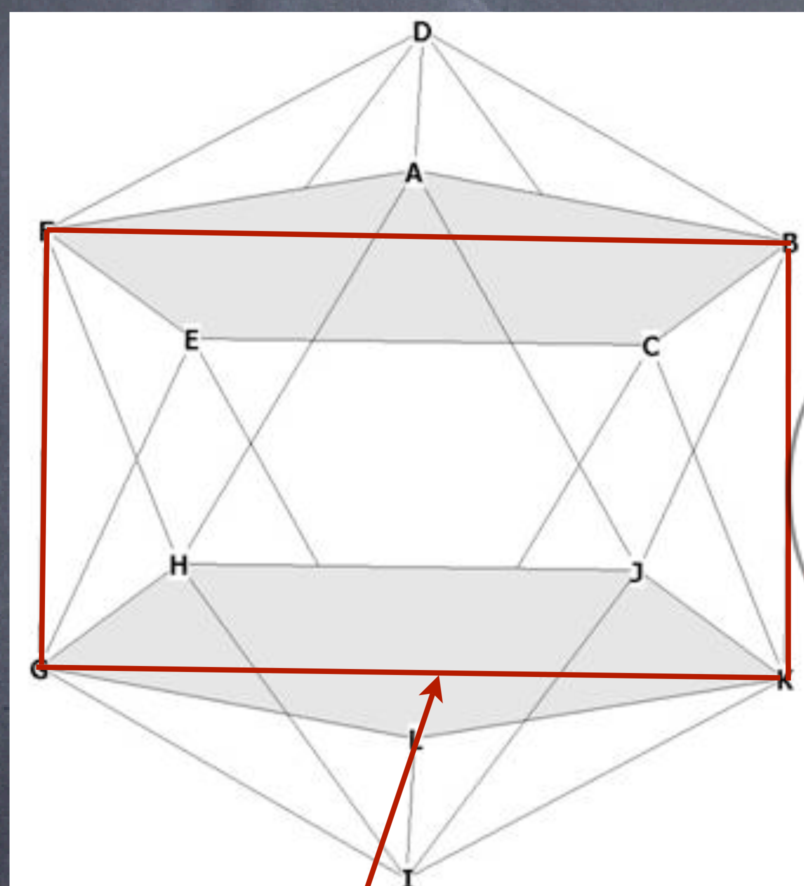

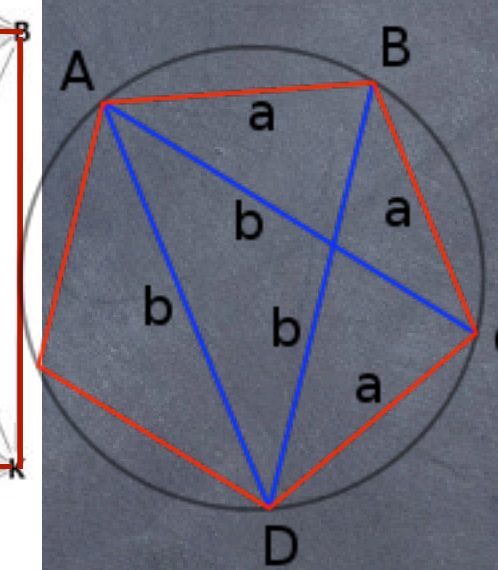

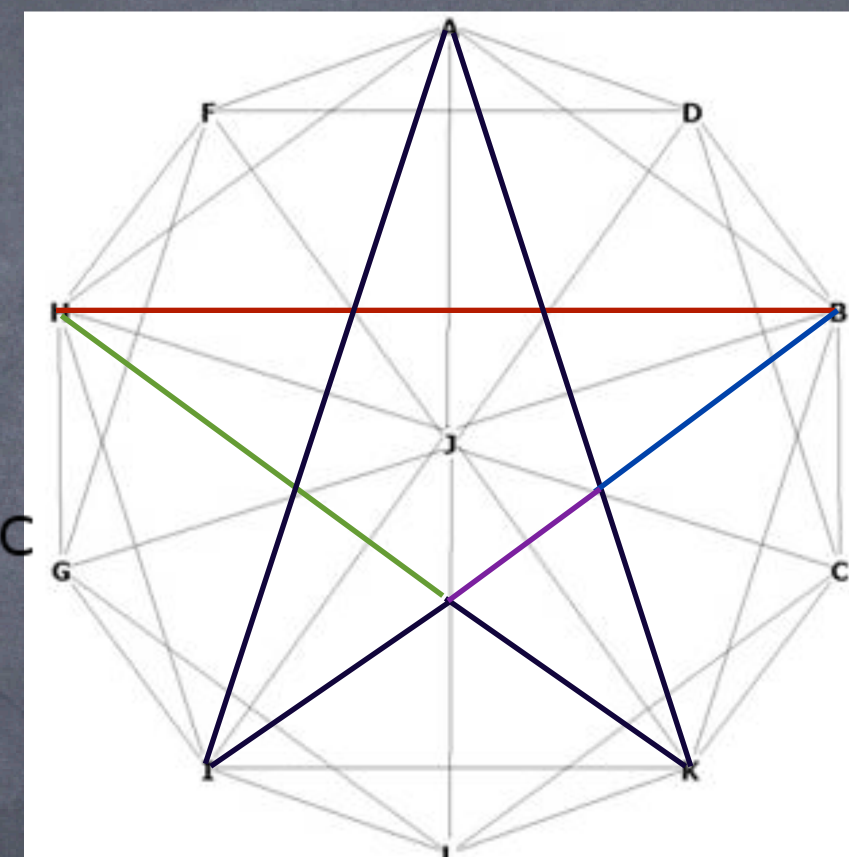

Φ = b/a

Φ = red/green = green/blue = blue/purple

Golden ratio Φ = FB/FG

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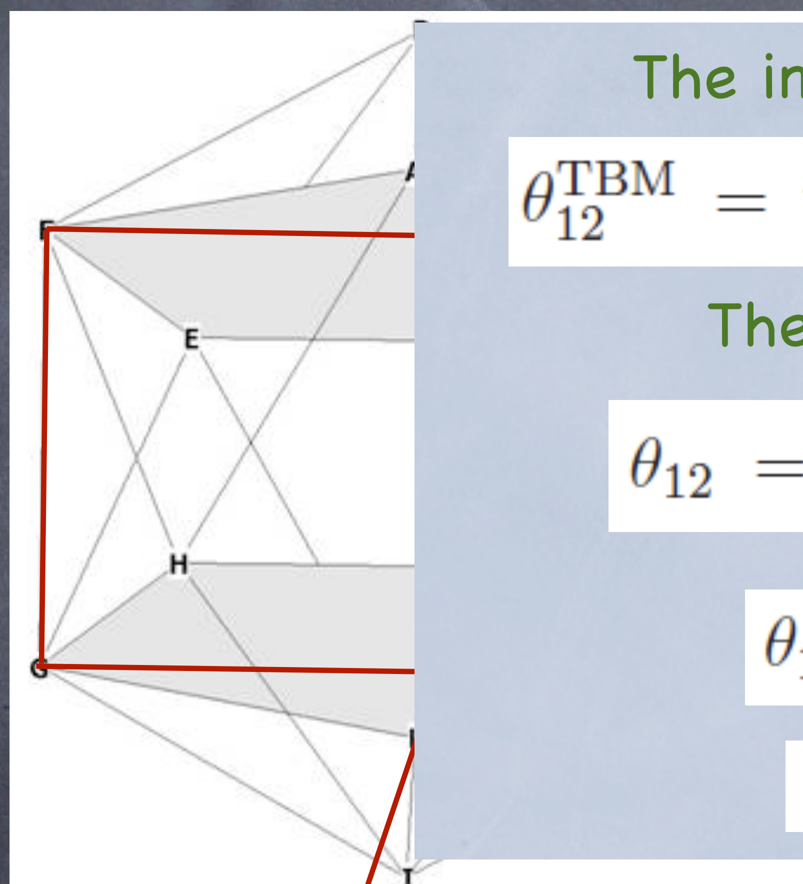


The interesting relations

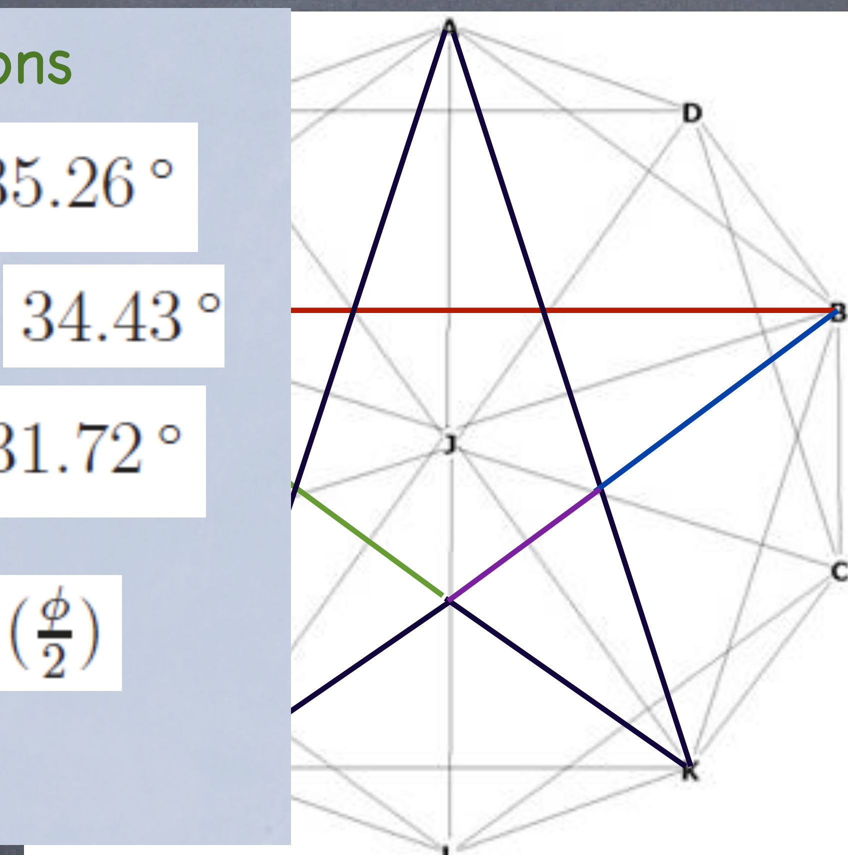
$$\theta_{12}^{\mathrm{TBM}} = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = 35.26^\circ$$

The best fit value : 34.43°

$$\theta_{12} = \tan^{-1}\left(\frac{1}{\phi}\right) = 31.72^\circ$$

$$\theta_{12} = \frac{\pi}{5} = \cos^{-1}\left(\frac{\phi}{2}\right)$$

$$\phi = (1+\sqrt{5})/2$$



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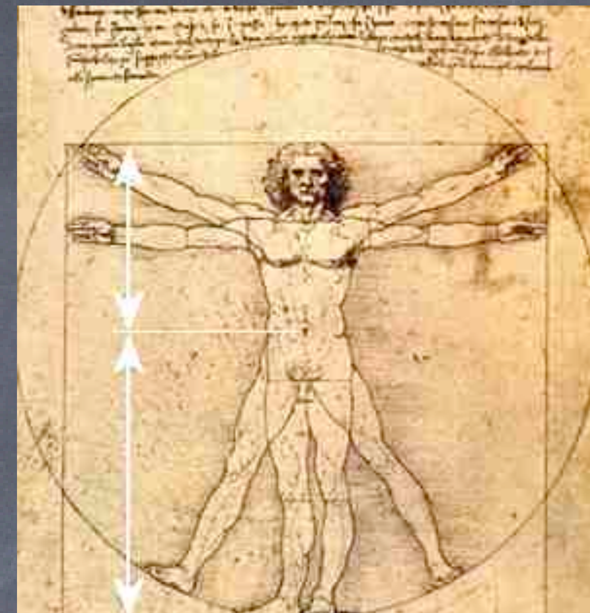
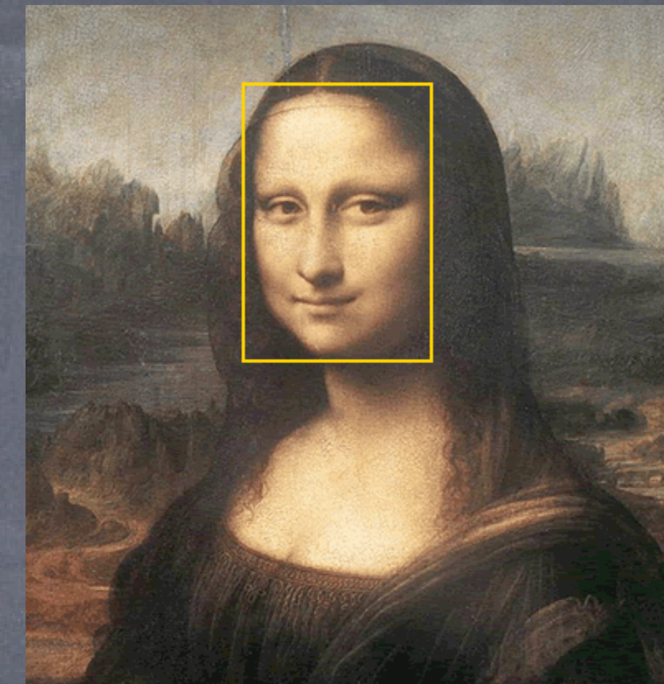
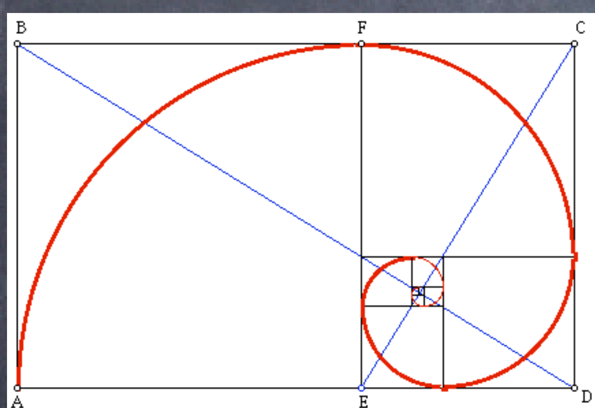
$$\frac{1}{\phi} = \phi - 1$$

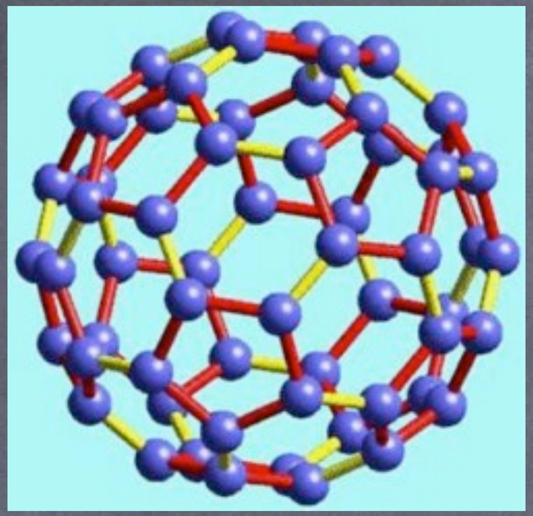
- And also







- if we cut the vertices of icosahedron, we obtain



Buckminsterfullerene, C60
1996 Nobel Chemistry Prize

1967 world Expo - U.S. Pavilion

- Let's go back to the group theory : a icosahedron consists of 20 identical equilateral triangular faces, 30 edges, and 12 vertices. The A_5 elements correspond to all the proper rotations of the icosahedron. They can be classified into five types of rotations

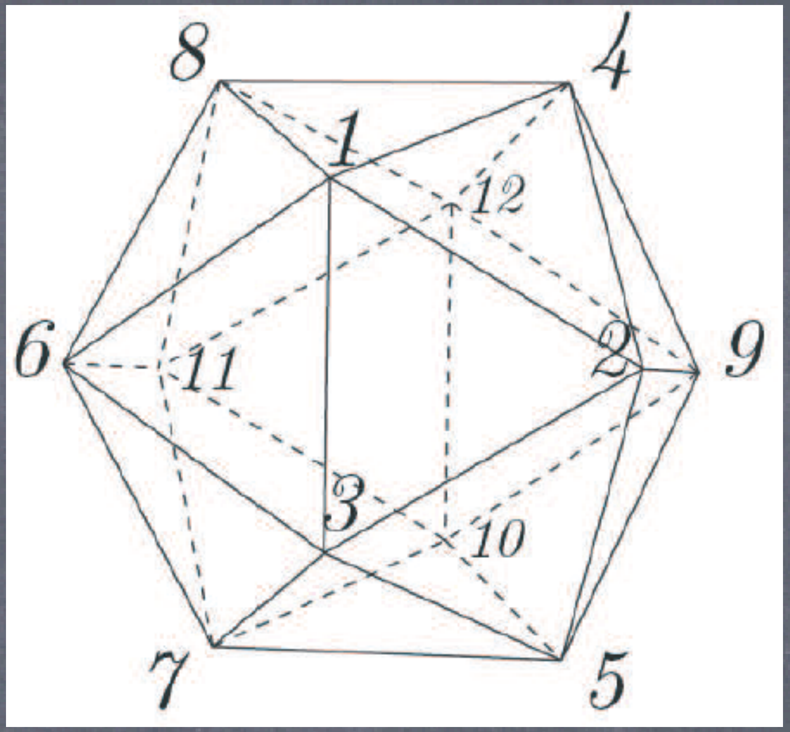
  - no rotation (identity)
  - π rotations about the midpoint of each edge
  - rotations by $2\pi/3$ about axes through the center of each face
  - rotations by $2\pi/5$ and $4\pi/5$ about an axis through each vertex

- We define two elements a and b :

  a : rotation by π about midpoint

  of the edge between 1 and 2

  (1,2,3,4,5,6,7,8,9,10,11,12) → (2,1, 3,4,5,6,7,8,9,10,11,12)



  b : rotation $2\pi/3$ about the axis

  through the center of the face 10-11-12

- (1,2,3,4,5,6,7,8,9,10,11,12) → (2,,3,1,5,6,4,8,9,7,11,12,10)

- ab : (1,2,3,4,5,6,7,8,9,10,11,12) → (3,2,5,1,9,7,10,6,4,12,11,8) , rotation by $2\pi/5$ about the axis through the vertex 2

$$a^2 = b^3 = (ab)^5 = e$$

- The 60 elements of A_5 are classified into five classes

C_1 : {e} ,

C_{15} :
{a(12),a(13),a(14),a(16),a(18),a(23),a(24),a(25),a(29),a(35),a(36),a(37),a(48),a(49),a(59)}

C_{20} :
{b(123),b(124),b(126),b(136),b(168),b(235),b(249),b(259),b(357),b(367), and the inverse elements}

C_{12} : {ab(1),ab(2),ab(3),ab(4),ab(5),ab(6), and the inverse elements}

C'_{12} : {a^2b^2(1),a^2b^2(2),a^2b^2(3),a^2b^2(4),a^2b^2(5),a^2b^2(6), and the inverse elements}

- Irreducible representations of A_5 are
- one singlet **1**, two triplets **3** and **3′**, one quartet **4**, and one quintet **5**
- Multiplication rules

A_5	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}'$	$\mathbf{4}$	$\mathbf{5}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}'$	$\mathbf{4}$	$\mathbf{5}$
$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}\oplus\mathbf{3}\oplus\mathbf{5}$	$\mathbf{4}\oplus\mathbf{5}$	$\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{5}$	$\mathbf{3}\oplus\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{5}$
$\mathbf{3}'$	$\mathbf{3}'$	$\mathbf{4}\oplus\mathbf{5}$	$\mathbf{1}\oplus\mathbf{3}'\oplus\mathbf{5}$	$\mathbf{3}\oplus\mathbf{4}\oplus\mathbf{5}$	$\mathbf{3}\oplus\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{5}$
$\mathbf{4}$	$\mathbf{4}$	$\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{5}$	$\mathbf{3}\oplus\mathbf{4}\oplus\mathbf{5}$	$\mathbf{1}\oplus\mathbf{3}\oplus\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{5}$	$\mathbf{3}\oplus\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{5}\oplus\mathbf{5}$
$\mathbf{5}$	$\mathbf{5}$	$\mathbf{3}\oplus\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{5}$	$\mathbf{3}\oplus\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{5}$	$\mathbf{3}\oplus\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{5}\oplus\mathbf{5}$	$\mathbf{1}\oplus\mathbf{3}\oplus\mathbf{3}'\oplus\mathbf{4}\oplus\mathbf{4}\oplus\mathbf{5}\oplus\mathbf{5}$

The model

- $A_5 \times Z_2 \times Z_3$ flavor symmetry of four generation leptons
- The left-handed lepton doublets

$$\underbrace{\begin{pmatrix} \nu_4 \\ l_4 \end{pmatrix}_L \begin{pmatrix} \nu_3 \\ l_3 \end{pmatrix}_L \begin{pmatrix} \nu_2 \\ l_2 \end{pmatrix}_L \begin{pmatrix} \nu_1 \\ l_1 \end{pmatrix}_L}_{L_{L4}(\mathbf{4},+1,\omega)}$$

$$\omega = e^{i2\pi/3}$$

- Right-handed charged leptons

$$l_{R5}(\mathbf{5},-1,+1) \quad , \quad l^c_{R3}(\mathbf{3},-1,+1) \quad \text{and} \quad l^{(1)}_{R1}(\mathbf{1},-1,+1) \quad , \quad l^{(2)}_{R1}(\mathbf{1},-1,+1)$$

- Right-handed neutrinos

$$N_{R5}(\mathbf{5},+1,+1) \quad \text{and} \quad N^{(1)}_{R}(\mathbf{1},+1,+1)$$

- Higgs sector :

  A_5 quartet and SM gauge singlet $S_4(\mathbf{4},+1,+1)$

  A_5 triplet and SM gauge doublet $\Phi_3(\mathbf{3},+1,\omega^2)$

  A_5 quartets and SM gauge doublet $H_4(\mathbf{4},+1,\omega^2)$, $H_4'(\mathbf{4},-1,\omega^2)$

- The Yukawa interactions are

$$
\begin{aligned}
L_{\text{Yukawa}} &= \frac{1}{2}M_1 N_R^{(1)} N_R^{(1)} + \frac{1}{2}M_5 N_{R5} N_{R5} + Y_{S1}(S_4 N_{R5} N_{R5}) + Y_{S2}(S_4 (l_{R3}^-)^c l_{R5}^-) \\
&+ Y_1(L_{L4} N_R^{(1)} H_4) + Y_2(L_{L4} N_{R5} H_4) + Y_3(L_{L4} N_{R5} \Phi_3) \\
&+ Y_4(L_{L4} l_{R5} H_4') + Y_5(L_{L4} l_{R1}^{(1)} H_4') + Y_6(L_{L4} l_{R1}^{(2)} H_4') + \text{H.c.}
\end{aligned}
$$

- Let S_4 develops a VEV $\langle S_4 \rangle = (V_S, 0, 0, 0)$, the discrete symmetry A_5 will break into A_4 causing the irreducible representations of A_5 to decompose as

$$\begin{aligned} A_5 &\to A_4 \\ \mathbf{1} &\to \mathbf{1} \\ \mathbf{3} &\to \mathbf{3} \\ \mathbf{3}' &\to \mathbf{3} \\ \mathbf{4} &\to \mathbf{1} + \mathbf{3} \\ \mathbf{5} &\to \mathbf{1}' + \mathbf{1}'' + \mathbf{3} \end{aligned}$$

- Leptons become

$$L_{L_4} \to L_{L1}(\mathbf{1}, +1, \omega) + L_{L3}(\mathbf{3}, +1, \omega) \ ,$$

$$l_{R5} \to l_{R3}(\mathbf{3}, -1, +1) + l_{R1'}(\mathbf{1}', -1, +1) + l_{R1''}(\mathbf{1}'', -1, +1) \ ,$$

$$N_{R5} \to N_{R3}(\mathbf{3}, +1, +1) + N_R^{(2)}(\mathbf{1}', +1, +1) + N_R^{(3)}(\mathbf{1}'', +1, +1) \ .$$

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$$
\begin{array}{ll}
A_5 & \rightarrow A_4 \\
\mathbf{1} & \rightarrow \mathbf{1} \\
\mathbf{3} & \rightarrow \mathbf{3} \\
\mathbf{3}' & \rightarrow \mathbf{3} \\
\mathbf{4} & \rightarrow \mathbf{1} + \mathbf{3} \\
\mathbf{5} & \rightarrow \mathbf{1}' + \mathbf{1}'' + \mathbf{3}
\end{array}
$$

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$$L_{L_4} \rightarrow L_{L1}(\mathbf{1}, +1, \omega) + L_{L3}(\mathbf{3}, +1, \omega) \quad ,$$

$$l_{R5} \rightarrow l_{R3}(\mathbf{3}, -1, +1) + l_{R1'}(\mathbf{1}', -1, +1) + l_{R1''}(\mathbf{1}'', -1, +1) \quad ,$$

$$N_{R5} \rightarrow N_{R3}(\mathbf{3}, +1, +1) + N_R^{(2)}(\mathbf{1}', +1, +1) + N_R^{(3)}(\mathbf{1}'', +1, +1) \quad .$$

- Scalar fields decompose as

$$S_4 \to S_1(\mathbf{1},+1,+1) + S_3(\mathbf{3},+1,+1) \ ,$$
$$H_4 \to H_1(\mathbf{1},+1,\omega^2) + H_3(\mathbf{3},+1,\omega^2) \ ,$$
$$H'_4 \to H'_1(\mathbf{1},-1,\omega^2) + H'_3(\mathbf{3},-1,\omega^2) \ ,$$
$$\Phi_3 \to \Phi_3(\mathbf{3},+1,\omega^2) \ .$$

- The Yukawa interactions after $A_5 \to A_4$ become

$$\begin{aligned}
L^{A_4}_{Yukawa} &= \frac{1}{2}M_1 N_R^{(1)} N_R^{(1)} + \frac{1}{2}M_5(N_R^{(2)} N_R^{(3)} + N_{R3}N_{R3}) \\
&+ Y_{S1}\left[(V_S + S_1)(N_R^{(2)} N_R^{(3)} + N_{R3}N_{R3}) + S_3(N_{R3}N_{R3})\right] \\
&+ Y_{S2}\left[(V_S + S_1)(l_{R3})^c l_{R3} + S_3(l_{R3})^c(l_{R3} + l_{R1'} + l_{R1''})\right] \\
&+ Y_1\left[L_{L1}N_R^{(1)}H_1 + L_{L3}N_R^{(1)}H_3\right] \\
&+ Y_2\left[L_{L1}N_{R3}H_3 + L_{L3}N_{R3}(H_1 + H_3) + L_{L3}(N_R^{(2)} + N_R^{(3)})H_3\right] \\
&+ Y_3\left[L_{L1}N_{R3}\Phi_3 + L_{L3}N_{R3}\Phi_3 + L_{L3}(N_R^{(2)} + N_R^{(3)})\Phi_3\right] \\
&+ Y_4\left[L_{L1}l_{R3}H'_3 + L_{L3}l_{R3}(H'_1 + H'_3) + L_{L3}(l_{R1'} + l_{R1''})H'_3\right] \\
&+ Y_5\left[L_{L1}l_{R1}^{(1)}H'_1 + L_{L3}l_{R1}^{(1)}H'_3\right] + Y_6\left[L_{L1}l_{R1}^{(2)}H'_1 + L_{L3}l_{R1}^{(2)}H'_3\right] + \mathrm{H.c.}
\end{aligned}$$

Results

- Charged lepton masses
- The subsequent breaking of A_4 and SM gauge symmetries due to $\langle H_4 \rangle$, $\langle H'_4 \rangle$, $\langle \Phi_3 \rangle$ and $\langle S_3 \rangle$
- Mass terms relate to charged leptons

$$
\begin{aligned}
L_l = & \; Y_{S2}\left[(V_S + \langle S_1 \rangle)(l_{R3})^c l_{R3} + \langle S_3 \rangle (l_{R3})^c (l_{R3} + l_{R1'} + l_{R1''})\right] \\
& + Y_4\left[L_{L1} l_{R3} \langle H'_3 \rangle + L_{L3} l_{R3} (\langle H'_1 \rangle + \langle H'_3 \rangle) + L_{L3}(l_{R1'} + l_{R1''}) \langle H'_3 \rangle\right] \\
& + Y_5\left[L_{L1} l^{(1)}_{R1} \langle H'_1 \rangle + L_{L3} l^{(1)}_{R1} \langle H'_3 \rangle\right] + Y_6\left[L_{L1} l^{(2)}_{R1} \langle H'_1 \rangle + L_{L3} l^{(2)}_{R1} \langle H'_3 \rangle\right]
\end{aligned}
$$

- Taking $\langle H'_3 \rangle = (V'_{3_1}, V'_{3_2}, V'_{3_3}) \quad \text{and} \quad \langle H'_1 \rangle = V'_1$

- The 7×7 charged lepton mass matrix of the form

$$M_l = \left(\begin{array}{cccc|ccc} Y_5V'_1 & Y_6V'_1 & 0 & 0 & Y_4V'_{3_1} & Y_4V'_{3_2} & Y_4V'_{3_3} \\ Y_5V'_{3_1} & Y_6V'_{3_1} & Y_4V'_{3_1} & Y_4V'_{3_1} & Y_4V'_1 & Y_4V'_{3_3} & Y_4V'_{3_2} \\ Y_5V'_{3_2} & Y_6V'_{3_2} & \omega Y_4V'_{3_2} & \omega^2Y_4V'_{3_2} & Y_4V'_{3_3} & Y_4V'_1 & Y_4V'_{3_1} \\ Y_5V'_{3_3} & Y_6V'_{3_3} & \omega^2Y_4V'_{3_3} & \omega Y_4V'_{3_3} & Y_4V'_{3_2} & Y_4V'_{3_1} & Y_4V'_1 \\ \hline 0 & 0 & Y_{S2}\langle S_3\rangle_1 & Y_{S2}\langle S_3\rangle_1 & Y_{S2}(V_S+\langle S_1\rangle) & Y_{S2}\langle S_3\rangle_3 & Y_{S2}\langle S_3\rangle_2 \\ 0 & 0 & \omega Y_{S2}\langle S_3\rangle_2 & \omega^2 Y_{S2}\langle S_3\rangle_2 & Y_{S2}\langle S_3\rangle_3 & Y_{S2}(V_S+\langle S_1\rangle) & Y_{S2}\langle S_3\rangle_1 \\ 0 & 0 & \omega^2 Y_{S2}\langle S_3\rangle_3 & \omega Y_{S2}\langle S_3\rangle_3 & Y_{S2}\langle S_3\rangle_2 & Y_{S2}\langle S_3\rangle_1 & Y_{S2}(V_S+\langle S_1\rangle) \end{array}\right)$$

under the left- and right-handed bases

$(L_{L1}, L_{L3} = (L_{L3_1}, L_{L3_2}, L_{L3_3}), l^c_{R3} = (l^c_{R3_1}, l^c_{R3_2}, l^c_{R3_3}))$ and

$(l^{(1)}_{R1}, l^{(2)}_{R1}, l_{R1'}, l_{R1''}, l_{R3_1}, l_{R3_2}, l_{R3_3})^T$

$\mathrm{Det}(M_l) = 0$ → one massless eigenstate : m_e

- If $V'_{3_1} = V'_{3_2} = V'_{3_3} = V'$, and A_5 breaking scale is much higher than A_4 and SM gauge symmetries breaking scales $V_S \gg \langle S_3 \rangle, \langle H'_1 \rangle$, and $\langle H'_3 \rangle$. The four generation charged lepton masses are given by

$$m_e = 0 \quad , \quad m_\mu = \sqrt{3} Y_4 V' \quad ,$$
$$m_\tau = \frac{1}{2}\left[(Y_5 V'_1 + (Y_6 - Y_4)V') - \sqrt{(Y_5 V'_1 + (Y_6 - Y_4)V')^2 + 4Y_4(Y_5 V'_1 + 3Y_6 V')V'}\right]$$

and

$$m_{\tau'} = \frac{1}{2}\left[(Y_5 V'_1 + (Y_6 - Y_4)V') + \sqrt{(Y_5 V'_1 + (Y_6 - Y_4)V')^2 + 4Y_4(Y_5 V'_1 + 3Y_6 V')V'}\right]$$

($\geq$ 100.8 GeV, 95% C.L.)

- Calculable electron mass m_e

- Many attempts try to derive the ratio $m_e/m_\mu \approx \frac{\alpha}{\pi}$

H. Georgi and S.L. Glashow 1973; S.M.Barr and A.Zee 1977,1978; B.S.Balakrishna 1988; Babu&Ma 1989;X.G.He et al. 1990; ……

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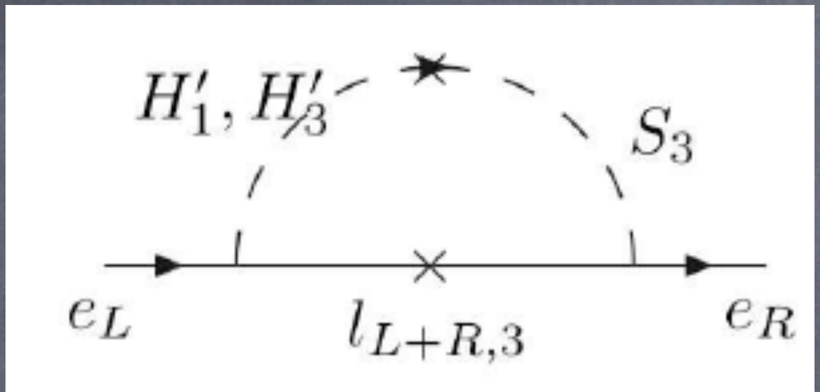
- Electron mass can be generated via 1-loop



$$m_e \sim \frac{Y_4^2}{16\pi^2}\frac{m_H^2}{m_{L+R,3}} \approx \frac{Y_4^2}{16\pi^2 Y_{S2}}\frac{m_H^2}{V_S}$$

- The construction provides another example of electron-muon universality proposed by S.M.Barr and A.Zee 1977. Finally

$$m_e/m_\mu \approx \frac{Y_4 m_H^2}{16\pi^2 Y_{S2} V_S V'} \approx \frac{\alpha}{\pi}.$$

in our model.

- Neutrino mixings and masses
- The full neutrino mass matrix of the model is a 10×10 matrix

$$M_\nu = \left(\begin{array}{c|c} M'_{\nu_L}(4\times 4) = 0 & M_D(4\times 6) \\ \hline M_D^T(6\times 4) & M_{N_R}(6\times 6) \end{array} \right)$$

- Right-handed neutrino masses after A_5 breaking are given by

$$M_{N_R} = \begin{pmatrix} M_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_2 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2M_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2M_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2M_2 \end{pmatrix}$$

$$M_2 = \tfrac{1}{2} M_5 + Y_{S1} V_S$$

$(N_R^{(1)}, N_R^{(2)}, N_R^{(3)}, N_{R3_1}, N_{R3_2}, N_{R3_3})$ basis

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$$M_2 = \tfrac{1}{2} M_5 + Y_{S1} V_S$$

$(N_R^{(1)}, N_R^{(2)}, N_R^{(3)}, N_{R3_1}, N_{R3_2}, N_{R3_3})$ basis

A_4 breaking effects $\langle s_3 \rangle$

Dirac mass terms of neutrinos

$$L_{Dirac} = Y_1 \left[L_{L1} N_R^{(1)} H_1 + L_{L3} N_R^{(1)} H_3 \right] + Y_2 L_{L3} N_{R3} H_1 + L_{L1} N_{R3} \underline{[Y_2 H_3 + Y_3 \Phi_3]}$$
$$+ L_{L3} N_{R3} \underline{[Y_2 H_3 + Y_3 \Phi_3]} + L_{L3}(N_R^{(2)} + N_R^{(3)}) \underline{[Y_2 H_3 + Y_3 \Phi_3]} \quad .$$

Ignoring H_3 3 terms, the Dirac mass matrix is

$$\nu_L M_D N_R^T =$$
$$\begin{pmatrix} \nu_{L1} & \nu_{L3_1} & \nu_{L3_2} & \nu_{L3_3} \end{pmatrix} \begin{pmatrix} Y_1 v_{H_1} & 0 & 0 & Y_3 v_{\phi_1} & Y_3 v_{\phi_2} & Y_3 v_{\phi_3} \\ 0 & Y_3 v_{\phi_1} & Y_3 v_{\phi_1} & Y_2 v_{H_1} & Y_3 v_{\phi_3} & Y_3 v_{\phi_2} \\ 0 & \omega Y_3 v_{\phi_2} & \omega^2 Y_3 v_{\phi_2} & Y_3 v_{\phi_3} & Y_2 v_{H_1} & Y_3 v_{\phi_1} \\ 0 & \omega^2 Y_3 v_{\phi_3} & \omega Y_3 v_{\phi_3} & Y_3 v_{\phi_2} & Y_3 v_{\phi_1} & Y_2 v_{H_1} \end{pmatrix} \begin{pmatrix} N_R^{(1)} \\ N_R^{(2)} \\ N_R^{(3)} \\ N_{R3_1} \\ N_{R3_2} \\ N_{R3_3} \end{pmatrix}$$

with VEVs $\langle \Phi_3 \rangle = (v_{\phi 1}, v_{\phi 2}, v_{\phi 3})$ and $\langle H_1 \rangle = v_{H_1}$

- Now we can calculate left-handed Majorana neutrino masses through seesaw mechanism

$$M_{\nu_L} = M_D M_{N_R}^{-1} M_D^T$$

$$\begin{aligned}
M_{\nu_L(\nu_{L1},\nu_{L1})} &= \frac{Y_1^2 v_{H_1}^2}{M_1} + \frac{Y_3^2(v_{\phi_1}^2 + v_{\phi_2}^2 + v_{\phi_3}^2)}{2M_2} , \\
M_{\nu_L(\nu_{L1},\nu_{L3_1})} &= \frac{Y_2 Y_3 v_{\phi_1} v_{H_1} + 2Y_3^2 v_{\phi_2} v_{\phi_3}}{2M_2} , \\
M_{\nu_L(\nu_{L1},\nu_{L3_2})} &= \frac{Y_2 Y_3 v_{\phi_2} v_{H_1} + 2Y_3^2 v_{\phi_1} v_{\phi_3}}{2M_2} , \\
M_{\nu_L(\nu_{L1},\nu_{L3_3})} &= \frac{Y_2 Y_3 v_{\phi_3} v_{H_1} + 2Y_3^2 v_{\phi_1} v_{\phi_2}}{2M_2} , \\
M_{\nu_L(\nu_{L3_1},\nu_{L3_1})} &= \frac{Y_2^2 v_{H_1}^2 + Y_3^2(4v_{\phi_1}^2 + v_{\phi_2}^2 + v_{\phi_3}^2)}{2M_2} ,
\end{aligned}$$

$$\begin{aligned}
M_{\nu_L(\nu_{L3_1},\nu_{L3_2})} &= \frac{2Y_2 Y_3 v_{H_1} v_{\phi_3} - Y_3^2 v_{\phi_1} v_{\phi_2}}{2M_2} , \\
M_{\nu_L(\nu_{L3_1},\nu_{L3_3})} &= \frac{2Y_2 Y_3 v_{H_1} v_{\phi_2} - Y_3^2 v_{\phi_1} v_{\phi_3}}{2M_2} , \\
M_{\nu_L(\nu_{L3_2},\nu_{L3_2})} &= \frac{Y_2^2 v_{H_1}^2 + Y_3^2(v_{\phi_1}^2 + 4v_{\phi_2}^2 + v_{\phi_3}^2)}{2M_2} , \\
M_{\nu_L(\nu_{L3_2},\nu_{L3_3})} &= \frac{2Y_2 Y_3 v_{H_1} v_{\phi_1} - Y_3^2 v_{\phi_2} v_{\phi_3}}{2M_2} , \\
M_{\nu_L(\nu_{L3_3},\nu_{L3_3})} &= \frac{Y_2^2 v_{H_1}^2 + Y_3^2(v_{\phi_1}^2 + v_{\phi_2}^2 + 4v_{\phi_3}^2)}{2M_2} .
\end{aligned}$$

Denoted

$$M_{\nu_L} = \begin{pmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{pmatrix}$$

$(\nu_{\tau'}, \nu_\tau, \nu_\mu, \nu_e)$ basis

- By taking $v_{\phi_1} = v_{\phi_2} = v_{\phi_3} \equiv v$ or say $\langle \Phi_3 \rangle = (v, v, v)$ and

$$v_{H_1} = -\frac{2Y_3}{Y_2} v$$

- We can diagonalize $M_{\nu L}$ via the unitary transformation

$$M_{diag} = U^{4g}_{TBM} M_{\nu_L} U^{4g^T}_{TBM}$$

with

$$U^{4g}_{TBM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} \\ 0 & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} & \sqrt{\frac{2}{3}} \end{pmatrix}$$

- And the four neutrino masses are written

$$
\begin{aligned}
m_{\nu_4} &= \frac{Y_1^2 v_{H_1}^2}{M_1} + \frac{3Y_3^2 v^2}{2M_2} \\
m_{\nu_3} &= \frac{15Y_3^2 v^2}{2M_2} \quad , \\
m_{\nu_2} &= 0 \quad , \\
m_{\nu_1} &= m_{\nu_3} \quad .
\end{aligned}
$$

- The results are phenomenological unacceptable, therefore, the deviations from tribimaximal mixings are needed.

<S3> in right-handed Majorana neutrinos and <H3> in Dirac masses

- Retain the A_4 perturbations $\langle S_3 \rangle = (\delta_1, \delta_2, \delta_3)$ in right-handed Majorana neutrino mass matrix

$$M_{N_R} = \begin{pmatrix} M_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_2 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2M_2 & Y_{S1}\delta_3 & Y_{S1}\delta_2 \\ 0 & 0 & 0 & Y_{S1}\delta_3 & 2M_2 & Y_{S1}\delta_1 \\ 0 & 0 & 0 & Y_{S1}\delta_2 & Y_{S1}\delta_1 & 2M_2 \end{pmatrix}$$

- The three light neutrino masses become

$$\begin{aligned} m_{\nu_3} &\approx \frac{6Y_3^2 v^2}{M_2} + \frac{3}{2M_2 + Y_{S1}\delta_1} Y_3^2 v^2 \ , \\ m_{\nu_2} &\approx -\frac{3Y_3^2 v^2}{2M_2} + \frac{2Y_3^2 v^2}{2M_2 - Y_{S1}\delta_1} + \frac{(2M_2 + Y_{S1}\delta_1) Y_3^2 v^2}{M_2(2M_2 - Y_{S1}\delta_1)} \ , \\ & \quad -\frac{(2M_2 + Y_{S1}\delta_1) Y_3^2 v^2}{2M_2^2} + \frac{(2M_2 + Y_{S1}\delta_1)^2 Y_3^2 v^2}{2M_2(4M_2^2 - Y_{S1}^2\delta_1^2)} \ , \\ m_{\nu_1} &\approx \frac{9Y_3^2 v^2}{2M_2} + \frac{2Y_3^2 v^2}{2M_2 - Y_{S1}\delta_1} + \frac{(2M_2 + Y_{S1}\delta_1) Y_3^2 v^2}{(2M_2 - Y_{S1}\delta_1) M_2} \ , \\ & \quad +\frac{(2M_2 + Y_{S1}\delta_1) Y_3^2 v^2}{4M_2^2} + \frac{(2M_2 + Y_{S1}\delta_1)^2 Y_3^2 v^2}{2M_2(4M_2^2 - Y_{S1}^2\delta_1^2)} \ , \end{aligned}$$

for $\delta_1 \neq 0$ and $\delta_2 = \delta_3 = 0$.

$$
\begin{aligned}
m_{\nu_3} &\approx \frac{9Y_3^2v^2}{2M_2} + \left[\frac{6M_2}{4M_2^2 - Y_{S1}^2\delta_2^2} + \frac{3}{2M_2 - Y_{S1}\delta_2}\right] Y_3^2v^2 \ , \\
m_{\nu_2} &\approx -\frac{3Y_3^2v^2}{2M_2} + \frac{2Y_3^2v^2}{2M_2 - Y_{S1}\delta_2} + \frac{(2M_2 + Y_{S1}\delta_2)Y_3^2v^2}{M_2(2M_2 - Y_{S1}\delta_2)} \ , \\
&\quad -\frac{(2M_2 + Y_{S1}\delta_2)Y_3^2v^2}{2M_2^2} + \frac{(2M_2 + Y_{S1}\delta_2)^2Y_3^2v^2}{2M_2(4M_2^2 - Y_{S1}^2\delta_2^2)} \ , \\
m_{\nu_1} &\approx \frac{9Y_3^2v^2}{2M_2} + \frac{2Y_3^2v^2}{2M_2 - Y_{S1}\delta_2} + \frac{(2M_2 + Y_{S1}\delta_2)Y_3^2v^2}{M_2(2M_2 - Y_{S1}\delta_2)} + \frac{(2M_2 + Y_{S1}\delta_2)^2Y_3^2v^2}{2M_2(4M_2^2 - Y_{S1}^2\delta_2^2)}
\end{aligned}
$$

for the case of $(0, \delta_2, 0)$, and $(0, 0, \delta_3)$

$$
\begin{aligned}
m_{\nu_3} &\approx \frac{3Y_3^2v^2}{M_2} + \frac{9Y_3^2v^2}{2M_2 - Y_{S1}\delta_3} \ , \\
m_{\nu_2} &\approx -\frac{3Y_3^2v^2}{2M_2} + \frac{2Y_3^2v^2}{2M_2 + Y_{S1}\delta_3} + \frac{Y_3^2v^2(2M_2 + Y_{S1}\delta_3)^2}{8M_2^3} \ , \\
m_{\nu_1} &\approx \frac{6Y_3^2v^2}{M_2} + \frac{Y_3^2v^2}{2M_2 + Y_{S1}\delta_3} + \frac{3Y_3^2v^2(2M_2 + Y_{S1}\delta_3)}{4M_2^2} + \frac{Y_3^2v^2(2M_2 + Y_{S1}\delta_3)^2}{8M_2^3}
\end{aligned}
$$

with the fourth neutrino mass

$$
m_{\nu_4} = \frac{Y_1^2v_{H_1}^2}{M_1} + \left(\frac{2}{2M_2 + Y_{S1}\delta_i} + \frac{1}{2M_2}\right)Y_3^2v^2 > M_Z/2
$$

- Also the non-zero <H$_3$> in Dirac masses,

$$L_{Dirac} = Y_1 \left[L_{L1} N_R^{(1)} H_1 + L_{L3} N_R^{(1)} H_3 \right] + Y_2 L_{L3} N_{R3} H_1 + L_{L1} N_{R3} [Y_2 H_3 + Y_3 \Phi_3] + L_{L3} N_{R3} [Y_2 H_3 + Y_3 \Phi_3] + L_{L3} (N_R^{(2)} + N_R^{(3)}) [Y_2 H_3 + Y_3 \Phi_3] \ .$$

- Assuming new physics to be around O(1) TeV, we take

$v_{H_1} = 220 \text{ GeV},\ v = 10 \text{ GeV}$, $Y_{S1}\delta_i = 100 \sim 500 \text{ GeV}$, $M_1 = 500 \text{ GeV},\ M_2 = 10^8 \text{ GeV}$

$Y_1 = 1$ and $Y_3 = 10^{-2} \sim 10^{-4}$

$m_{\nu_4} \approx 96.8 \text{ GeV}$

≥90.3 GeV for Dirac coupling and
≥80.5 GeV for Majorana coupling

A huge parameter space for $m_{\nu 1} \sim m_{\nu 2} \sim m_{\nu 3} \sim 0.1$ eV ,
degenerate spectrum is preferred

Conclusion

- The existence of 4^{th} generation is not excluded by EW precision data and LHC has the potential to discover or fully exclude SM4
- We study a model of four generation leptons under the A_5 symmetry, where the best features of the three family A_4 model survive
- Electron mass is predicted to be massless at tree level but calculable through quantum corrections
- A degenerate spectrum of three light neutrinos is preferred, and the splitting of the neutrino masses can be obtained as a result of the breaking of A_5 down to A_4