B decays in SCET

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Introduction to SCET

The main idea of effective field theories is the presence of lower limit of the distances that can be resolved through a process of given energy. In this limit, the heavy modes can be integrated out and the non-local interactions mediated by these heavy modes are reduced to local interactions.

- SCET is an effective field theory describing the dynamics of highly energetic particles moving close to the light-cone interacting with a background field of soft quanta
- SCET provides a systematic and rigorous approach for calculating processes with several relevant energy scales. For instance, in B decays to light mesons we have the B energy scale, the jet scale and the low energy QCD scale.
- SCET has the ability to sum up all large radiative corrections appears in high energy scattering processes and thus preserving the perturbation theory at each order. As an example, in processes with highly energetic hadron jets SCET can sum up the enhanced corrections which are proportional to large logarithms of ratios of mass scales .
- The systematic power counting in SCET reduce the complexity of calculations: we start by defining a small parameter λ as the ratio of the lowest and largest energy scales in the process under consideration and then we make a scaling for the momenta and fields in terms of λ and finally, the Lagrangian, Effective Hamiltonian can be expanded into terms with different orders in λ.

Momenta and field scaling in SCET

- Consider a process in which B meson decays into two light energetic quarks. In the rest frame of the B meson and due to the conservation of momentum, the two quarks will be emitted in opposite directions which can be chosen as Z direction for simplicity.
- As the motion of the emitted quarks is in the Z direction, it is appropriate to define two vectors $n^\mu = (1, 0, 0, 1)$ and $\bar{n}^\mu = (1, 0, 0, -1)$. The two emitted quarks move in n^μ and $\bar{n}^\mu$ directions and they are called collinear quarks.
- In terms of n^μ and $\bar{n}^\mu$ we can write any vector P^μ as $P^\mu = (n \cdot P)\, \frac{\bar{n}^\mu}{2} + (\bar{n} \cdot P)\, \frac{n^\mu}{2} + P_\perp^\mu = P_+^\mu + P_-^\mu + P_\perp^\mu$
- The notation $P = (P^+, P^-, P_\perp)$ is usually used and this decomposition of momentum is referred as light cone decomposition.
- For any given momentum, the scaling of any of its light cone components depends on a small parameter λ.
- Usually, λ is defined as $\lambda = (\frac{lowest\ energy\ scale}{largest!energy\ scale})^n$ where n can be $1/2$ or 1 depending on the process under consideration and so we will have two types of SCET as we will show in the following.

- Consider the energetic collinear quark moves along the n^μ direction: its different light cone components are widely separated, with $P_- \sim E$ being large, $P_\perp \sim \lambda E$ being small and $P_+ \sim \lambda^2 E$ being very small where we have used $P_+ P_- \sim P_\perp^2$ for fluctuations near the mass shell. So the scaling of the momentum of the collinear quark moves in n^μ direction is : $p^\mu = n \cdot p \frac{\bar{n}^\mu}{2} + \bar{n} \cdot p \frac{n^\mu}{2} + p_\perp^\mu = \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^0) + \mathcal{O}(\lambda^1) \equiv (\lambda^2, 1, \lambda) E$.
- The other partons in the B meson carry momenta that scale like $(\Lambda_{QCD}, \Lambda_{QCD}, \Lambda_{QCD})$. If we choose $\lambda = \sqrt{\Lambda_{QCD}/E}$ we can write the scaling of these partons as $(\lambda^2, \lambda^2, \lambda^2)E$ and the momentum is referred as ultrasoft momentum mode.
- If we choose $\lambda = \frac{\Lambda_{QCD}}{E}$ we can write the scaling of these partons as $(\lambda, \lambda, \lambda)E$ and the momentum is referred as soft momentum mode.
- We can classify two different effective theories SCET$_I$ and SCET$_{II}$ according to the momenta modes in the process under consideration:
- SCET$_I$: When we have only collinear and ultrsoft momentum modes as inclusive decay of a heavy meson such as $B \to X_s^* \gamma$ at the end point region and $e^- p \to e^- X$ at the threshold region.
- SCET$_{II}$: When we have only collinear and soft momentum modes as in semi-inclusive or exclusive decays of a heavy meson such as $B \to D\pi$, $B \to K\pi$,....etc.

- In order to write the scaling of the collinear quark field we rewrite the momentum of the collinear quark as

(1) $$p = \tilde{p} + k$$

where

(2) $$\tilde{p} \equiv \frac{1}{2}(\bar{n} \cdot p) n^{\mu} + p_{\perp} \, .$$

- We can remove the large momenta $\tilde{p}$ by defining a new field $\psi_{n,p}$ for the collinear quark as follows

(3) $$\psi(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \psi_{n,p}$$

- $\psi_{n,p}$ contains only the component k that will be treated as a dynamical degree of freedom while $\tilde{p}$ becomes a label on the field.

- The four component field $\psi_{n,p}$ can be expressed in terms of two two-components spinors $\xi_{n,p}$ and $\xi_{\bar{n},p}$ defined as follows

(4)
$$\begin{aligned} \xi_{n,p} &= \frac{\not{n}\,\not{\bar{n}}}{4}\psi_{n,p} \\ \xi_{\bar{n},p} &= \frac{\not{\bar{n}}\,\not{n}}{4}\psi_{n,p} \end{aligned}$$

The scaling $\xi_{n,p}$ and $\xi_{\bar{n},p}$ can be obtained using

(5)
$$\langle 0|T\left\{\psi_i(x),\bar{\psi}_j(y)\right\}|0\rangle = \int \frac{d^4p}{(2\pi)^4}\,\frac{i\,\not{p}_{ij}}{p^2+i\epsilon}\,e^{-ip\cdot(x-y)}\ .$$

Assuming collinear momentum scaling $\sim(\lambda^2,1,\lambda)$, one finds that $p^2\sim O(\lambda^2)$ and d^4p scales like $O(\lambda^4)$.

- Thus, the two-component spinor fields scale as $\xi_{n,p}\sim O(\lambda)$ and $\xi_{\bar{n},p}\sim O(\lambda^2)$.
- The scaling momenta modes and their corresponding fermion fields they can be obtained in a similar way and it is given in Table (1).

Momenta mode	Momentum scaling	Fermion field scaling
Hard (h)	(1,1,1)E	
Collinear (c)	$(\lambda^2, 1, \lambda)E$	λ
Hard-collinear (hc)	$(\lambda, 1, \lambda^{1/2})E$	$\lambda^{1/2}$
Soft (s)	$(\lambda, \lambda, \lambda)E$	$\lambda^{3/2}$
Ultrasoft (us)	$(\lambda^2, \lambda^2, \lambda^2)E$	λ^3
Soft-collinear (sc)	$(\lambda^2, \lambda, \lambda^{3/2})E$	λ^2

Table 1: Scaling of different momenta modes and their corresponding fermion fields

SCET Lagrangian

Few Remarks about construction of the SCET Lagrangian:

- Consideration of the kinematics:

  Kinematics allows only collinear collinear and collinear ultrasof interactions between quarks or gluons or a quark and gluon. So we have Lagrangian for collinear collinear interaction and Lagrangian for collinear ultrasof interaction.

- Matching with the QCD Lagrangian and Doing expansion in orders of λ and keeping order 0,1,2 terms.

- integrating out the off shell fluctuation through introducing Wilson lines.

Recalling that the QCD Lagrangian for massless quarks and gluons is given by

$$\mathcal{L}_{\rm QCD} = \bar{\psi}\, i \not{D}\, \psi - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} \,, \tag{6}$$

where the covariant derivative D_μ is defined as $D_\mu = \partial_\mu - igT^a A^a_\mu$, and $G_{\mu\nu}$ is the gluon field strength. Using

$$\gamma^\mu = n^\mu \, \not{\bar{n}}/2 + \bar{n}^\mu \, \not{n}/2 + \gamma^\mu_\perp \tag{7}$$

the quark part in the Lagrangian (6) can be expressed in terms of $\xi_{n,p}$ and $\xi_{\bar{n},p}$ as follows

$$\mathcal{L} = \sum_{\tilde{p},\tilde{p}'} e^{-i(\tilde{p}-\tilde{p}')\cdot x}\Big[\bar{\xi}_{n,p'}\frac{\bar{\not n}}{2}\Big(in\cdot D\Big)\xi_{n,p} + \bar{\xi}_{\bar{n},p'}\frac{\not n}{2}\Big(\bar{n}\cdot p + i\bar{n}\cdot D\Big)\xi_{\bar{n},p}$$

$$+\bar{\xi}_{n,p'}\Big(\not p_\perp + i\not D_\perp\Big)\xi_{\bar{n},p} + \bar{\xi}_{\bar{n},p'}\Big(\not p_\perp + i\not D_\perp\Big)\xi_{n,p}\Big]. \quad (8)$$

Using the equation of motion, one can eliminate the small component $\xi_{\bar{n},p}$ in favor of $\xi_{n,p}$

$$\mathcal{L} = \sum_{\tilde{p},\tilde{p}'} e^{-i(\tilde{p}-\tilde{p}')\cdot x}\bar{\xi}_{n,p'}\left[n\cdot iD + (\not p_\perp + i\not D_\perp)\frac{1}{\bar{n}\cdot p + \bar{n}\cdot iD}(\not p_\perp + i\not D_\perp)\right]\frac{\bar{\not n}}{2}\xi_{n,p}. \quad (9)$$

The covariant derivative D^μ includes only collinear and ultrasoft gluons, $A^\mu = A^\mu_c + A^\mu_{us}$ as the interaction of soft gluons and collinear quarks is forbidden kinematically.

- it is convenient to separate the collinear and ultrasoft gluon field such that the covariant derivative D^μ contains only the ultrasoft collinear gluons. We define the collinear gluon as $A^\mu_c(x) = e^{-i\tilde{q}\cdot x}A^\mu_{n,q}(x)$ in a similar way to the collinear quark field and so Eq. (9)can be written as

$$\mathcal{L} = \sum_{\tilde{p},\tilde{p}',\tilde{q}} e^{-i(\tilde{p}-\tilde{p}')\cdot x}\bar{\xi}_{n,p'}\Bigg[n\cdot iD + ge^{-i\tilde{q}\cdot x}n\cdot A_{n,q} + \left(\not{p}_\perp + i\not{D}_\perp + ge^{-i\tilde{q}\cdot x}\not{A}^\perp_{n,q}\right)$$

$$\frac{1}{\bar{n}\cdot p + \bar{n}\cdot iD + ge^{-i\tilde{q}\cdot x}\bar{n}\cdot A_{n,q}}\left(\not{p}_\perp + i\not{D}_\perp + ge^{-i\tilde{q}\cdot x}\not{A}^\perp_{n,q}\right)\Bigg]\frac{\not{\bar{n}}}{2}\xi_{n,p} \tag{10}$$

- Expanding Eq. (10) in powers of gA_c, we obtain at order λ^0

$$\begin{aligned}\mathcal{L}_{cus} &= \bar{\xi}_{n,p}\frac{p_\perp^2}{\bar{n}\cdot p}\frac{\not{\bar{n}}}{2}\xi_{n,p} + \bar{\xi}_{n,p}n\cdot iD\frac{\not{\bar{n}}}{2}\xi_{n,p} \\ &\quad +\bar{\xi}_{n,p+q}\Bigg[gn\cdot A_{n,q} + g\not{A}^\perp_{n,q}\frac{\not{p}_\perp}{\bar{n}\cdot p} + \frac{\not{p}_\perp + \not{q}_\perp}{\bar{n}\cdot(p+q)}g\not{A}^\perp_{n,q} \\ &\quad - \frac{\not{p}_\perp + \not{q}_\perp}{\bar{n}\cdot(p+q)}g\bar{n}\cdot A_{n,q}\frac{\not{p}_\perp}{\bar{n}\cdot p}\Bigg]\frac{\not{\bar{n}}}{2}\xi_{n,p} + \ldots + \mathcal{O}(\lambda)\end{aligned} \tag{11}$$

The first term in Eq. (11) gives the propagator for the collinear quarks, the second term gives its interaction with an ultrasoft gluon and the third term gives its interaction with a collinear gluons.

- The order λ lagrangian of the quark collinear field is given by

(12) $$\mathcal{L}_{\xi\xi}^{(1)} = \bar{\xi}_n i \not{D}_\perp^{us} \frac{1}{i\bar{n}\cdot D_c} i \not{D}_\perp^c \frac{\not{\bar{n}}}{2} \xi_n \text{ + h.c.}$$

- We need also to carry the matching of the full QCD lagrangian to the mixed usoft quark-collinear quark Lagrangian.

We split the quark field into the ultrasoft quark field q_{us} and the collinear quark field

(13) $$\psi = \xi_n + \xi_{\bar{n}} + q_{us}$$

Decompose the Dirac matrix γ^μ in light-cone coordinates as before and using $\not{n}\xi_n = \not{\bar{n}}\xi_{\bar{n},p} = 0$, the kinetic term in the Lagrangian $\mathcal{L} = \bar{\psi}\, i \not{D}\, \psi + \ldots$ can be matched to

(14) $$\begin{aligned} \mathcal{L}_{\xi q} &= \left[\bar{\xi}_n g \not{A}_n\, q_{us} + \bar{\xi}_n \frac{\not{\bar{n}}}{2} i \not{D}_\perp \frac{1}{i\bar{n}\cdot D} g \not{A}_n\, q_{us}\right] + \left[\bar{q}_{us}\, g \not{A}_n\, \xi_n \right. \\ &+ \left. \bar{q}_{us}\, g \not{A}_n \frac{1}{i\bar{n}\cdot D} i \not{D}_\perp \frac{\not{\bar{n}}}{2}\, \xi_n\right] \end{aligned}$$

Expanding Eq. (14) to second order in λ gives

$$\mathcal{L}^{(1)}_{\xi q} = \bar{\xi}_n \left(g \not{A}^c_\perp - i \not{D}^c_\perp \frac{1}{i\bar{n}\cdot D_c} g\bar{n}\cdot A_c \right) q_{us} \text{ + h.c.}$$

$$\mathcal{L}^{(2)}_{\xi q} = \bar{\xi}_n \frac{\not{\bar{n}}}{2} \left(g n\cdot A^c + i \not{D}^c_\perp \frac{1}{i\bar{n}\cdot D_c} g \not{A}^c_\perp \right) q_{us} - \bar{\xi}_n i \not{D}^{us}_\perp \frac{1}{i\bar{n}\cdot D_c} g\bar{n}\cdot A^c q_{us} \text{ + h.c.}$$

(15)

Introducing the label operators $\bar{\mathcal{P}} \sim \lambda^0$ and $\mathcal{P}^\mu_\perp \sim \lambda$ such that, $\bar{\mathcal{P}}\,\xi_{n,p} = (\bar{n}\cdot p)\,\xi_{n,p}$ allows us to redefine the collinear covariant derivatives as follows

(16) $$i\bar{n}\cdot D_c = \bar{\mathcal{P}} + g\bar{n}\cdot A_n\,, \qquad iD^{\perp\mu}_c = \bar{\mathcal{P}}^\mu_\perp + gA^{\perp\mu}_n\,,$$

The ultrasoft covariant derivatives can be written as

(17) $$i\bar{n}\cdot D_{us} = i\bar{n}\cdot\partial + g\bar{n}\cdot A_{us},\ iD^{\perp\mu}_{us} = i\partial^\mu_\perp + gA^{\perp\mu}_{us}\,.$$

- In $SCET$, the off-shell fluctuations resulting from attaching gluons to quarks can be integrated out with the help of the so-called Wilson lines.

- For example, attaching collinear gluons to a heavy quark results in intermediate states (propagators) which are off-shell by an amount of order E. These intermediate states must be integrated out as SCET is an effective theory below the energy scale E. This can be done by a field redefinition $q(x) \to W(x)q^0(x)$ where q(x) is the quark field and W is the collinear Wilson line defined as

(18) $$W = \Big[\sum_{\text{perms}} \exp\Big(-\frac{g}{\bar{\mathcal{P}}}\, \bar{n}\cdot A_{n,q}(x) \Big)\Big],$$

where the label operators only act on fields inside the square brackets. Performing this field redefinition will lead to a lagrangian that contains the field $q^0(x)$ which no longer couples to the collinear gluons.

we can write eq.(15) as:

(19) $$\mathcal{L}^{(1)}_{\xi q} = ig\,\bar{\xi}_n \,\frac{1}{i\bar{n}\cdot D_c}\, \bar{\not{B}}^c_\perp W q_{us} \text{ + h.c.}$$

$$\mathcal{L}^{(2a)}_{\xi q} = ig\,\bar{\xi}_n \frac{1}{i\bar{n}\cdot D_c}\, \bar{\not{M}}\, W\, q_{us} \text{ + h.c. },$$

(20)

(21) $$\mathcal{L}_{\xi q}^{(2b)} = ig\,\bar{\xi}_n \frac{\bar{\not n}}{2} i\,\not D_\perp^{\,c} \frac{1}{(i\bar{n}\cdot D_c)^2}\ \bar{\not B}_\perp^c W\, q_{us} \text{ + h.c.}$$

where the following operators are introduced

(22) $$ig\,\bar{\not B}_\perp^c = [i\bar{n}\cdot D^c, i\,\not D_\perp^c].$$

and

(23) $$ig\,\bar{\not M} = [i\bar{n}\cdot D^c, i\,\not D^{\,us} + \frac{\bar{\not n}}{2} g n\cdot A^c].$$

Finally, we give here the Lagrangian for the collinear gluons

(24) $$\mathcal{L}_{cg} = \frac{1}{2g^2} tr\left\{[iD^\mu, iD^\nu]^2\right\}$$

- The order λ collinear gluons Lagrangian which can be obtained by Expanding eq.(24) in powers of λ

(25) $$\mathcal{L}_{cg}^{(1)} = \frac{2}{g^2} tr\left\{[i\mathcal{D}^\mu, iD_c^{\perp\,\nu}][i\mathcal{D}_\mu, iD_{us\nu}^{\perp}]\right\}$$

where

(26) $$\mathcal{D}^\mu = D_c^\mu + \bar{n}^\mu n\cdot\frac{D_{us}}{2}$$

SCET Hamiltonian for $\Delta B = 1$ decays

We start by writing the Weak Effective Hamiltonian in the SM describing the weak interactions of the B meson namely $\Delta B = 1$ decays.

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(D)} \left(C_1\, Q_1^p + C_2\, Q_2^p + \sum_{i=3,\ldots,10} C_i\, Q_i + C_{7\gamma}\, Q_{7\gamma} + C_{8g}\, Q_{8g} \right) \qquad + \text{h.c.}$$

(27)

where $\lambda_p^{(D)} \equiv V_{pb} V_{pD}^*$ with $D = d, s$ and

$$\begin{aligned}
Q_1^p &= (\bar{p} b)_{V-A} (\bar{D} p)_{V-A}\,, & Q_2^p &= (\bar{p}_\alpha b_\beta)_{V-A} (\bar{D}_\beta p_\alpha)_{V-A}\,, \\
Q_3 &= (\bar{D} b)_{V-A} \sum\nolimits_q (\bar{q} q)_{V-A}\,, & Q_4 &= (\bar{D}_\alpha b_\beta)_{V-A} \sum\nolimits_q (\bar{q}_\beta q_\alpha)_{V-A}\,, \\
Q_5 &= (\bar{D} b)_{V-A} \sum\nolimits_q (\bar{q} q)_{V+A}\,, & Q_6 &= (\bar{D}_\alpha b_\beta)_{V-A} \sum\nolimits_q (\bar{q}_\beta q_\alpha)_{V+A}\,, \\
Q_7 &= (\bar{D} b)_{V-A} \sum\nolimits_q \tfrac{3}{2} e_q (\bar{q} q)_{V+A}\,, & Q_8 &= (\bar{D}_\alpha b_\beta)_{V-A} \sum\nolimits_q \tfrac{3}{2} e_q (\bar{q}_\beta q_\alpha)_{V+A}\,, \\
Q_9 &= (\bar{D} b)_{V-A} \sum\nolimits_q \tfrac{3}{2} e_q (\bar{q} q)_{V-A}\,, & Q_{10} &= (\bar{D}_\alpha b_\beta)_{V-A} \sum\nolimits_q \tfrac{3}{2} e_q (\bar{q}_\beta q_\alpha)_{V-A}
\end{aligned}$$

(28)

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{D}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b\,, \qquad Q_{8g} = \frac{g_s}{8\pi^2} m_b \bar{D}_\alpha\sigma_{\mu\nu}(1+\gamma_5)t^A_{\alpha\beta}G^A_{\mu\nu}b_\beta$$

(29)

where $C_i(M_W)$ are the Wilson coefficients α and β stand for color indices, $t^A_{\alpha\beta}$ are the $SU(3)_c$ color matrices and $\sigma^{\mu\nu} = \frac{1}{2}i[\gamma^\mu, \gamma^\nu]$. e_q are quark electric charges in units of e, $(\bar{q}q)_{V\pm A} \equiv \bar{q}\gamma_\mu(1\pm\gamma_5)q$, and q runs over u, d, s, c, and b quark labels.

Matching of the weak effective Hamiltonian $\mathcal{H}_{\rm eff}$ to the corresponding SCET gauge invariant operators requires two step matching:

- First the full QCD effective weak Hamiltonian is matched to the corresponding weak Hamiltonian in $SCET_I$ by integrating out the hard scale m_b. Hence, after integrating out the $\bar{b}b$ pairs in the electroweak Penguin operators, we can write

$$\begin{aligned} Q_9 &= \frac{3}{2}Q_2^u + \frac{3}{2}Q_2^c - \frac{2}{2}Q_3 \\ Q_{10} &= \frac{3}{2}Q_1^u + \frac{3}{2}Q_1^c - \frac{2}{2}Q_4. \end{aligned} \quad (30)$$

which leads to remove the operators Q_9 and Q_{10} from the effective Hamiltonian.

- Second, the $SCET_I$ weak Hamiltonian is matched to the weak Hamiltonian $SCET_{II}$ by integrating out the hard collinear modes with $p^2 \sim \Lambda m_b$ as we will show in details below.

At leading power in $(1/m_b)$ expansion, the effective Hamiltonian in eq.(27) is matched into $SCET_I$ Hamiltonian as follows

$$H_W = \frac{2G_F}{\sqrt{2}} \sum_{n,\bar{n}} \Big\{ \sum_i \int [d\omega_j]_{j=1}^{3} c_i^{(f)}(\omega_j) Q_{if}^{(0)}(\omega_j) + \sum_i \int [d\omega_j]_{j=1}^{4} b_i^{(f)}(\omega_j) Q_{if}^{(1)}(\omega_j) + Q_{c\bar{c}} + ... \Big\} \tag{31}$$

where $f = d$ or s, $c_i^{(f)}$ and $b_i^{(f)}$ are Wilson coefficients corresponding to $O(\lambda^0)$ operators ($Q_{if}^{(0)}$) and $O(\lambda)$ operator ($Q_{if}^{(1)}$). $Q_{c\bar{c}}$ denotes the operator corresponding to the long distance charm Penguin. For $B \to \pi\pi$ decays as an example, the operators $Q_{if}^{(0)}$ and $Q_{if}^{(1)}$ in Eq.(31) are given by

$$Q_{1f}^{(0)} = [\bar{u}_{n,\omega_1} \bar{\not{n}} P_L b_\nu][\bar{f}_{\bar{n},\omega_2} \not{n} P_L u_{\bar{n},\omega_3}] \tag{32}$$

$$Q^{(0)}_{2f,3f} = [\bar{f}_{n,\omega_1}\bar{\not n}P_L b_\nu][\bar{u}_{\bar{n},\omega_2}\not n P_{L,R} u_{\bar{n},\omega_3}],$$
$$Q^{(0)}_{4f} = [\bar{q}_{n,\omega_1}\bar{\not n}P_L b_\nu][\bar{f}_{\bar{n},\omega_2}\not n P_L q_{\bar{n},\omega_3}],$$

(33)

$$Q^{(1)}_{1f} = \frac{-2}{m_b}[\bar{u}_{n,\omega_1} ig\not{\mathcal{B}}^{\perp}_{n,\omega_4} P_L b_\nu][\bar{f}_{\bar{n},\omega_2}\not n P_L u_{\bar{n},\omega_3}],$$
$$Q^{(1)}_{2f,3f} = \frac{-2}{m_b}[\bar{f}_{n,\omega_1} ig\not{\mathcal{B}}^{\perp}_{n,\omega_4} P_L b_\nu][\bar{u}_{\bar{n},\omega_2}\not n P_{L,R} u_{\bar{n},\omega_3}],$$
$$Q^{(1)}_{4f} = \frac{-2}{m_b}[\bar{q}_{n,\omega_1} ig\not{\mathcal{B}}^{\perp}_{n,\omega_4} P_L b_\nu][\bar{f}_{\bar{n},\omega_2}\not n P_L q_{\bar{n},\omega_3}],$$

(34)

where a sum over $q = u, d, s$ is understood and

(35) $$q_{n,\omega} = [\delta(\omega - \bar{n}.P)W_n^\dagger \xi_n^{(q)}],$$

(36) $$ig\mathcal{B}^{\perp\mu}_{n,\omega} = \frac{1}{(-\omega)}[W_n^\dagger[i\bar{n}.D_{c,n}, iD^\mu_{n,\perp}]W_n\delta(\omega - \bar{n}.P^\dagger)].$$

with $\bar{n}.P$ is the operator that project out the large momentum component of the collinear quark field and P operates only inside the square brackets. The b_ν is the standard heavy quark effective field (HQET). The Wilson coefficients corresponding to the operators $Q_{if}^{(0)}$ are given by

$$\begin{aligned} c_{1,2}^{(f)} &= \lambda_u^{(f)}\left[C_{1,2}+\frac{1}{N}C_{2,1}\right]-\lambda_t^{(f)}\frac{3}{2}\left[\frac{1}{N}C_{9,10}+C_{10,9}\right]+\Delta c_{1,2}^{(f)} \\ c_3^{(f)} &= -\frac{3}{2}\lambda_t^{(f)}\left[C_7+\frac{1}{N}C_8\right]+\Delta c_3^{(f)} \\ c_4(f) &= -\lambda_t^{(f)}\left[\frac{1}{N}C_3+C_4-\frac{1}{2N}C_9-\frac{1}{2}C_10\right]+\Delta c_4^{(f)} \end{aligned} \tag{37}$$

and for $Q_{if}^{(1)}$ we have

$$\begin{aligned} b_{1,2}^{(f)} &= \lambda_u^{(f)}\left[C_{1,2}+\frac{1}{N}\left(1-\frac{m_b}{\omega_3}\right)C_{2,1}\right]-\lambda_t^{(f)}\frac{3}{2}\left[C_{10,9}+\frac{1}{N}\left(1-\frac{m_b}{\omega_3}\right)C_{9,10}\right]+\Delta b_{1,2}^{(f)} \\ b_3^{(f)} &= -\lambda_t^{(f)}\frac{3}{2}\left[C_7+\left(1-\frac{m_b}{\omega_2}\right)\frac{1}{N}C_8\right]+\Delta b_3^{(f)} \\ b_4^{(f)} &= -\lambda_t^{(f)}\left[C_4+\frac{1}{N}\left(1-\frac{m_b}{\omega_3}\right)C_3\right]+\lambda_t^{(f)}\frac{1}{2}\left[C_{10}+\frac{1}{N}\left(1-\frac{m_b}{\omega_3}\right)C_9\right]+\Delta b_4^{(f)} \end{aligned} \tag{38}$$

where $\omega_2 = m_b u$ and $\omega_3 = -m_b \bar{u}$. u and $\bar{u} = 1 - u$ are momentum fractions for the quark and antiquark $\bar{n}$ collinear fields. The $\Delta c_i^{(f)}$ and $\Delta b_i^{(f)}$ denote terms depending on α_s generated by matching from H_W.

- The matching of $SCET_I$ onto $SCET_{II}$ is performed by integrating out the hard collinear modes with $p^2 \sim \Lambda m_b$. Therefore, the collinear n and collinear $\bar{n}$ sectors are decoupled and the operators $Q_{if}^{(0,1)}$ factor into

(39) $$Q_{if}^{(0,1)} = \tilde{Q}_{if}^{(0,1)} \tilde{Q}_{if}^{\bar{n}}.$$

where, after dropping f , $\tilde{Q}_{if}^{(0,1)}$ and $\tilde{Q}_{if}^{\bar{n}}$ are given by

(40) $$\tilde{Q}_i^{(0)} = \left[\bar{q}_{n,\omega_1}^i \, \bar{\not{n}} P_L b_v\right] \qquad \tilde{Q}_i^{(1)} = \frac{-2}{m_b}\left[\bar{q}_{n,\omega_1}^i \, ig \, \not{\mathcal{B}}_{n,\omega_4}^{\perp} P_L b_v\right]$$

and

(41) $$\tilde{Q}_i^{\bar{n}} = \begin{cases} \bar{q}_{\bar{n},\omega_2}^i \, \not{n} P_L q'^i_{\bar{n},\omega_3} & i = 1,2,4,5 \\ \bar{q}_{\bar{n},\omega_2}^i \, \not{n} P_R q'^i_{\bar{n},\omega_3} & i = 3,6 \end{cases}$$

$B \to M_1 M_2$ decay amplitudes in SCET

The leading order amplitude can be generated through the time ordered products of the operators $Q_i^{(0)}$ and $Q_i^{(1)}$ and the subleading Lagrangians

$$\begin{aligned} T_1[\tilde{Q}_i^{(0)}] &= \int d^4y\, d^4y'\, T[\tilde{Q}_i^{(0)}(0), i\mathcal{L}^{(1)}_{\xi_n \xi_n}(y') + i\mathcal{L}^{(1)}_{cg}(y'), i\mathcal{L}^{(1)}_{\xi_n q}(y)] \\ &+ \int d^4y T[Q_{if}^{(0)}(0), i\mathcal{L}^{(1,2)}_{\xi_n q}(y)], \\ T_2[\tilde{Q}_i^{(1)}] &= \int d^4y T[\tilde{Q}_i^{(1)}(0), i\mathcal{L}^{(1)}_{\xi_n q}(y)]. \end{aligned} \tag{42}$$

The matrix elements of the time ordered products ($T_{1,2}$)and $\tilde{Q}_i^{\bar{n}}$ can be expressed in terms of the following hadronic parameters

$$\begin{aligned} \left\langle M_n \middle| T_1\left[\bar{q}^L_{n\omega_1}\ \bar{n}\!\!\!/ b_v\right] \middle| B\right\rangle &= \mathcal{C}^{BM}_{q_L} \bar{\delta}_{\omega_1}\, m_B\, \zeta^{BM}, \\ \left\langle M_n \middle| T_2\left[\bar{q}^L_{n\omega_1} ig\, \mathcal{B}\!\!\!/^{\perp}_{n\omega_4} b_v\right] \middle| B\right\rangle &= -\mathcal{C}^{BM}_{q_L} \bar{\delta}_{\omega_1\omega_4} \frac{m_B}{2} \zeta^{BM}_J(z), \\ \left\langle M_{\bar{n}} \middle| \bar{q}'^L_{\bar{n}\omega_2}\ n\!\!\!/ q^L_{\bar{n}\omega_3} \middle| 0\right\rangle &= \frac{i}{2}\, \mathcal{C}^M_{q'_L q} \bar{\delta}_{\omega_2\omega_3}, f_M \phi_M(u), \\ \left\langle M_{\bar{n}} \middle| \bar{q}'^R_{\bar{n}\omega_2}\ n\!\!\!/ q^R_{\bar{n}\omega_3} \middle| 0\right\rangle &= \frac{i}{2}\, \mathcal{C}^M_{q'_R q} \bar{\delta}_{\omega_2\omega_3} f_M \phi_M(u). \end{aligned} \tag{43}$$

The parameters ζ^{BM}, ζ_J^{BM} are treated as hadronic parameters that can be determined through the fit to the non leptonic decay data. Finally, the amplitude at leading order in $1/m_b$ expansion and to all orders in $\alpha_S(m_b)$ is given by

$$\begin{aligned} A^{LO}(\bar{B} \to M_1 M_2) &= -i\Big\langle M_1 M_2 \Big| H_W \Big| \bar{B} \Big\rangle \\ &= \frac{G_F m_B^2}{\sqrt{2}} f_{M_1} \Bigg[\int_0^1 du dz T_{1J}(u,z) \zeta_J^{BM_2}(z) \phi_{M_1}(u) \\ &+ \zeta^{BM_2} \int_0^1 du T_{1\zeta}(u) \phi_{M_1}(u) \Bigg] + \lambda_c^{(f)} A_{cc}^{M_1 M_2} + (1 \leftrightarrow 2). \end{aligned} \tag{44}$$

The hard kernels $T_{1\zeta}$ and T_{1J} are functions of linear combinations of the matching coefficients $c_i^{(f)}(u)$ and $b_i^{(f)}(u,z)$ and depend on the final state mesons. $T_{1\zeta}$ and T_{1J} for some specific decay channels and $T_{1\zeta}$ and T_{1J} are given in Table(2).

Decay Mode $(M_1 M_2)$	$T_{1\xi}(u)$	$T_{2\xi}(u)$
$\pi^0 K^-$	$\frac{1}{\sqrt{2}}(c_2^{(s)} - c_3^{(s)})$	$\frac{1}{\sqrt{2}}(c_1^{(s)} + c_4^{(s)})$
$\pi^+ K^-$	0	$c_1^{(s)} + c_4^{(s)}$
$\pi^0 \bar{K}^0$	$\frac{1}{\sqrt{2}}(c_2^{(s)} - c_3^{(s)})$	$-\frac{1}{\sqrt{2}} c_4^{(s)}$
$\pi^- \bar{K}^0$	0	$-c_4^{(s)}$
$\bar{K}^0 \phi$	0	$c_4^{(s)} + c_5^{(s)} + c_6^{(s)}$
$\bar{K}^0 \eta_s$	$c_4^{(s)}$	$c_4^{(s)} + c_5^{(s)} - c_6^{(s)}$
$\bar{K}^0 \eta_q$	$\frac{1}{\sqrt{2}} c_4^{(s)}$	$\frac{1}{\sqrt{2}}(c_2^{(s)} - c_3^{(s)} + 2c_5^{(s)} - 2c_6^{(s)})$

Table 2: Hard kernels for $\Delta S = 1$ decays of B^-, $\bar{B}^0$ and $\bar{B}_s^0$ into some final states. The coefficients $T_{1J,2J}(u,z)$ for all these states are *identical* to $T_{1\zeta,2\zeta}(u)$ with each $c_i^{(f)}(u)$ replaced by $b_i^{(f)}(u,z)$.