Seminar given at NTHU/NCTS Journal Club, 2010 Sep. 21

Holographic Anyons in the ABJM theory

Shoichi Kawamoto (NTNU, Taiwan)
with Feng-Li Lin (NTNU)

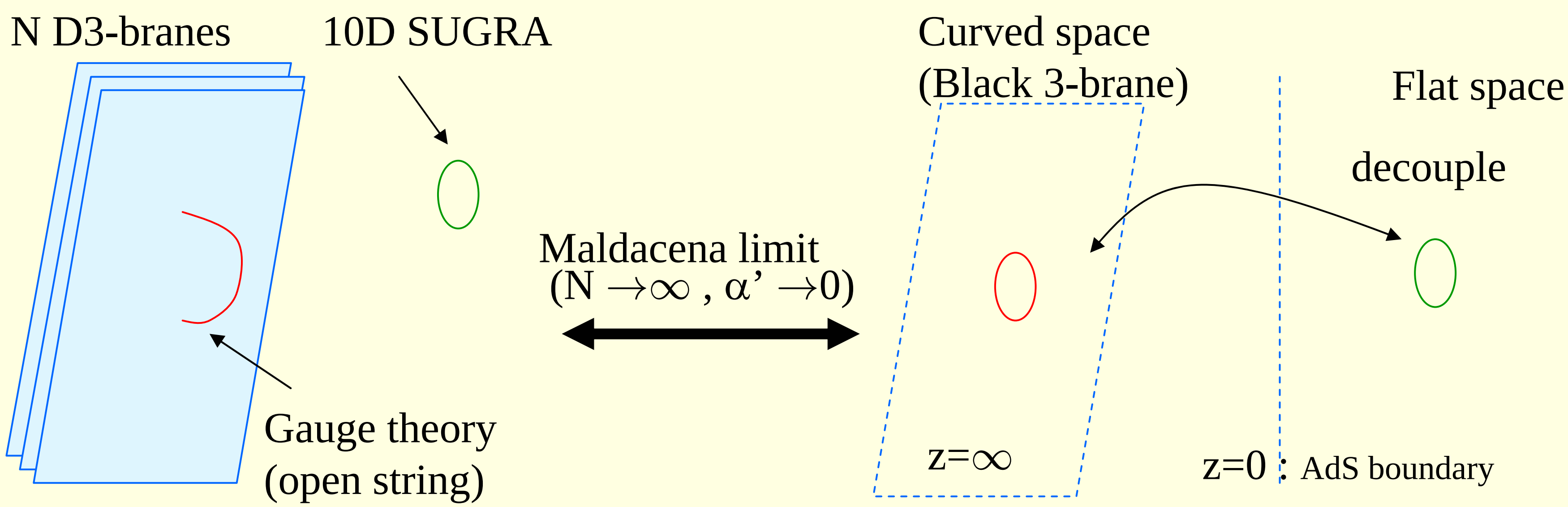
Based on JHEP 1002:059(2010)

Motivation

- AdS/CFT correspondence provides a useful tool to probe strong coupling dynamics of (gauge) field theory.
- Recently, this is applied to the analysis of the problems of condensed matter theory.
- For example, holographic superconductors.
- Today, we construct another tool for this application (not application itself, today)

AdS/CFT correspondence

<u>Best known case: D3-branes and N=4 super Yang-Mills theory</u>



N=4 Supersymmetric Yang-Mills ⟷ **String (SUGRA) in $AdS_5 * S^5$**

(Strong coupling regime: $\lambda = g_{YM}^2 N = \infty$)

(weak curvature: $L^4 = \lambda \alpha'^2 = \infty$)

Correspondence: Symmetry, States, correlation functions,....

Various AdS/CFT correspondence

There are several variants of "AdS/CFT" correspondence (or rather "gauge/gravity").

Symmetry

Superconformal	4D N=4 super YM (AdS_5), 3D N=6 CS (AdS_4)
Less SUSY	orbifolding, etc.
No SUSY, conformal	finite temperature (AdS-BH),
Not SUSY, nonconformal	holographic QCD,
"phenomenological" (not from string theory)	holographic superconductor

more reliable correspondence, field theory side is known

Multiple M2 and AdS_4

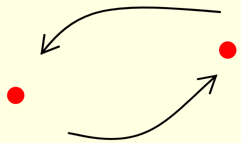
- It is known that M2-brane background also admits AdS_4 description in a near-horizon limit.
- However, no superconformal theory on M2 has been known for long time.
- Recently, there are much progress on multiple M2 theory: BLG model and ABJM model.
- Both are superconformal Chern-Simons theory in 2+1 dimenions.

→ Curiosity in 2+1D: Anyonic states

What is Anyon?

Anyon is a particles that obeys fractional statistics
– between bosons and fermions

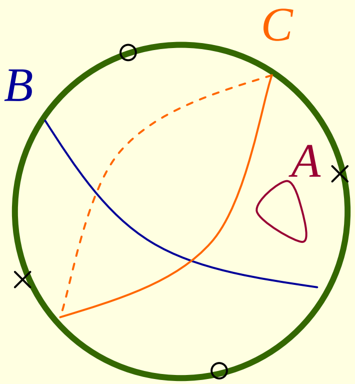
Consider the (adiabatic) exchange of the two identical particles:



relative configuration space of these two particles in (d+1)-dim.

$$\frac{\mathbf{R}^{d} \setminus \{0\}}{\mathbf{Z}_2} \cong \mathbf{RP}^{d-1}$$

Introduction to Anyons

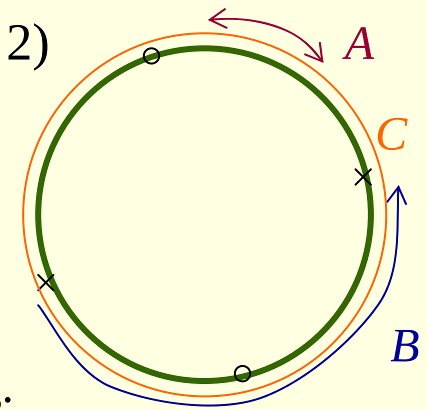


$\mathbf{RP}^{d-1}(d \geq 3)$

- *A* Trivial Path
- *B* One exchange (noncontractable)
- *C* Exchange twice (contractable)

$\pi_1(\mathbf{RP}^{d-1}) = Z_2$ ⟶ ± 1 phase is allowed (boson and fermion)

$\mathbf{RP}^1(d = 2)$



- *A* Trivial Path
- *B* One exchange (noncontractable)
- *C* Exchange twice (noncontractable)

$\pi_1(\mathbf{RP}^2) = Z$ ⟶ $e^{\pm in\theta}$ phase for n winding.

Anyon!!

Anyons and Chern-Simons theory

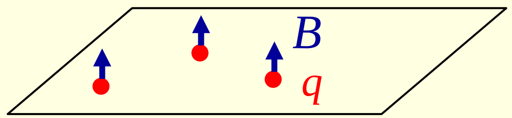
Abelian Chern-Simons Lagrangean with the source term:

$$\mathcal{L} = \frac{k}{4\pi} \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - A_\mu J^\mu$$

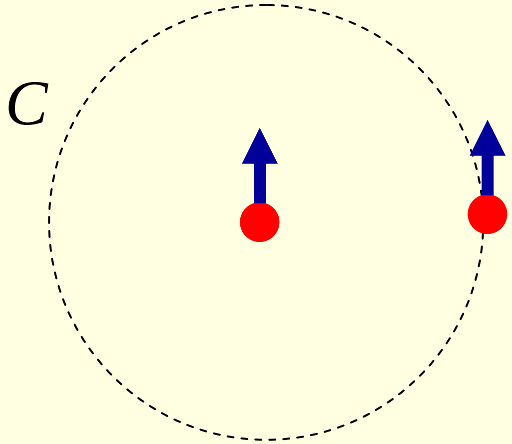
For point charge distribution $J = (\rho, 0, 0)$ $\rho = q \sum \delta(x - x_i)$

Equation of motion: $B = \frac{\rho}{k} = \frac{q}{k} \sum \delta(x - x_i)$

Chern-Simons term attach a magnetic flux to each particle.



Anyon phase as AB phase



Consider that a particle is adiabatically going around the other particle.

Due to the magnetic flux, the particle gets the Aharanov-Bohm phase as,

$$\exp\left(iq\int_C A\right) = \exp\left(\frac{iq^2}{k}\right)$$

Exchange of these two particle is understood as “half” of this procedure.

Anyon phase is given by $\theta = \frac{\phi_{AB}}{2} = \frac{q^2}{2k}$

Holographic realization of Anyon

Motivation:

•Anyon is not just a mathematical construct, but appears in a real physical system like Fractional Quantum Hall System. So....

- Question: Can we realize anyons holographically?
- Setup: Want to have 2+1D Chern-Simons theory and its dual
- ABJM: 2+1D N=6 Chern-Simons-matter theory and its dual description of $AdS_4 * CP^3$ is well studied.
- A good starting point!

Plan

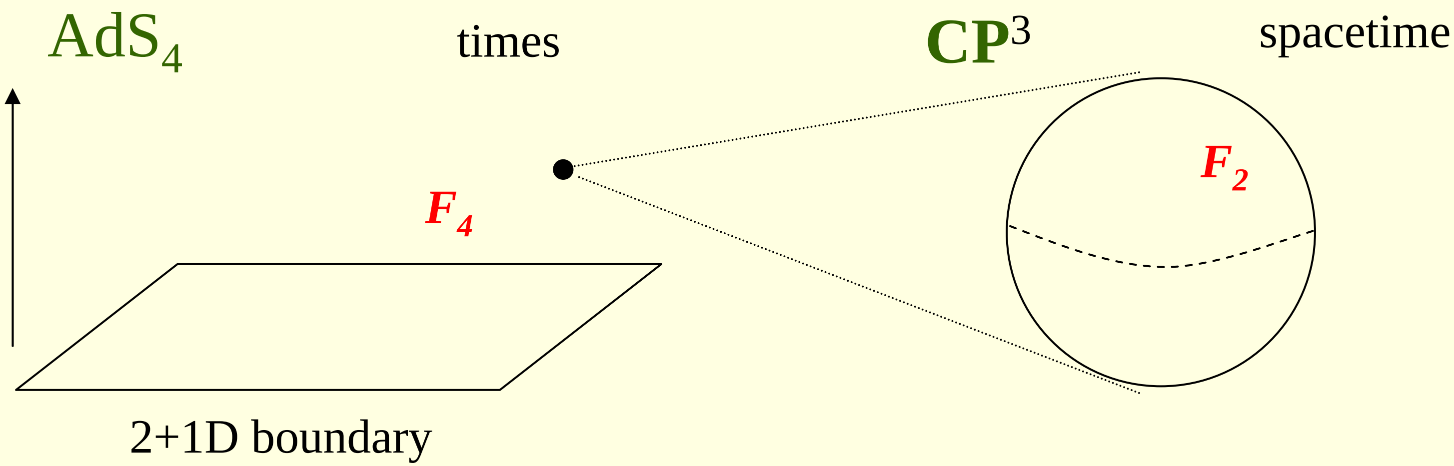
1. Introduction – What is Anyon?
2. Gravity Side
 — IIA Sugra on AdS space —
3. Field Theory Side
 — N=6 Superconformal CS —
4. Conclusion

Gravity Side

— IIA Supergravity on $AdS_4 \times \mathbf{CP}^3$ —

$AdS_4 \times \mathbf{CP}^3$ background

We start with the gravity side. The background geometry we work with is



metric

This space-time has RR four-form and two-form fluxes.

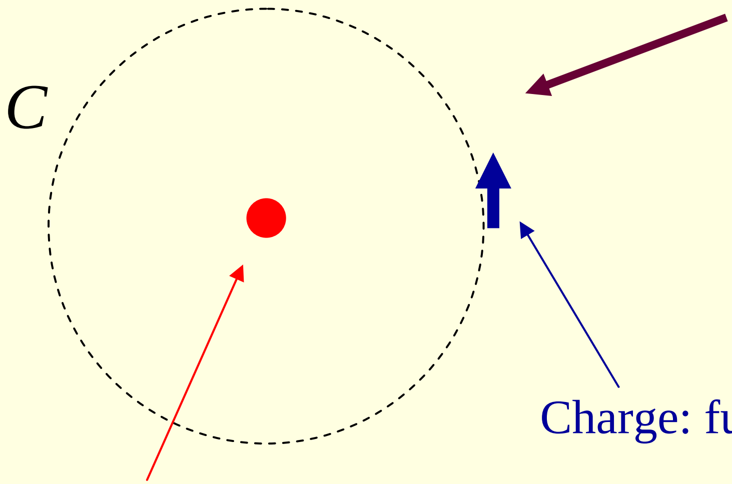
$$F_4 = \frac{3}{8} R^3 d\Omega_{AdS_4}, \quad F_2 = k d\omega, \qquad \left(d\omega \propto \text{Kahler form of } \mathbf{CP}^3\right)$$

The curvature radius is $\frac{R^3}{k} = 2^{5/2}\pi\sqrt{\lambda}$ $\left(\lambda = \frac{N}{k}\right)$ and IIA picture is valid for $\lambda \gg 1$, $g_s^2 = N^{-2}\lambda^{5/2} \ll 1$

Strategy

We are constructing an analogue of the "source-charge" pair.

"Encircling" will pick up a phase



Charge: fundamental string

$$\int B_2$$

charged under NSNS 2-form field

Source: D0-brane

together with the background 4-form,
it generates $H_3 = dB_2$

Supergravity equations of motion

(A part of) Type IIA supergravity with D0-brane sources

$$S = -\frac{1}{4\kappa_{10}^2}\int e^{-\Phi} H_3 \wedge *H_3 + e^{\frac{3\Phi}{2}} F_2 \wedge *F_2 + e^{\frac{\Phi}{2}} \tilde{F}_4 \wedge *\tilde{F}_4 + B_2 \wedge F_4 \wedge F_4$$

$$-\mu_0 \int_{D0} C_1$$

$\tilde{F}_4 = F_4 - C_1 \wedge H_3$

C_1 equation of motion $\quad d * \left(e^{\frac{3\Phi}{2}} F_2 \right) - H_3 \wedge * \left(e^{\frac{\Phi}{2}} \tilde{F}_4 \right) + 2\kappa_{10}^2 \mu_0 \, \delta(x_{D0}) = 0$

with the background four-form field F_4, D0-brane induces the NS-NS 2-form field B_2.

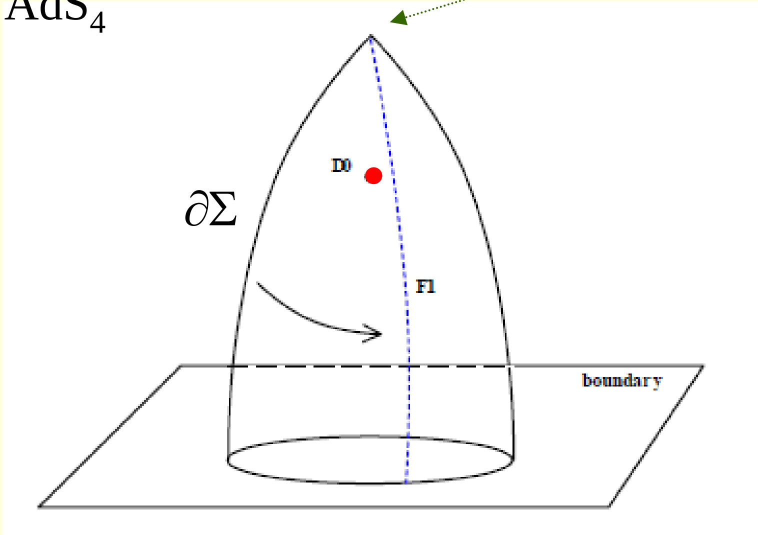
linearized equation for induced fluctuation by D0 $\quad F_n \to F_n^{(0)} + F_n$

$$H_3 \wedge \hat{\epsilon}_6 = \frac{k}{6R^6}\left((2\pi)^7 \delta^9(x_{D0}) + e^{3\Phi^{(0)}/2} d * F_2 + \frac{3}{2} d * \left(e^{3\Phi^{(0)}/2} \Phi F_2^{(0)} \right) \right)$$

D0 - F1 pair

(we discuss this end later)

AdS$_4$



We consider a fundamental string going around D0-brane.

What we need to calculate is

$$\int_C A \leftrightarrow \int_{\partial\Sigma} B_2$$

An approximation: Minimal distance between D0 and F1 is long enough to neglect the position of $\mathbf{CP}^3$.

(Originally considered by Hartnoll for D1-F1 in $AdS_5 * S^5$ and M2 in $AdS_7 * S^4$)

AB phase from D0 source

$$
\begin{aligned}
\Delta\phi_{F1D0} &= -T_{F1}\int_{\partial\Sigma} B_2 \\
&= -T_{F1}\frac{1}{vol_{\mathbf{CP}^3}}\int_{\partial\Sigma\times\mathbf{CP}^3} B_2\wedge\hat{\epsilon}_6 \\
&= -T_{F1}\frac{1}{vol_{\mathbf{CP}^3}}\int_{\Sigma\times\mathbf{CP}^3} H_3\wedge\hat{\epsilon}_6
\end{aligned}
$$

$$
= -T_{F1}\frac{1}{vol_{\mathbf{CP}^3}}\frac{k}{6R^6}\int_{\Sigma\times\mathbf{CP}^3}\left((2\pi)^7\delta^9(x_{D0}) + e^{3\Phi/2}d*F_2 + \frac{3}{2}d*(e^{3\Phi^{(0)}/2}\Phi F_2^{(0)})\right)
$$

$$
= -\frac{2\pi}{N} - \frac{2\pi}{N}\frac{1}{(2\pi)^7}e^{3\Phi/2}\int_{\partial\Sigma\times\mathbf{CP}^3} *F_2.
$$

average over $\mathbf{CP}^3$

liniearized eom

form structure does not match

F_2 is "massive" $\Delta F_2 - F_2 \propto \delta(x)$

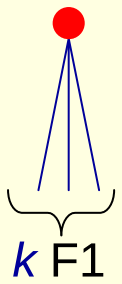
$$
\longrightarrow \quad \Delta\phi_{F1D0} = -\frac{2\pi}{N}
$$

D2 on **CP**^{1} and a survived phase

The phase we obtained is subleading in large *N*...

D2-brane wrapped on **CP**^{1} is a "baryon-vertex." (Witten)

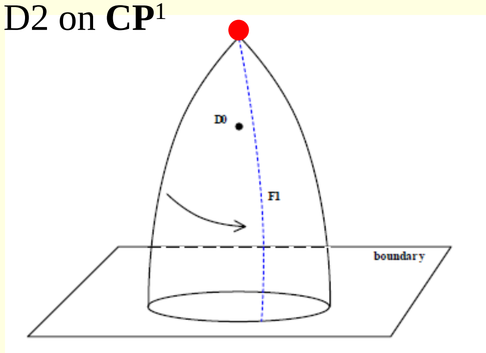
D2-brane WZ term: $\int_{t\times \mathrm{CP}^1} F_2 \wedge 2\pi\alpha' A \propto -k\int_t A_t$



To cancel this induced electric charge, we need to attach *k* fundamental strings

Then we exchange D2 (with *k* F1) and D0, we get the phase in total:

D2 on **CP**^{1}

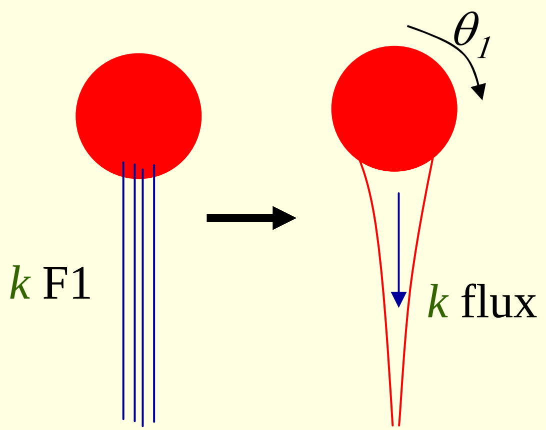


$$\Delta\phi = -2\pi\frac{-k}{N} = \frac{2\pi}{\lambda}$$

Anyon as D0-D2 bound state

So far, what we have calculated is an AB phase. Anyon is "equivalent particles" that gets fractional phase under exchange.

Can we construct BPS one?



(Callan-Maldacena, Gibbons, Drukker-Fiol)

We can construct BPS D2 + *k* F1 as a "spike" D2-brane.

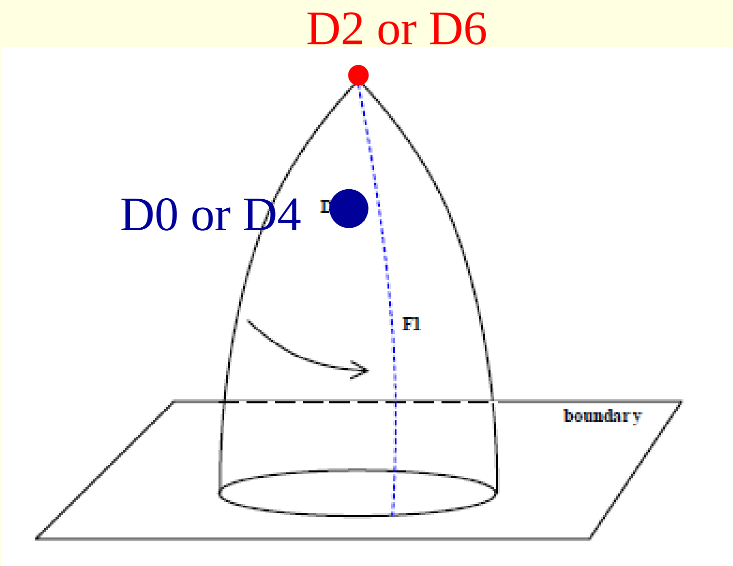
$$r(\theta_1) = A_t(\theta_1) = \frac{2r_0}{1+\cos\theta_1}$$

(SK, Lin)

However, turning on magnetic fields (D0-brane charge), BPS condition breaks down.

→ No BPS anyons.

Summary for Gravity part



details of D4-D6

F1 strings attached to D2 or D6 “baryon” pick up phase when they go around D0 or D4 source.

$$\Delta\phi_{D2-D0} = 2\pi\lambda^{-1} \qquad \Delta\phi_{D6-D4} = 2\pi\lambda$$

The other combinations will not have a trivial phase.

Note: •These phases are “long-range” effects, valid only when the source and F1s are separated enough. Otherwise, phases get modified and disappear.

•We can construct anyons by D0-D2-*k* F1 or D4-D6-N F1 bound state, though they are not BPS.

Field Theory Side

— $\mathcal{N}$=6 Chern-Simons-matter theory —

N=6 Chern-Simons-matter theory

Field theory side by ABJM is Supersymmetric Chern-Simons-matter theory:

$$\mathcal{L} = \mathcal{L}_{CS} + \mathcal{L}_{kin} + \mathcal{L}_{\psi\psi \mathrm{CC}} + \mathcal{L}_{6}$$

$$\mathcal{L}_{\mathrm{CS}} = \mathrm{Tr}\left(\frac{k}{4\pi}\left(AdA - \frac{2}{3}A^3 \right) - \frac{k}{4\pi}\left(\tilde{A}d\tilde{A} - \frac{2}{3}\tilde{A}^3 \right)\right)$$

- U(N) * U(N) Chern-Simons-matter theory in 2+1 dimensions
- N=6 supersymmetry and SU(4)*U(1) symmetry
- Symmetric under parity transformation

We mainly consider the operator made of

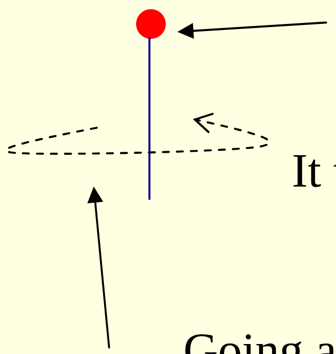
$\left(C, C^\dagger\right)$: bosons

They are $\left(\mathbf{N}, \overline{\mathbf{N}}; \mathbf{4}\right)_{+1}$ and $\left(\overline{\mathbf{N}}, \mathbf{N}; \overline{\mathbf{4}}\right)_{-1}$ of $\left(U(N)_k, U(N)_{-k}; SU(4)_R\right)_{U(1)_B}$

D0 and Chiral primary operator

The chiral primary operator that is dual to D0-brane is identified as (ABJM)

C^k But they are not gauge invariant. To make it gauge invariant, we attach a monopole operator *m*.



Magnetic flux (Dirac string)

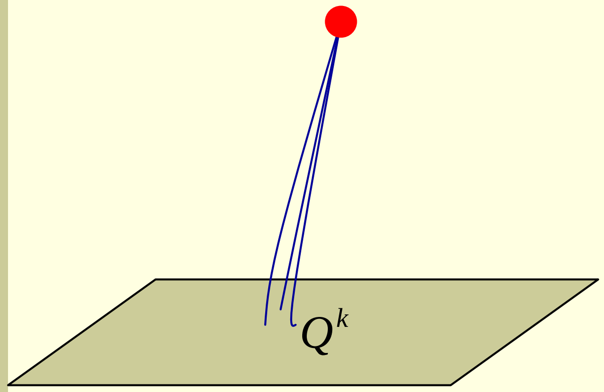
It transforms as $\left(\mathrm{Sym}\,\mathbf{N}^k, \mathrm{Sym}\,\overline{\mathbf{N}}^k\right)$

Going around this string, fields get gauge transformation, $\left(e^{\frac{i}{N}\phi}, e^{\frac{i}{N}\phi}\right)$

(ϕ : polar angle around the loop)

more

D2 baryon



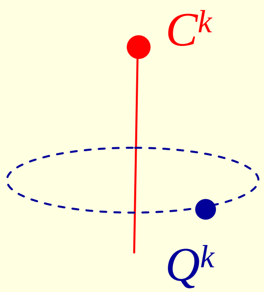
There are ***k*** fundamental strings attached to D2 on $\mathbf{CP}^1$.

The other end points need to end on the boundary, and give ***k*** fields in the fundamental representation.

Assume: They are *k* bound state of **N** of first U(N). Q^k

They are the dual of the fundamental strings, Wilson lines with *k* copies of the fundamental representation of the first U(N).

AB phase in the field theory side



Now D2-brane baryon (Q^k) is going around C^k and Dirac string.

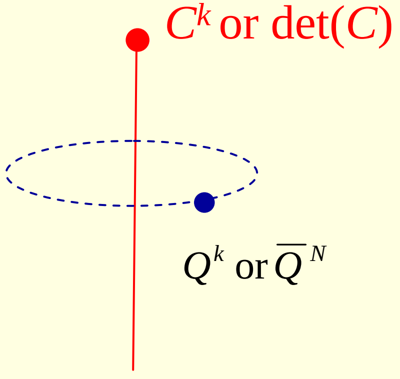
$$Q^k \to \left(e^{\frac{2\pi i}{N}} \right)^k Q^k = e^{2\pi i \frac{k}{N}} Q^k$$

Then the AB phase for this configuration is $\Delta\phi = \dfrac{k}{N} = \lambda^{-1}$

Matching with the gravity side result!!

Note: D6 baryon is *N* bound state of anti-*Q*, and gets no phase.

Summary for Field Theory part



The state corresponding to D0 or D4, we need to attach magnetic flux ('t Hooft operator) to make them gauge invariant.

Extra state from D2 or D6 baryons get the phase when going around the flux

$$\Delta\phi_{Q^k - C^k} = 2\pi\lambda^{-1} \qquad \Delta\phi_{\overline{Q}^N - \det(C)} = 2\pi\lambda$$

details of det(C) operator

The other combinations will not have a trivial phase.

Conclusion and Discussion

•We calculated the anyonic phase arising from Dp-Dq-F1 configuration.
•D0-D2-*k* F1 and D4-D6-*N* F1 pair lead anyonic phase.
•We may construct anyons by bound states of above combinations.
•We also calculated the same phase for (probably) corresponding field theory configuration.
•These phases are matched to D-brane results.

Future directions...

•Other brane configurations.
•More “realistic” setup??

Thank you!

$AdS_4\times \mathbf{CP}^3$ backgound

$$ds^2 = \tilde{R}^2 \left(ds^2_{AdS_4} + 4 ds^2_{\mathbf{CP}^3} \right)$$

$$ds^2_{AdS_4} = r^2 \left(-dt^2 + dx^2 + dy^2 \right) + \frac{dr^2}{r^2} \, ,$$

$$\begin{aligned} ds^2_{\mathbf{CP}^3} =& \frac{1}{4} \Big[d\alpha^2 + \sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}\left(d\chi + \cos\theta_1 d\varphi_1 - \cos\theta_2 d\varphi_2\right)^2 \\ & + \cos^2\frac{\alpha}{2}\left(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2\right) \\ & + \sin^2\frac{\alpha}{2}\left(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2\right) \Big] \end{aligned}$$

$$(0 \leq \alpha, \theta_1, \theta_2 \leq \pi \, , \qquad 0 \leq \varphi_1, \varphi_2 < 2\pi \, , \qquad 0 \leq \chi < 4\pi)$$

$$\tilde{R}^2 = \frac{R^3}{4k} \, , \qquad e^{2\Phi} = \frac{R^3}{k^3} \, ,$$

$$\frac{R^3}{k} = 2^{5/2}\pi\sqrt{\frac{N}{k}} \, , \qquad F_4 = \frac{3}{8} R^3 d\Omega_{AdS_4} \, , \qquad F_2 = k\, d\omega \, ,$$

$$\omega = \frac{1}{4}\left(\cos\alpha\, d\chi + 2\cos^2\frac{\alpha}{2}\cos\theta_1 d\varphi_1 + 2\sin^2\frac{\alpha}{2}\cos\theta_2 d\varphi_2 \right) .$$

back

The ABJM action

Aharony-Bergman-Jafferis-Maldacena's $\mathcal{N}$=6 Superconformal Chern-Simons-matter action

$$\mathcal{L} = \mathcal{L}_{CS} + \mathcal{L}_{kin} + \mathcal{L}_{\psi\psi\mathrm{CC}} + \mathcal{L}_6$$

$$\mathcal{L}_{\mathrm{CS}} = \mathrm{Tr}\left(\frac{k}{4\pi}\left(AdA - \frac{2}{3}A^3 \right) - \frac{k}{4\pi}\left(\tilde{A}d\tilde{A} - \frac{2}{3}\tilde{A}^3 \right) \right)$$

$$\mathcal{L}_{\mathrm{kin}} = \mathrm{Tr}\left(\left| D_\mu C_I \right|^2 + i\overline{\psi}^I \gamma^\mu D_\mu \psi_I \right)$$

$$\mathcal{L}_{\mathrm{six}} = -\frac{8\pi^2}{k^3}\mathrm{Tr}\left[-\left(C_{I_1} C^{\dagger[I_1} C_{I_2} C^{\dagger I_2} C_{I_3} C^{\dagger I_3]} \right) + \left(C_{I_1} C^{\dagger[I_1} C_{I_2} C^{\dagger I_3]} C_{I_3} C^{\dagger I_2} \right) \right]$$

Comment on D6-D4 pair

Fluctuation by D4 brane source can also be considered, and the linear perturbation is

$$H_3 \wedge F_2^{(0)} = d * \left(e^{-\Phi^{(0)}/2} F_6\right) - \frac{1}{2} d * \left(e^{-\Phi^{(0)}/2} \Phi F_6^{(0)}\right) + 2\kappa_{10}^2 \mu_4 \delta^5(x_{D4})$$

Consider D4 wrapping on $J \wedge J$, where J is proportional two Kahler two form on $\mathbf{CP}^3$.

$$\begin{aligned}
\Delta\phi_{F1D4} &= -T_{F1} \int_{\partial\Sigma} B_2 \\
&= -T_{F1} \frac{1}{-48 vol_{\mathbf{CP}^3}} \int_{\Sigma \times \mathbf{CP}^3} H_3 \wedge J \wedge J \wedge J \\
&= T_{F1} \frac{1}{(2\pi)^3} \int_{\Sigma \times \mathbf{CP}^3} H_3 \wedge \frac{1}{k} F_2^{(0)} \wedge J \wedge J \\
&= T_{F1} \frac{1}{(2\pi)^3} \frac{1}{k} \int_{\Sigma \times \mathbf{CP}^3} \left((2\pi)^3 \frac{-1}{2\pi} J + d * \left(e^{-\Phi^{(0)}/2} F_6\right) \right) \wedge J \wedge J \\
&= \frac{2\pi}{k} + T_{F1} \frac{1}{(2\pi)^3} \frac{1}{k} \int_{\partial\Sigma \times \mathbf{CP}^3} * \left(e^{-\Phi^{(0)}/2} F_6\right) \wedge J \wedge J \,.
\end{aligned}$$

$$\longrightarrow \qquad \Delta\phi = 2\pi \frac{1}{k}$$

The phase from D4-D6 pair

We then now consider D6 "Baryon". Due to the magnetic six-form on **CP**^{3}, $F_6 = *F_4$

we have $\int F_6 \wedge A_t \propto N \int A_t$ Then N anti-fundamental strings are attached to it.

Thus we get the phase for D4-D6 pair,

$$\Delta\phi_{D4-D6} = 2\pi \frac{N}{k} = 2\pi\lambda$$

Note: D2-D4 pair and D6-D0 pair have *trivial* phase!

back to the summary

The operator corresponding to D4

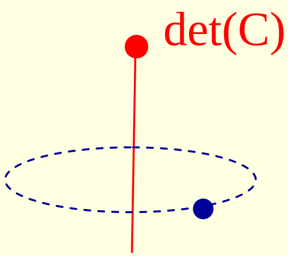
$\det(C)$ Carrying (N, -N) $U(1)_b$ charge.

⟷ dual to D4 on **CP²** (ABJM, Park)

Need to cancel U(1) charge.

Attach U(1) Wilson line or ‘t Hooft operator (equivalent)

monopole operator



To cancel N U(1) charge, ‘t Hooft operator causes the gauge transformation

$$\left(e^{-\frac{i}{k}\phi}, e^{-\frac{i}{k}\phi}\right)$$

⟶ D2 baryon cannot detect it.

The phase from det(C)-D6 Baryon

We consider the similar baryonic state from D6-brane.

N strings need to attach it and the direction is opposite to D2 ones.

→ Assume we have $\overline{Q}^N$ (*N* bound state of anti-fundamental)

$$\overline{Q}^N \to \left(e^{-\frac{2\pi i}{k}} \right)^{-N} \overline{Q}^N = e^{2\pi i \frac{N}{k}} \overline{Q}^N$$

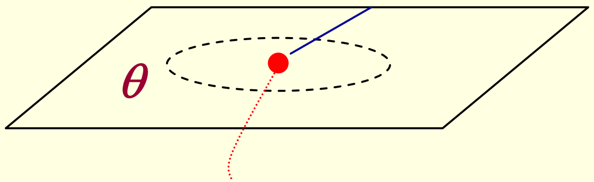
The AB phase for this pair is $\Delta\phi = \frac{N}{k} = \lambda$

back to the summary

Monopole operator (I)

In 2+1D, ‘t Hooft loop becomes a local operator, ‘t Hooft operator.

(localized magnetic flux)



It generates a gauge transformation of the center

$$T_h \approx e^{ih\theta} \qquad h = \mathrm{diag}\left(\frac{1}{N}, \cdots, \frac{1}{N}, \frac{1}{N} - 1\right)$$ (Park)

With Chern-Simons action, this is equivalent to the insertion of Wilson line

$$\int Dg\, T_h\, e^{S_{CS}} \approx \int Dg\, e^{S_{CS} + \delta S_{CS}} = W e^{S_{CS}}$$

For SU(N) Chern-Simons of level k, this Wilson line is in $\mathrm{Sym}\mathbf{N}^k$ (Itzhaki)

representation and then can be used to make C^k gauge invariant.

back

Monopole operator (added)

$\mathbf{Z}_N$ in the previous page is not the center for each U(N) in ABJM.

We may define through U(1) subgroup of U(N)? (Kapustin-Witten, Klebanov et al, Imamura,...)

It generates a gauge transformation around it: $e^{i\alpha\theta}$

(localized magnetic flux)

label for Cartan

For nonabelian group *U(N)*, we can consider *U(1)*^{r} Cartan rotations $A_\mu^i = \lambda^i d\theta$

Fact: There is $U(N)_k$ Chern-Simons action, this monopole belongs to the representation whose highest weight is given by $k\lambda^i$

$\lambda^i = (1,0,\cdots,0) \longrightarrow$ Sym$\mathbf{N}^k$ representation

Not sure that it generates the desired gauge rotation.

Monopole operator (II)

Diagonal U(1) part $A^D_\mu = A_\mu + \tilde{A}_\mu$ couples only through $\int A^B \wedge F^D$

→ Dualize *dual photon* $d\alpha = A^B$

$e^{i\alpha}$ is a monopole operator.

This changes the flux F_D 1 unit → magnetic flux

It transforms under $U(1)_B$ $(kN, -kN)$

Thus taking *1/N* unit of anti-flux, we can make C^k gauge invariant.

note: Since no field is charged under $U(1)_D$, flux quantization condition can be relaxed. However, it may be problematic for the moduli space structure.

(Park)

return