“Non-commutative theory”
in the M-theory

Tomohisa Takimi (NTU)

2010 Oct 19th @ NCTS journal club

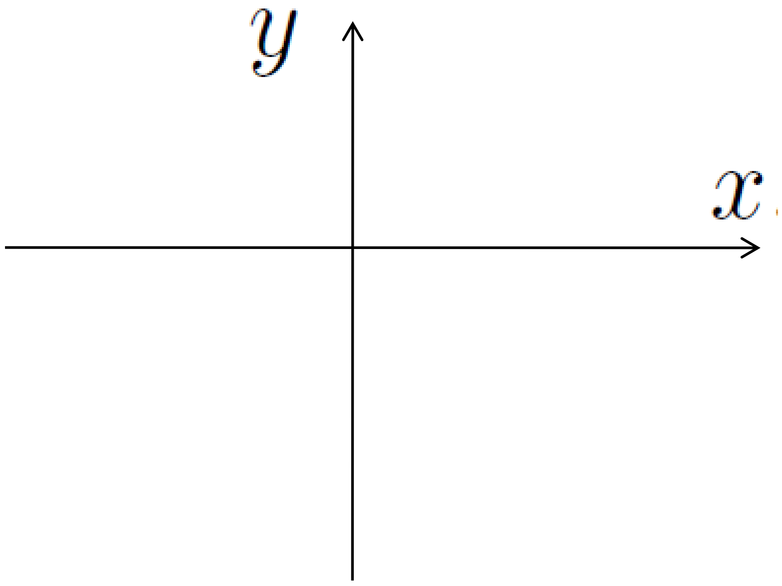
Ref.) Chen-Ho-T.T (JHEP 1003:104,2010.)

0-1. About the Non-commutative geometry.

What is Non-commutative geometry (NC)?

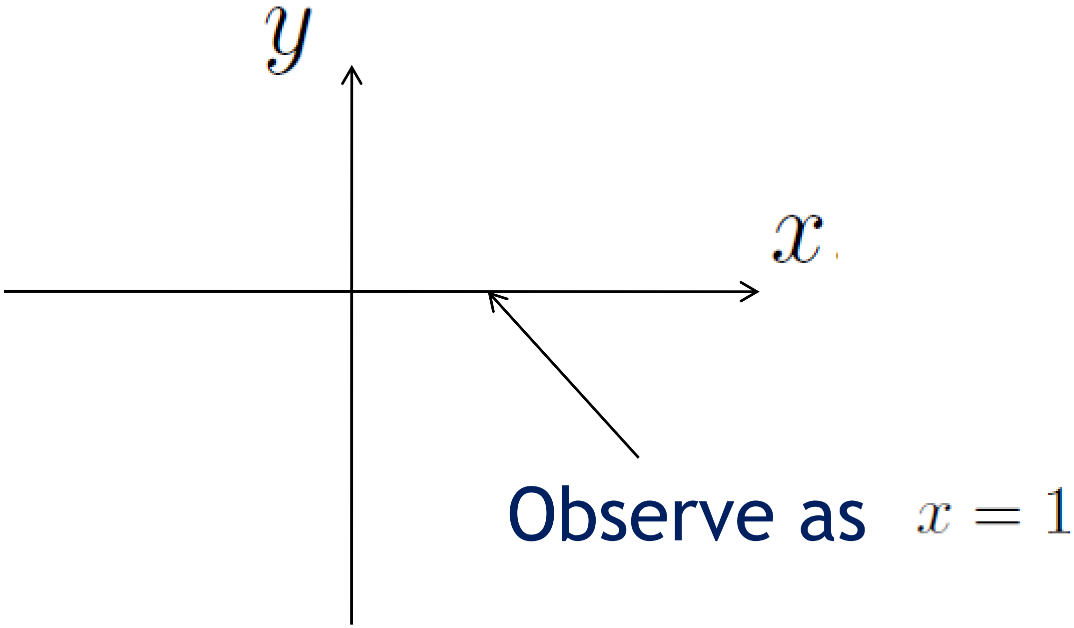
- Quantum space $[x^{\mu}, x^{\nu}] = \theta^{\mu\nu}$ $\theta \Leftrightarrow \hbar$

When we observe the x-coordinate of the location of the object, the information for the y-coordinate is disturbed !



Uncertainty between x and y

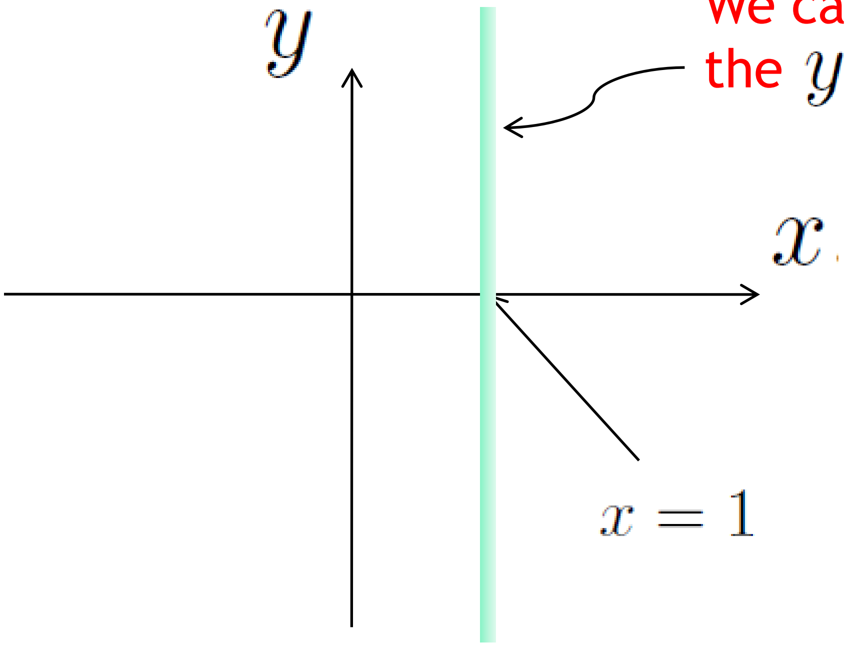
$$[x^\mu, x^\nu] = \theta^{\mu\nu}$$



Observe as $x = 1$

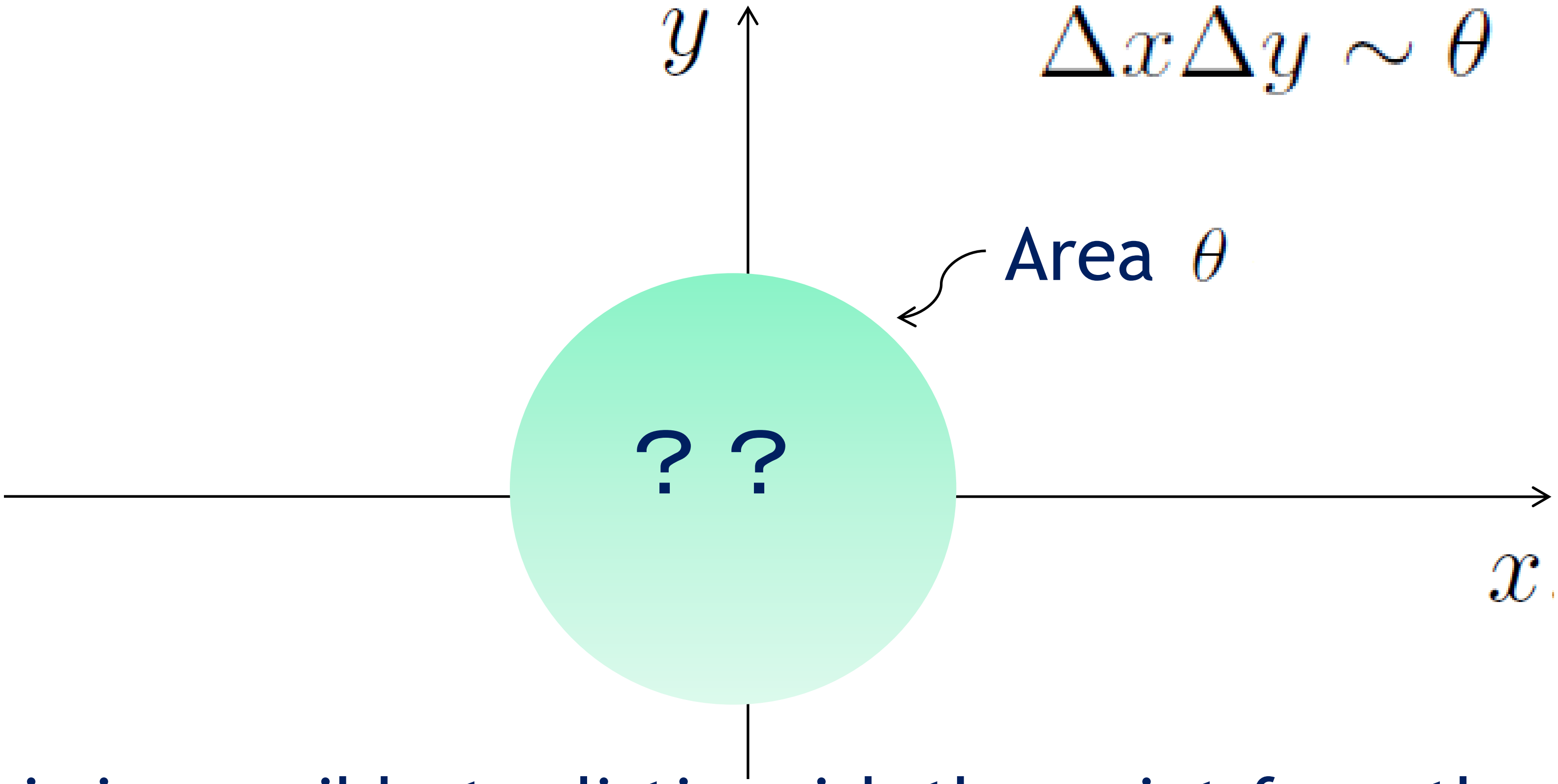
Uncertainty between x and y

$$[x^\mu, x^\nu] = \theta^{\mu\nu}$$



We cannot specify the y direction

Uncertainty principle



$$\Delta x \Delta y \sim \theta$$

It is impossible to distinguish the point from the other points inside the domain with area θ

→ Somehow space is naturally regularized by the parameter θ

Motivation of NC

To consider the Quantum gravity,
due to the serious divergence,
it might be necessary to consider
the "Generalized geometry"
beyond the usual
Riemannian geometry

→ Non-commutative geometry is a natural choice of generalized geometry

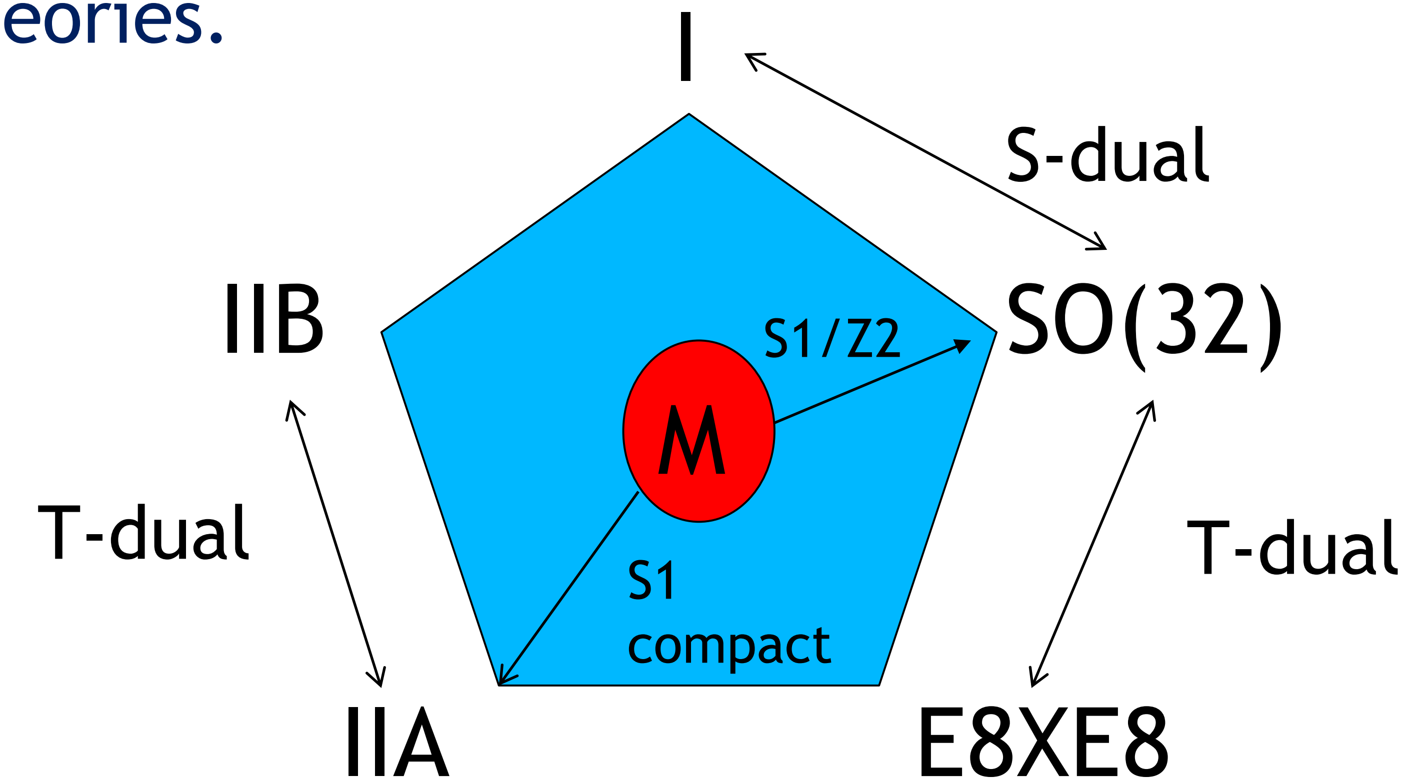
Consistent NC from string theory

There are so many candidates of NC theory. But it is non-trivial whether each NC-theory is consistent or not.

If we consider the NC geometry from string theory, it is already known that the NC from string theory is guaranteed as consistent !

Consistent NC from M-theory ?

It is believed that the M-theory is the most fundamental theory which is origin of string theories.



Consistent NC from M-theory ?

Since the string theory naturally derive the consistent NC, so we should know how the M-theory derives such NC descriptions.

↓

Today's motivation

1. NC from string theory

1-0. Basic stuff for the string theory

Basic (Powerful) formalism of string theory

World-Sheet action

＝ Area of 2-dimensional surface representing the track of string (1-dimensional object)

$$S = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \sqrt{-\det g}\, g^{ij}\partial_i X^\mu \partial_j X^\nu G_{\mu\nu}(X)$$

$$i = \tau, \sigma$$

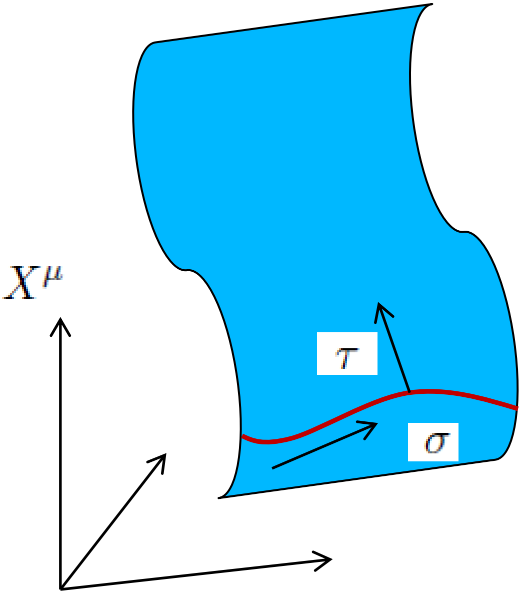
τ : Time of world-sheet

σ : world-sheet space

g^{ij} : world-sheet metric

X^μ : target space coordinate

$G_{\mu\nu}(X)$: target space metric



1-1 How the NC geometry shows up from the string theory

We add the anti-symmetric tensor to the metric.

↓

It becomes the generalized geometry beyond Riemannian geometry

World-Sheet action with anti-symmetric B-field

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left[g_{\mu\nu}\partial^a X^\mu \partial_a X^\nu - 2\pi\alpha'\epsilon^{ab}\boxed{B_{\mu\nu}}\partial_a X^\mu \partial_b X^\nu\right]$$

$$\updownarrow$$

without B-field $$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left[g_{\mu\nu}\partial^a X^\mu \partial_a X^\nu\right]$$

Coordinate on the boundary

Momentum

⇨ Not only time derivative but also space derivative term exists

$$P_\mu = g_{\mu\nu}\partial_\tau X^\nu - 2\pi\alpha' B_{\mu\nu}\boxed{\partial_\sigma X^\nu}$$

Due to the effect of the B-field, the canonical momentum can include the coordinate also.

Action: $$S=-\frac{1}{4\pi\alpha'}\int d^2\sigma\left[g_{\mu\nu}\partial^a X^\mu\partial_a X^\nu-2\pi\alpha'\epsilon^{ab}B_{\mu\nu}\partial_a X^\mu\partial_b X^\nu\right]$$

1. Boundary conditions: $$g_{ij}\partial_\sigma X^j+2\pi\alpha' B_{ij}\partial_t X^j=0$$

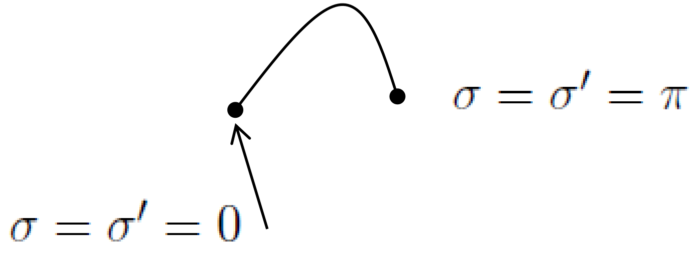
2. Commutation relation: $$[X^i(\sigma,\tau),P^j(\sigma',\tau)]=ig^{ij}\delta(\sigma-\sigma').$$

Quantize the world sheet theory by using 1 and 2

As a result, the non-commutative geometry shows up

$$[X^i(\sigma,\tau),X^j(\sigma',\tau)] = \begin{cases} i\theta^{ij} & \sigma=\sigma'=0 \\ -i\theta^{ij} & \sigma=\sigma'=\pi \\ 0 & \text{other} \end{cases}$$

End of string



$$\theta^{ij} = -(2\pi\alpha')^2\left(\frac{1}{g+2\pi\alpha' B}\boxed{B}\frac{1}{g-2\pi\alpha' B}\right)^{ij}$$

θ vanishes at B=0

Non-commutative effect comes from anti-symmetric tensor B-field

1-2. Non-commutative field theory as a low energy effective theory.

First we take following limit

$$\alpha' \sim \epsilon^{\frac{1}{2}} \longrightarrow 0 \qquad\qquad g_{\mu\nu} \sim \epsilon \longrightarrow 0$$

2-point function becomes $\langle X^i(z) X^j(z') \rangle \sim \frac{i}{2}\theta^{ij}\epsilon(z-z')$

Then the 2-point function of Fourier mode

$$: e^{ip\cdot X}(z) :: e^{iq\cdot X}(z') :\sim \boxed{e^{-\frac{i}{2}\theta^{ij}p_i q_j}}\, e^{i(p+q)\cdot X}(z')$$

the product becomes **”star product”**

$$\boxed{\begin{gathered} : e^{ip\cdot X}(z) :: e^{iq\cdot X}(z') :\sim e^{ip\cdot X} * e^{iq\cdot X} \\[2ex] f * g \equiv f e^{\frac{i}{2}\theta^{ij}\overleftarrow{\partial}_i \overrightarrow{\partial}_j} g \end{gathered}}$$

Low energy effective theory

→ U(1) Gauge theory on Non-commutative space written by the Moyal product

Moyal product

$$[f, g]_{\mathrm{Moyal}} \equiv f * g - g * f$$

U(1) gauge transformation : $\delta A_\mu = \partial_\mu \alpha$

→ Non-commutative U(1) transformation :

$$\delta A_\mu = \partial_\mu \alpha + [\alpha, A_\mu]_{\mathrm{Moyal}}$$

Non-commutative U(1) gauge theory is very similar to non-abelian gauge theory. Only difference is that Lie-product $[\,\cdot,\cdot\,]$ is rewritten as Moyal product $[\,\cdot,\cdot\,]_{\mathrm{Moyal}}$

Remarks

These Star and Moyal product is characterized by the parameter θ

$$\theta = 0$$

$$f * g = fg$$

$$[f, g]_{Moyal} = 0$$

$$\delta A_\mu = \partial_\mu \alpha$$

Commutative

$$\theta \neq 0$$

$$f * g = f e^{\frac{i}{2}\theta^{ij}\overleftarrow{\partial_i}\overrightarrow{\partial_j}} g$$

$$[f, g]_{\mathrm{Moyal}} \equiv f * g - g * f$$

$$\delta A_\mu = \partial_\mu \alpha + [\alpha, A_\mu]_{\mathrm{Moyal}}$$

Non-commutative

Remarks

Commutative theory and Non-commutative theory is continuously connected by the continuous parameter θ

$\theta = 0$

$$f * g = fg$$

$$[f, g]_{Moyal} = 0$$

$$\delta A_\mu = \partial_\mu \alpha$$

θ increases

$\theta \neq 0$

$$f * g = f e^{\frac{i}{2}\theta^{ij}\overleftarrow{\partial_i}\overrightarrow{\partial_j}} g$$

$$[f, g]_{\mathrm{Moyal}} \equiv f * g - g * f$$

$$\delta A_\mu = \partial_\mu \alpha + [\alpha, A_\mu]_{\mathrm{Moyal}}$$

Commutative

Non-commutative

Remarks

Realization of NC theory by tuning the θ is called as **Deformation Quantization**

Since $[x^i, x^j] = \theta^{ij}$ $\theta \Leftrightarrow \hbar$

$\boxed{\theta = 0}$

$$f * g = fg$$

$$[f, g]_{Moyal} = 0$$

$$\delta A_\mu = \partial_\mu \alpha$$

θ increases

Quantize

$\boxed{\theta \neq 0}$

$$f * g = f e^{\frac{i}{2}\theta^{ij}\overleftarrow{\partial_i}\overrightarrow{\partial_j}} g$$

$$[f, g]_{\mathrm{Moyal}} \equiv f * g - g * f$$

$$\delta A_\mu = \partial_\mu \alpha + [\alpha, A_\mu]_{\mathrm{Moyal}}$$

Commutative

Non-commutative

Part 2 Non-commutative theory in the M-theory.

Before entering into the M-theory..

Rough sketch how to construct NC

String theory

1. There is **world sheet action**

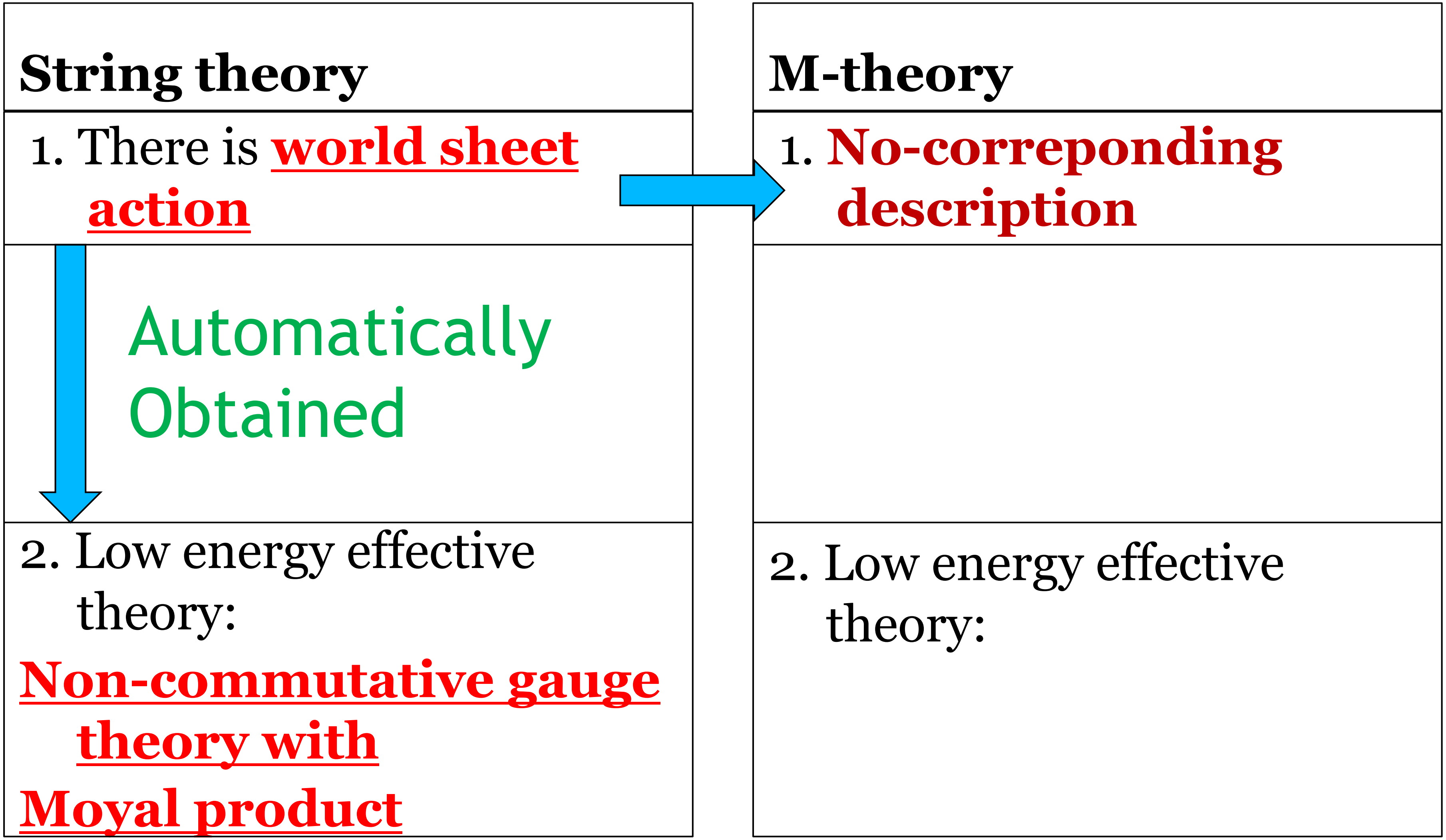
Powerful source

Automatically Obtained

2. Low energy effective theory:

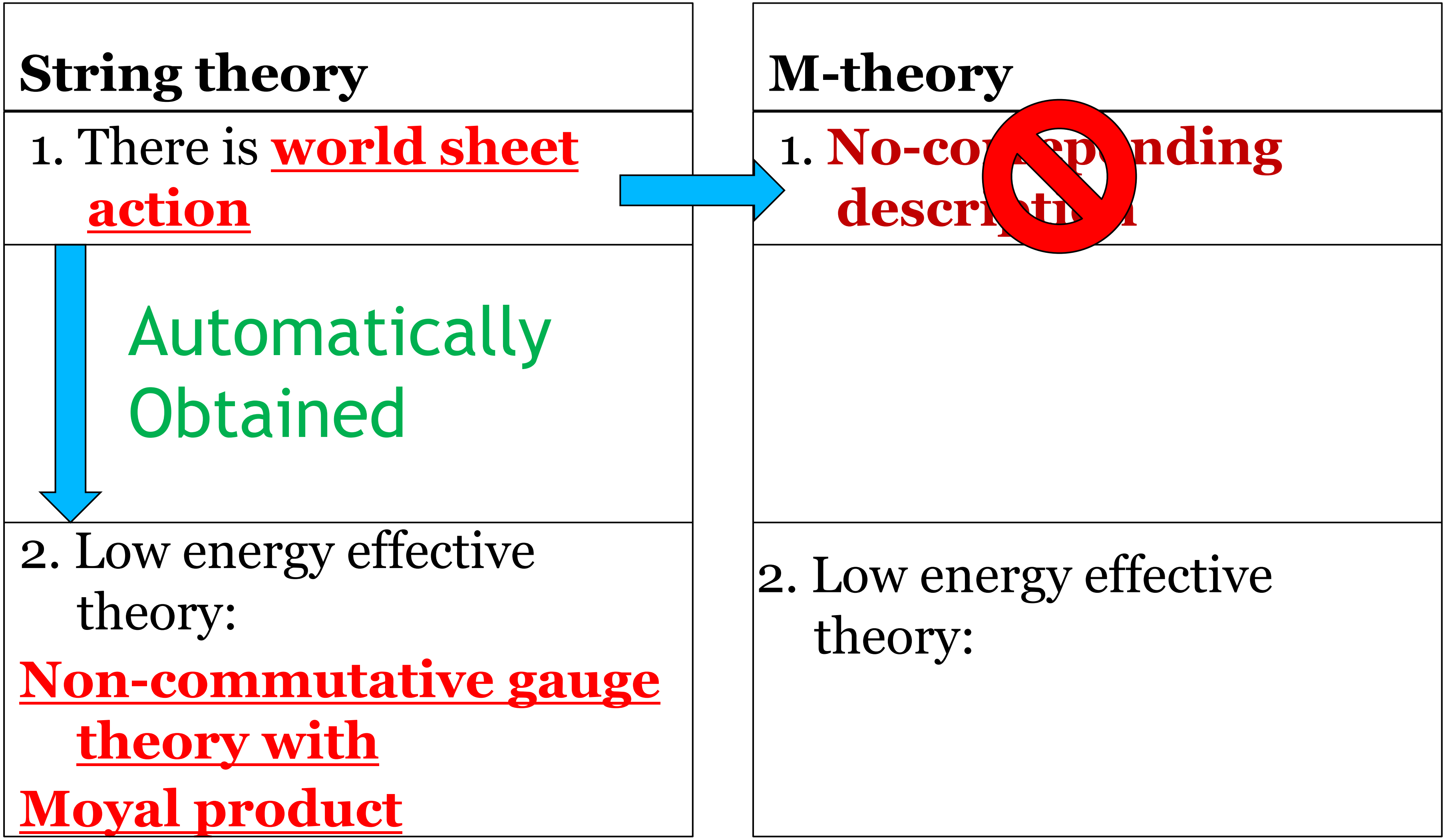
Non-commutative gauge theory with Moyal product

Rough sketch how to construct NC



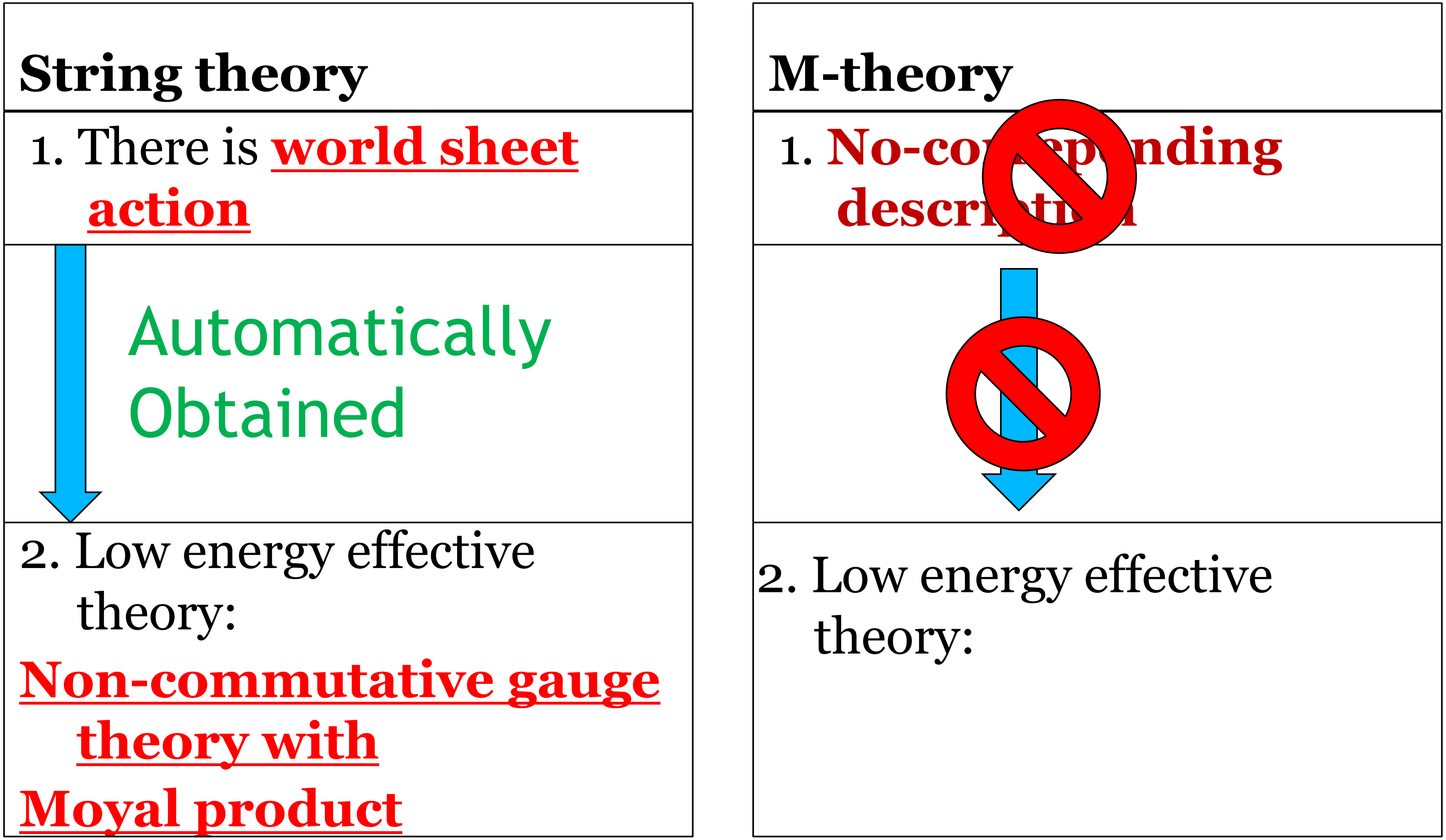
String theory	**M-theory**
1. There is **world sheet action**	1. **No-correponding description**
Automatically Obtained	
2. Low energy effective theory: **Non-commutative gauge theory with Moyal product**	2. Low energy effective theory:

Rough sketch how to construct NC



String theory	**M-theory**
1. There is **world sheet action**	1. **No-corresponding description**
Automatically Obtained	
2. Low energy effective theory: **Non-commutative gauge theory with Moyal product**	2. Low energy effective theory:

Rough sketch how to construct NC



String theory

1. There is **world sheet action**

Automatically Obtained

2. Low energy effective theory:

Non-commutative gauge theory with Moyal product

M-theory

1. **No-corresponding description**

2. Low energy effective theory:

Rough sketch how to construct NC

String theory

1. There is **world sheet action**

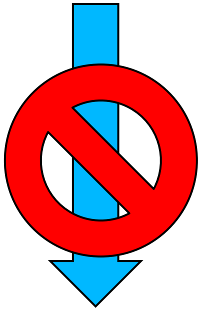
We have to find out the corresponding theory by try and error !!

theory:

Non-commutative gauge theory with Moyal product

M-theory

1. **No-corresponding description**



2. Low energy effective theory:

2—1. One candidate

→ Nambu-bracket theory (NP)

Poisson bracket＝2-dimensional anti-symmetric differential operator

$$\{f,g\}=\epsilon^{ij}\partial_i f\partial_j g$$

↓

Nambu-Bracket＝3-d extension of Poisson bracket

$$\{f,g,h\}=\epsilon^{ijk}\partial_i f\partial_j g\partial_k h$$

Map from 3 entry to 1 entry

$$\mathcal{A}\times\mathcal{A}\times\mathcal{A}\to\mathcal{A} \qquad \xcancel{\mathcal{A}\times\mathcal{A}\to\mathcal{A}}$$

2—2. How to obtain the M-theory model with NP structure

Ho-Matsuo (JHEP 0806:105,2008.)

Ho-Imamura-Matsuo-Shiba (JHEP 0808:014,2008.)

N=8 (1+2)-dimensional supersymmetric Lie 3-algebra gauge theory of multiple membranes (BLG model)

↓

∞ generators of Lie 3-algebra → NP bracket

+

Expand the scalar around the particular background (3-form C-field background)

↓

M5-brane theory with NP bracket shows up

2—3. BLG model

Usual quantum field theory: described by Lie-algebra

$$T^a, T^b \in V_2 \Rightarrow [T^a, T^b] \in V_2$$

N=8 3-d SUSY 'gauge' model described by the Lie 3-algebra describing multiple membranes

$$T^a, T^b, T^c \in V_3 \Rightarrow [T^a, T^b, T^c] = f^{abc}{}_d T^d \in V_3$$

Action

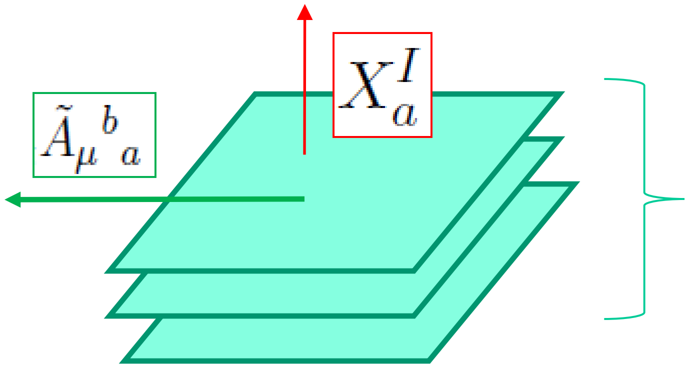
$$S = \int d^3x \, \mathcal{L},$$

$$\mathcal{L} = -\frac{1}{2}\langle D^\mu X^I, D_\mu X^I\rangle + \frac{i}{2}\langle\bar{\Psi}, \Gamma^\mu D_\mu \Psi\rangle + \frac{i}{4}\langle\bar{\Psi}, \Gamma_{IJ}[X^I, X^J, \Psi]\rangle - V(X) + \mathcal{L}_{CS}.$$

$$\mathcal{L}_{CS} = \frac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right).$$

$$V(X) = \frac{1}{12}\langle[X^I, X^J, X^K], [X^I, X^J, X^K]\rangle.$$

$$(D_\mu X^I(x))_a = \partial_\mu X^I_a(x) - \tilde{A}_\mu{}^b{}_a(x) X^I_b(x), \quad \tilde{A}_\mu{}^b{}_a \equiv A_{\mu cd} f^{cdb}{}_a,$$



Object in the M-theory

Multiple M2-branes in (1+10) dimensions.

The symmetries of the theory

* Gauge symmetry (Written by 3-algebra)

$$\delta_\Lambda X^I_a = \Lambda_{cd}[T^c, T^d, X^I]_a = \Lambda_{cd} f^{cde}{}_a X^I_e = \tilde{\Lambda}^e{}_a X^I_e,$$

$$\delta_\Lambda \Psi_a = \Lambda_{cd}[T^c, T^d, \Psi]_a = \Lambda_{cd} f^{cde}{}_a \Psi_e = \tilde{\Lambda}^e{}_a \Psi_e,$$

$$\delta_\Lambda \tilde{A}_\mu{}^b{}_a = \partial_\mu \tilde{\Lambda}^b{}_a - \tilde{\Lambda}^b{}_c \tilde{A}_\mu{}^c{}_a + \tilde{A}_\mu{}^b{}_c \tilde{\Lambda}^c{}_a, \quad \tilde{\Lambda}^b{}_a \equiv f^{cdb}{}_a \Lambda_{cd}.$$

* N=8 Supersymmetry

$$\delta_\epsilon X^I_a = i\bar{\epsilon}\Gamma^I \Psi_a,$$

$$\delta_\epsilon \Psi_a = D_\mu X^I_a \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X^I_b X^J_c X^K_d f^{bcd}{}_a \Gamma^{IJK} \epsilon,$$

$$\delta_\epsilon \tilde{A}_\mu{}^b{}_a = i\bar{\epsilon}\Gamma_\mu \Gamma_I X^I_c \Psi_d f^{cdb}{}_a,$$

From the BLG model we can obtain the M5-brane theory with NP structure.

Ho-Matsuo (JHEP 0806:105,2008)

Ho-Imamura-Matsuo-Shiba
(JHEP 0808:014,2008)

Nambu-Poisson structure in the Lie 3-algebra

***dimension of Lie 3-algebra = ∞**

Basis of Lie 3-algebra can be written by the Fourier modes along the internal 3-directions $T^a \rightarrow \chi^a(y) = e^{i\vec{p}_a\cdot\vec{y}}$

$$p^a = 0, \pm 1, \pm 2, \ldots \pm \infty$$

Field variable $\boxed{f(x) = f^a(x)T^a \rightarrow f(x,y) = f^a(x)\chi^a(y)}$

***3-product becomes Nambu-Poisson bracket.**

$$[T^a, T^b, T^c] \rightarrow \{\chi^a, \chi^b, \chi^c\} = \epsilon^{\mu\nu\rho}\partial_\mu\chi^a\partial_\nu\chi^b\partial_\rho\chi^c$$

Trace: $\mathrm{Tr}(T^aT^b) \rightarrow \langle\chi^a, \chi^b\rangle \equiv \int d^3y(\chi^a)^*(y)\chi^b(y)$

It seems to enhance

(1+2) dimensional world-volume

➡ (1+(2+3)) dimensional world-volume

But the kinetic term along the 3 enhanced direction is still lacking. ⟶ Not real space yet

How the internal 3-direction shows up as real world-volume by inducing the kinetic term along the directions ?

A. Expanding the field X^I around the background

$$X^I(x,y) = y^I + \sum_a X^I_a(x)\chi^a(y)$$

Then the potential term serves the kinetic term

$$\{X^I, X^J, X^K\}^2 \sim \left(\epsilon^{\mu\nu\rho}\delta^I_\mu\delta^J_\nu\partial_\rho X^K\right)^2$$

(1+2) dimensional theory is really enhanced to (1+5) dimensional theory.

Action $S = \int d^3x \, \mathcal{L},$

$$\mathcal{L} = -\frac{1}{2}\langle D^\mu X^I, D_\mu X^I\rangle + \frac{i}{2}\langle\bar{\Psi}, \Gamma^\mu D_\mu \Psi\rangle + \frac{i}{4}\langle\bar{\Psi}, \Gamma_{IJ}[X^I, X^J, \Psi]\rangle - V(X) + \mathcal{L}_{CS}.$$

$$V(X) = \frac{1}{12}\langle[X^I, X^J, X^K], [X^I, X^J, X^K]\rangle.$$

⬇

Nambu-Poisson structure

Expand around the background

$$V(X) = \frac{1}{12}\langle[X^I, X^J, X^K], [X^I, X^J, X^K]\rangle.$$

→ **Serves kinetic terms**

$$\{X^I, X^J, X^K\}^2 \sim \left(\epsilon^{\mu\nu\rho}\delta^I_\mu \delta^J_\nu \partial_\rho X^K\right)^2$$

Trace changed to integration over enhanced direction

$$\langle , \rangle \rightarrow \int d^3y$$

M5-brane action shows up !!

$$S = \frac{T_6}{T_{str}^2} \int d^3x d^3y \, \mathcal{L}$$

$$\mathcal{L} = \mathcal{L}_X + \mathcal{L}_{\rm pot} + \mathcal{L}_\Psi + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm CS}$$

$$\begin{aligned}
\mathcal{L}_X + \mathcal{L}_{\rm pot} &= -\frac{1}{2}(\mathcal{D}_\mu X^i)^2 - \frac{1}{2}(\mathcal{D}_{\dot\mu} X^i)^2 - \frac{1}{4}\mathcal{H}^2_{\lambda\dot\mu\dot\nu} - \frac{1}{12}\mathcal{H}^2_{\dot\mu\dot\nu\dot\rho} \\
&\quad -\frac{1}{2g^2} - \frac{g^4}{4}\{X^{\dot\mu}, X^i, X^j\}^2 - \frac{g^4}{12}\{X^i, X^j, X^k\}^2. \\
\mathcal{L}_\Psi + \mathcal{L}_{\rm int} &= \frac{i}{2}\bar\Psi\Gamma^\mu\mathcal{D}_\mu\Psi + \frac{i}{2}\bar\Psi\Gamma^{\dot\rho}\Gamma_{\dot1\dot2\dot3}\mathcal{D}_{\dot\rho}\Psi \\
&\quad + \frac{ig^2}{2}\bar\Psi\Gamma_{\dot\mu i}\{X^{\dot\mu}, X^i, \Psi\} + \frac{ig^2}{4}\bar\Psi\Gamma_{ij}\{X^i, X^j, \Psi\}
\end{aligned}$$

The M5-brane is divided by 2 parts

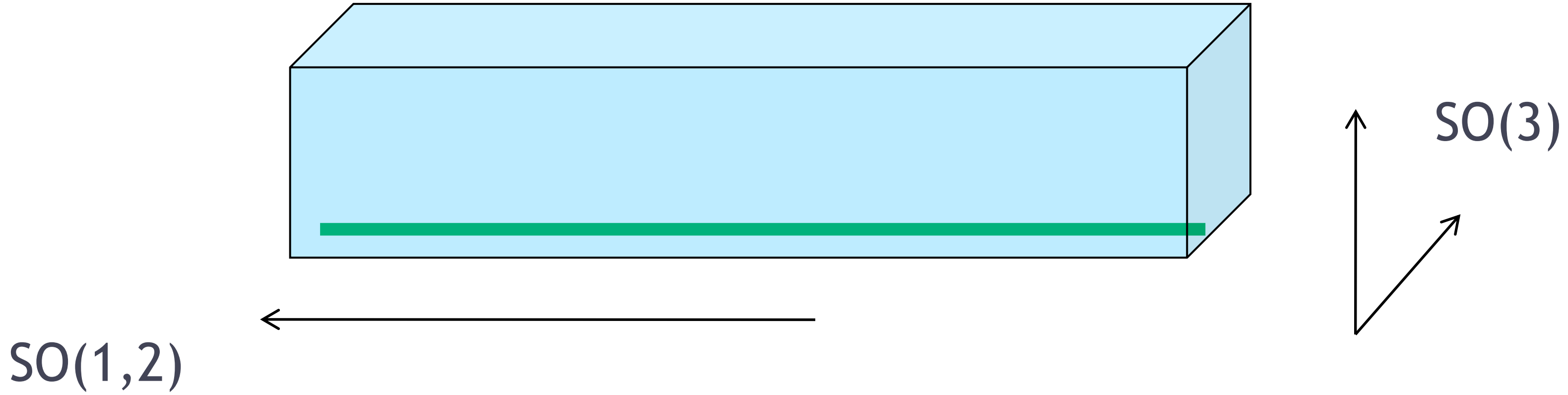
SO(1,2)-directions: original BLG membranes extends

SO(3)-directions: Enhanced directions,
which relates to gauge sym. generators

SO(1,2)

SO(1,2)-directions: original BLG membranes extends

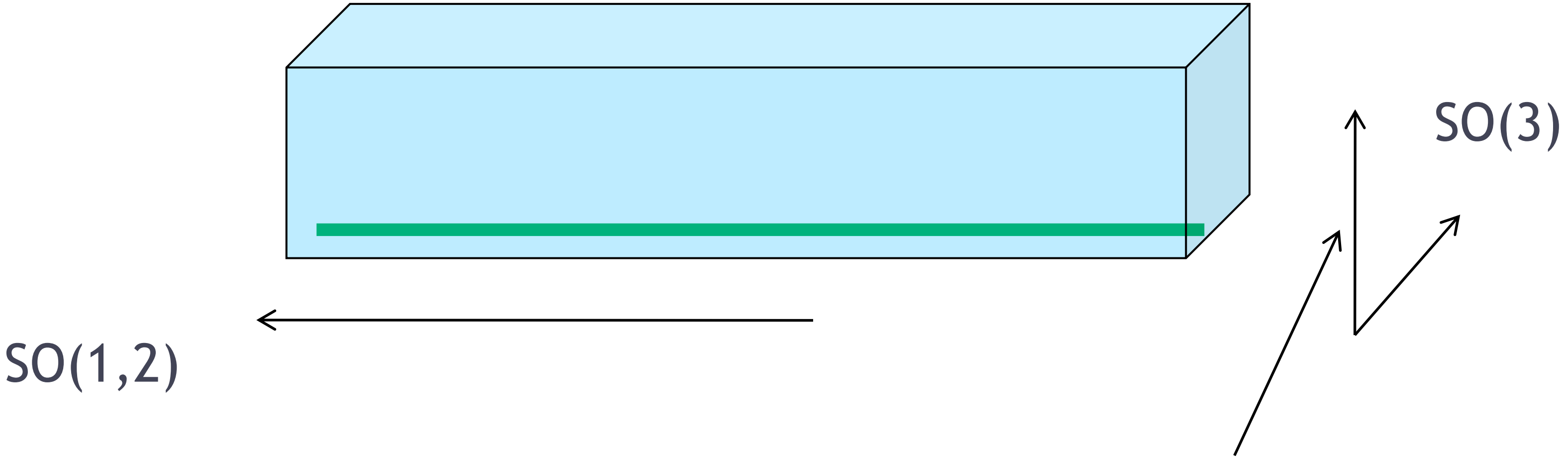
SO(3)-directions: Enhanced directions,
which relates to gauge sym. generators



The M5-brane world volume has
SO(1,2) ×SO(3) Lorenz symmetry

SO(1,2)-directions: original BLG membranes extends

SO(3)-directions: Enhanced directions, which relates to gauge sym. generators



Nambu-bracket shows up as derivative operator along SO(3)-directions

Gauge transformation in the BLG model

$$\delta_\Lambda X^I = \Lambda_{ab}[T^a, T^b, X^I]$$

➡ This can be rewritten by NP bracket

$$\epsilon^{\dot\mu\dot\nu\dot\rho}\partial_{\dot\mu}\Lambda_{\dot\nu}\partial_{\dot\rho}\phi = \{\Lambda_{\dot\nu}, y^{\dot\nu}, \phi\} = \kappa^{\dot\mu}\partial_{\dot\mu}\phi$$

=Volume preserving diffeo (VPD) along SO(3)-directions

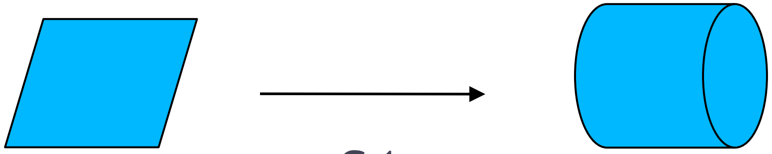
$$\boxed{\delta_\kappa\phi = \kappa^{\dot\mu}\partial_{\dot\mu}\phi \qquad \partial_{\dot\mu}\kappa^{\dot\mu} = 0}$$

$y^{\dot\mu}, y^{\dot\nu}, y^{\dot\rho}$: coordinates along SO(3) directions

How the NP M5-brane theory gives the string theory with NC geometry ?

If NP M5-brane theory is valid, the M5-theoy should reproduce the NC theory on string theory

Since the M-theory is believed that the origin of the string theory, which reproduce the string theory by circle (S1) compactification.



M-theory

S1

String theory

M5-brane theory
With NP structure

→

Gauge theory with
Poisson bracket
(Poisson theory)

$$[f,g]_{\mathrm{Moyal}} \sim \theta \epsilon^{ij} \partial_i f \partial_j g + O(\theta^3)$$

M5-brane theory
With NP structure

→

Gauge theory with
Poisson bracket
(Poisson theory)

$$[f,g]_{\mathrm{Moyal}} \sim \boxed{\theta\epsilon^{ij}\partial_i f\partial_j g} + \cancel{O(\theta^3)}$$

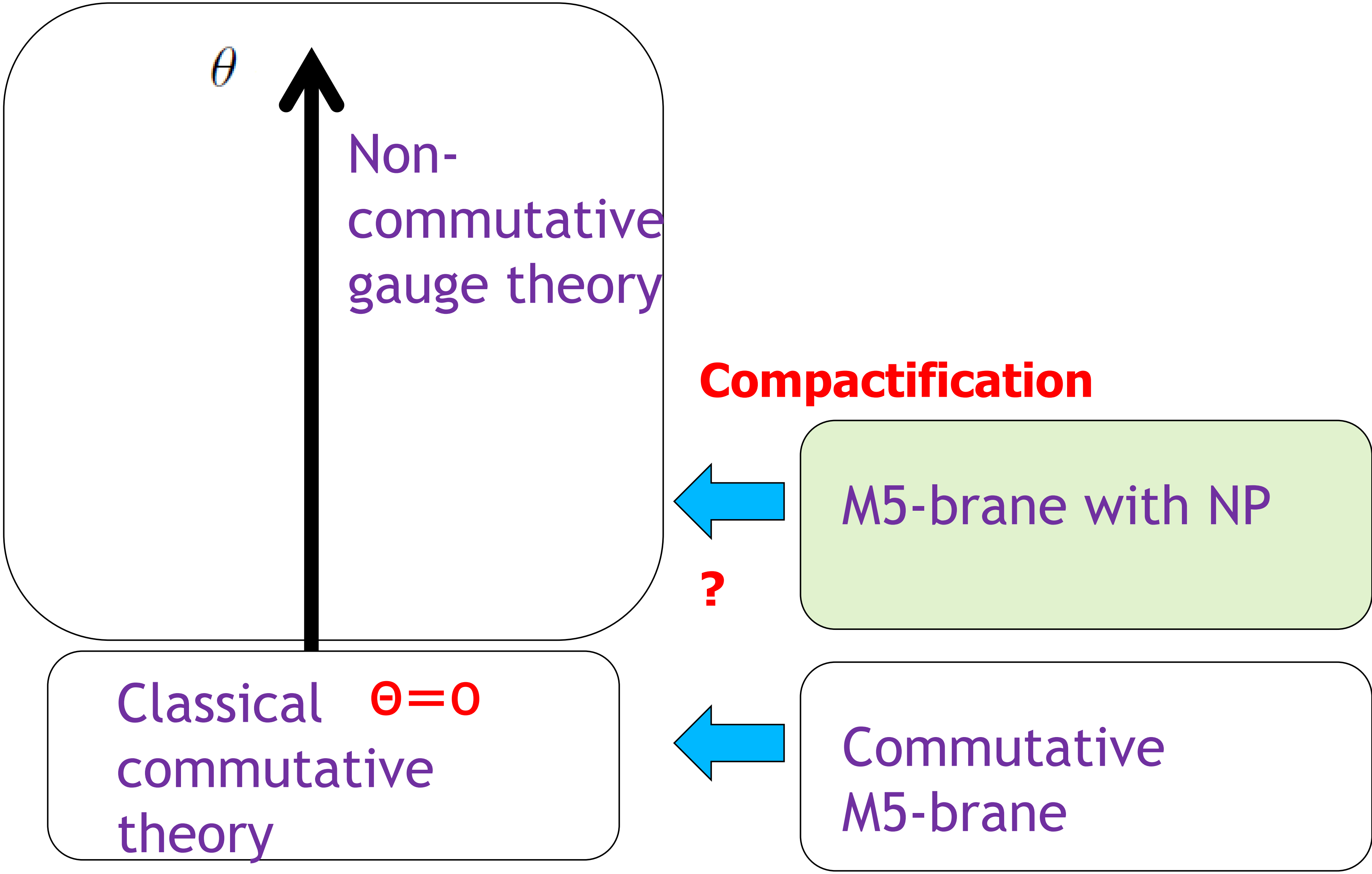
Picking up only first order of Moyal product and discard the higher order of Θ

Poisson bracket

$$\delta A_\mu = \partial_\mu \alpha + \theta\{A_\mu, \alpha\}$$

This is the Θ→0 limit of NC gauge theory

M5-brane with NP structure gives only the
Θ→0 classical limit, does not give finite Θ



θ

Non-commutative gauge theory

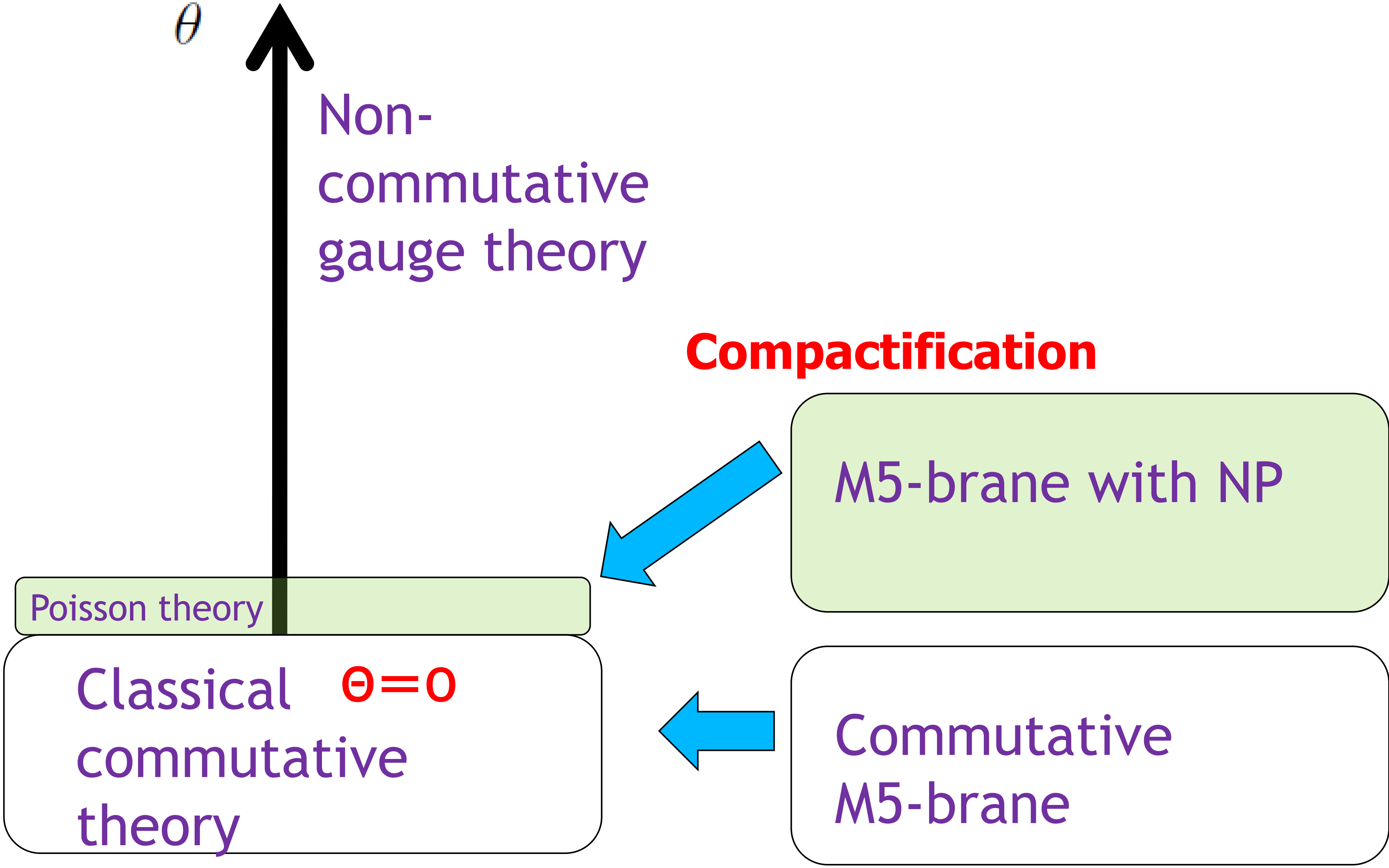
Compactification

M5-brane with NP

?

Classical commutative theory $\theta=0$

Commutative M5-brane



θ

Non-commutative gauge theory

Compactification

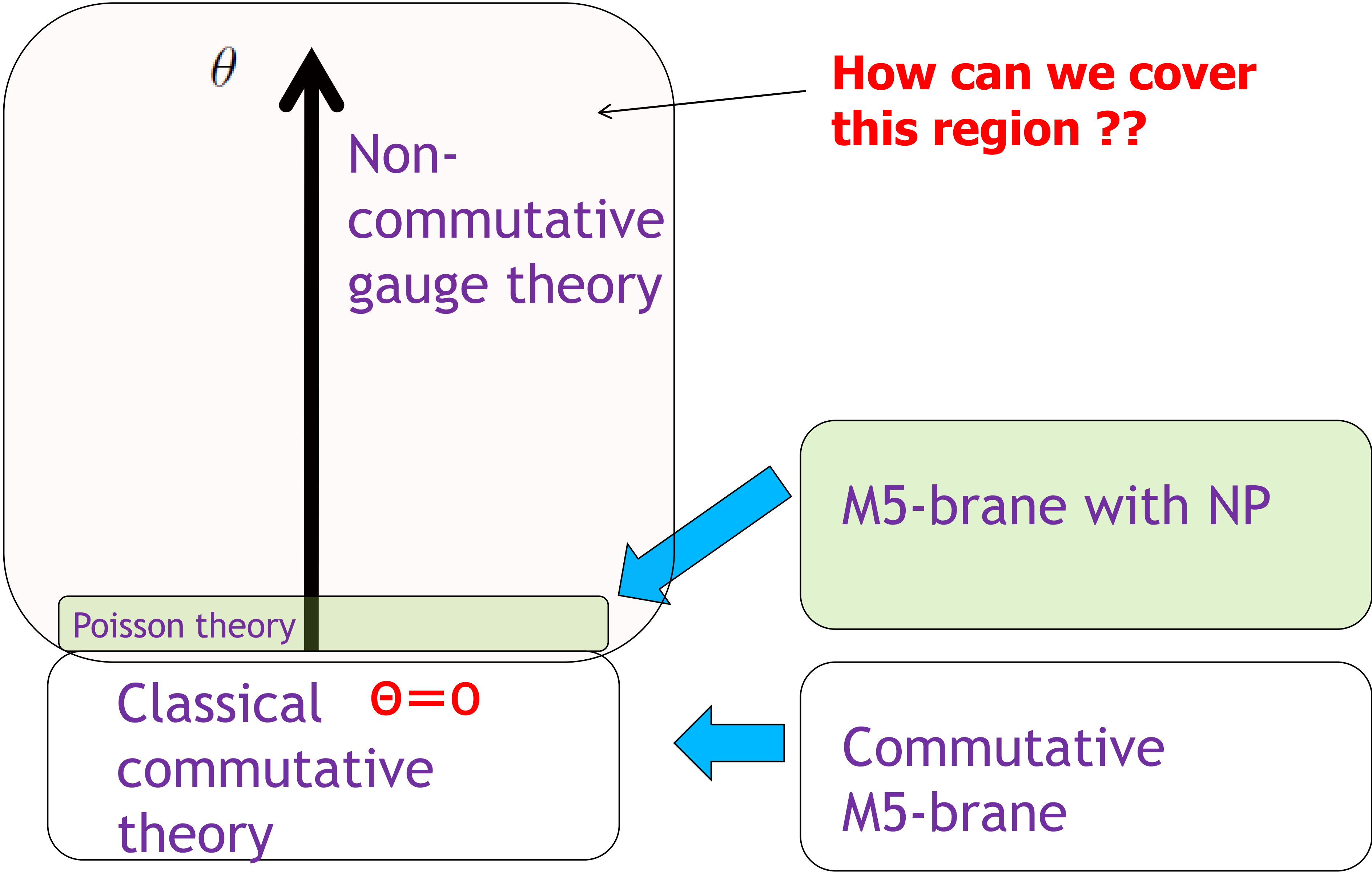
M5-brane with NP

Poisson theory

Classical commutative theory

$\theta=0$

Commutative M5-brane



θ

Non-commutative gauge theory

How can we cover this region ??

Poisson theory

M5-brane with NP

Classical commutative theory $\Theta=0$

Commutative M5-brane

What is the M5-brane theory recovering full order of Moyal product by the compactification ?

２－３. Main dish (Quantization of NP structure)

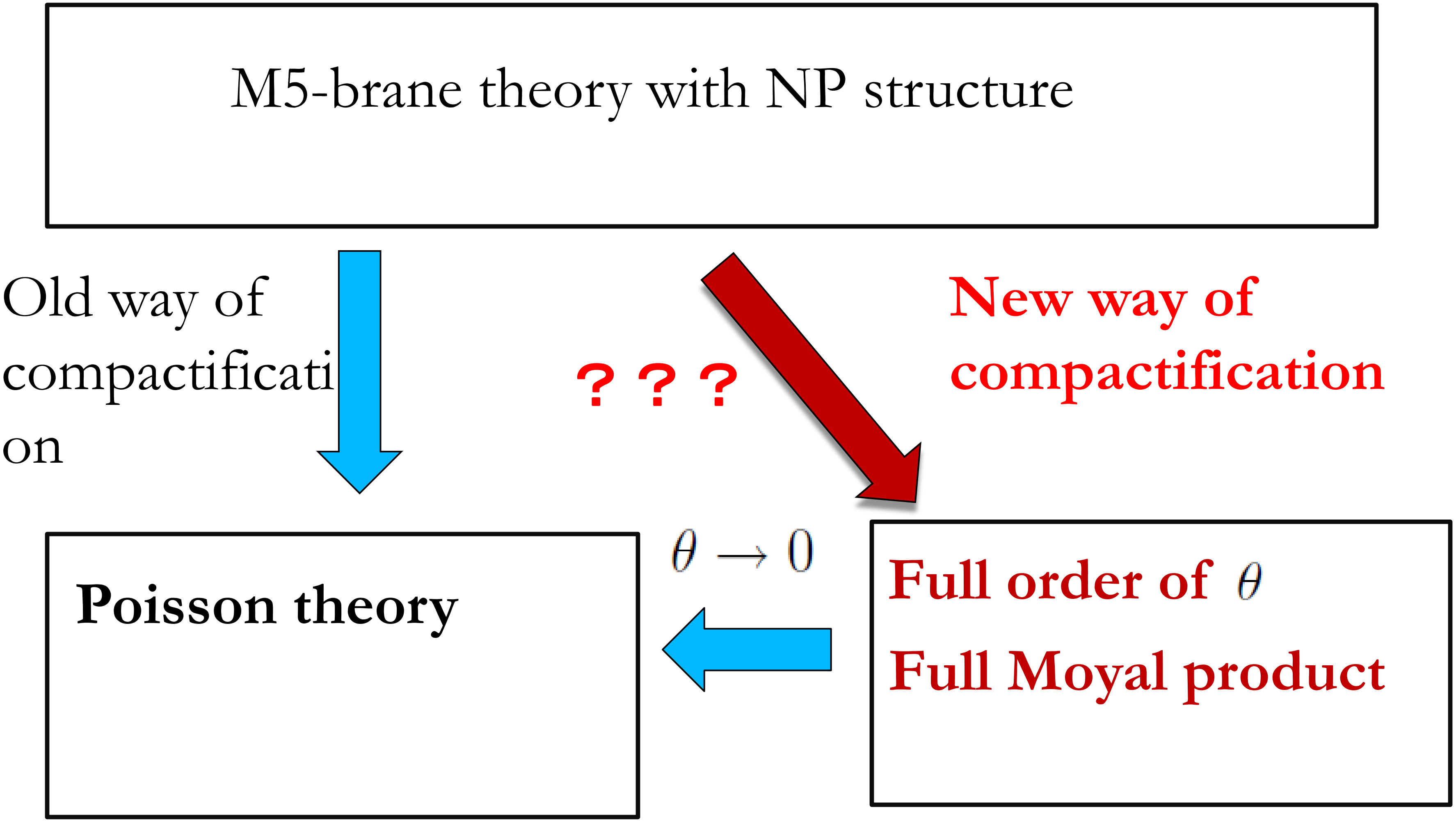
Possible 2 ways to recover the full NC theory

（A） **We keep NP M5-brane as it is. Just change the way of compactification.**

（B） We try to perform the deformation quantization of NP M5-brane theory

At Chen-Ho-T.T (JHEP 1003:104,2010.)
We challenge the above 2 ways.

About the strategy (A)



M5-brane theory with NP structure

Old way of compactification

? ? ?

New way of compactification

$\theta \to 0$

Poisson theory

Full order of θ

Full Moyal product

There is no compactification way !!

Chen-Ho-T.T (JHEP 1003:104,2010.)

Proof of impossibility

（1）If another compactification recover the full order of Moyal product, **Gauge algebra of the Moyal product must be induced to the gauge algebra of VPD at M5-brane.**

$$\kappa^{\dot{\mu}}(\alpha) = \kappa_0^{\dot{\mu}}(\alpha) + \theta\kappa_1^{\dot{\mu}}(\alpha) + \theta^2\kappa_2^{\dot{\mu}}(\alpha) + \dots$$

（2）So following equation must be satisfied.

$$\boxed{\kappa^{\dot{\mu}}([\alpha, \alpha']_{\mathrm{Moyal}}) = \theta(\kappa^{\dot{\nu}}(\alpha)\partial_{\dot{\nu}}\kappa^{\dot{\mu}}(\alpha') - \kappa^{\dot{\nu}}(\alpha')\partial_{\dot{\nu}}\kappa^{\dot{\mu}}(\alpha))}$$

(3)But if we include higher order terms,

$$\boxed{\kappa^{\dot{\mu}}([\alpha,\alpha']_{\mathrm{Moyal}})=\theta(\kappa^{\dot{\nu}}(\alpha)\partial_{\dot{\nu}}\kappa^{\dot{\mu}}(\alpha')-\kappa^{\dot{\nu}}(\alpha')\partial_{\dot{\nu}}\kappa^{\dot{\mu}}(\alpha))}$$

Cannot be satisfied !!

(3) But if we include higher order terms,

$$\kappa^{\dot{\mu}}([\alpha, \alpha']_{\mathrm{Moyal}}) \cancel{=} \theta(\kappa^{\dot{\nu}}(\alpha)\partial_{\dot{\nu}}\kappa^{\dot{\mu}}(\alpha') - \kappa^{\dot{\nu}}(\alpha')\partial_{\dot{\nu}}\kappa^{\dot{\mu}}(\alpha))$$

Cannot be satisfied !!

↓

There is no way of compactification to recover the full order of Moyal product

（A） We keep the M5-brane as it is. Just change the way of compactification.

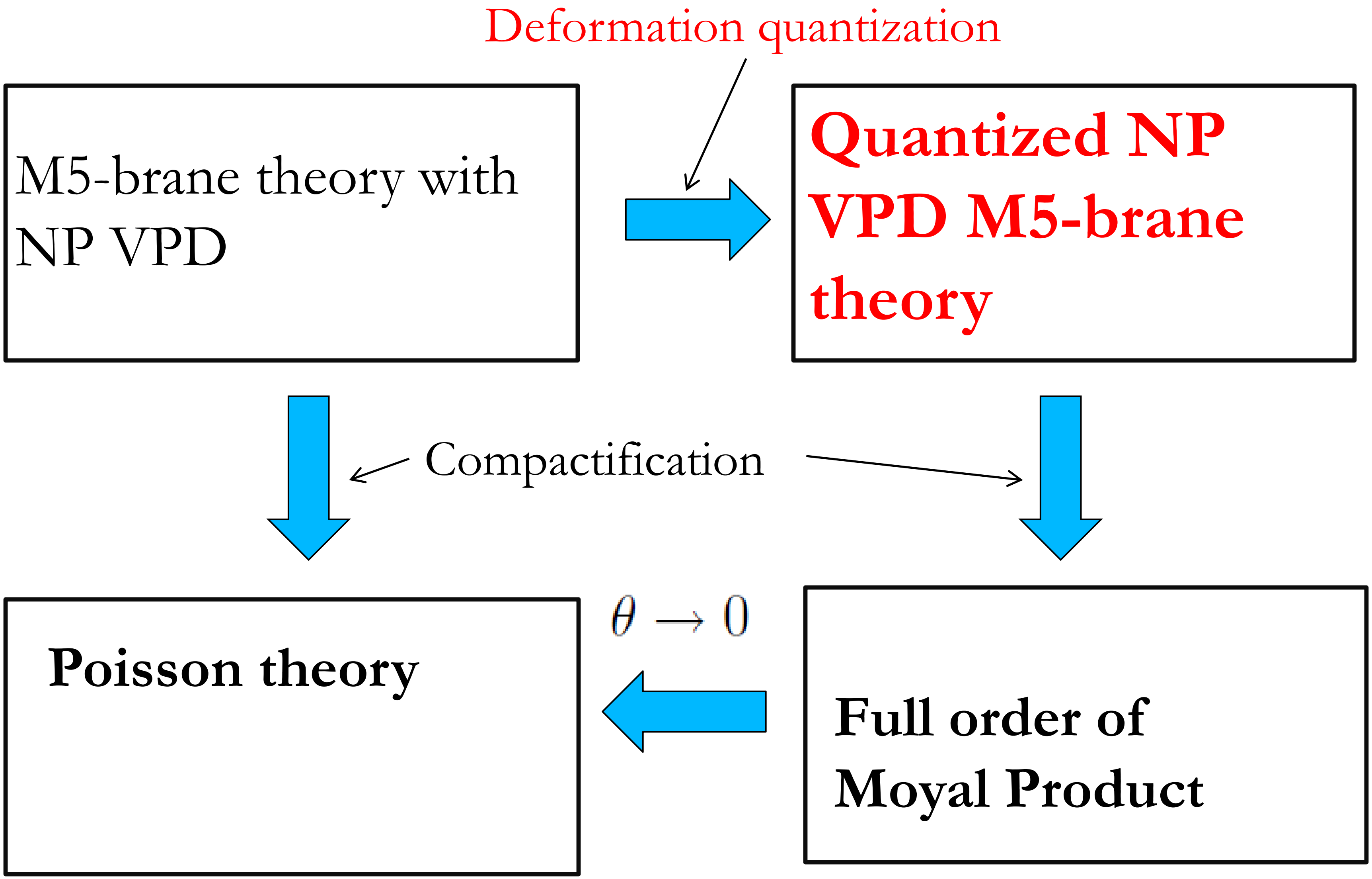
（B） We try to perform the deformation quantization of NP M5-brane theory

At Chen-Ho-T.T (JHEP 1003:104,2010.)
We challenge the above 2 ways.

（A）　We keep 5-brane as it is. Just change he wa of compactifica on.

（B）　We try to perform the deformation quantization of NP M5-brane theory

How about the (B) ??



Deformation quantization

M5-brane theory with NP VPD

Quantized NP VPD M5-brane theory

Compactification

Poisson theory

$\theta \rightarrow 0$

Full order of Moyal Product

Impossible

By Lecomte & Roger (1996, French paper)

Chen-Ho-T.T (JHEP 1003:104,2010.)

Lecomte & Roger (1996, French paper) (Mathematical paper)

In the compact manifold,
more than 3-dimensions,
There is no non-trivial deformation quantization of VPD symmetry

We pointed out due to this theorem,
It is impossible to perform the deformation quantization of VPD based on NP bracket.

Chen-Ho-T.T (JHEP 1003:104,2010.)

Today I omitted the proof

(A) We keep [illegible]5-brane as it is. Just change [illegible]he wa[illegible] of compactifica[illegible]on.

(B) We try to perform the deformation quantization of NP M5-brane theory

How about the (B) ??

（A） We keep 5-brane as it is. Just change he wa of compactifica on.

（B） We tr o pe form the deformation ua iz tion of NP M5-brane theo y.

How about the (B) ??

Basic reason of No-go

Star product $*$:Map from 2 to 1 component

$$\mathcal{A} \times \mathcal{A} \rightarrow \mathcal{A}$$

It is Ring with associativity !!

$$(\Lambda_1 * \Lambda_2) * h = \Lambda_1 * (\Lambda_2 * h)$$

Associativity guarantees the closure of algebra !!

$$[\Lambda_1, [\Lambda_2, \phi]_{\mathrm{Moyal}}]_{\mathrm{Moyal}} - [\Lambda_2, [\Lambda_1, \phi]_{\mathrm{Moyal}}]_{\mathrm{Moyal}} = [[\Lambda_1, \Lambda_2]_{\mathrm{Moyal}}, \phi]_{\mathrm{Moyal}}$$

$$\delta_{\Lambda_1}\delta_{\Lambda_2}\phi - \delta_{\Lambda_1}\delta_{\Lambda_2}\phi = \delta_{[\Lambda_1,\Lambda_2]}\phi$$

But NP is the map from 3 to 1.

$$\mathcal{A} \times \mathcal{A} \times \mathcal{A} \rightarrow \mathcal{A}$$

does not exist the ring structure with associativity.

By this, Fundamental identity (generalized Jacobi identity) is not satisfied, and the gauge algebra is not closed…

Origin of No-go result !!

Showing such No-go result is our work.

Future prospect

How to overcome ?

(C) To satisfy the Fundamental identity or associativity is key-point !

Zariski quantization.

Dito,Flato, Sternheimer,Takhtajan: Commun. Math. Phys. 183, 1-22 (1997)

(1) We deform the structure of NP itself from beginning.

$\mathcal{A} \times \mathcal{A} \to \mathcal{A}$ Product as Ring !!

$$\{f_1, f_2, f_3\} = \epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}}\ \partial_{\dot{\mu}} f_1 \cdot \partial_{\dot{\nu}} f_2 \cdot \partial_{\dot{\rho}} f_3$$

By deforming this product, we will be able to possess the property of fundamental identity and associativity.

Following is their deformation.

$$[f,g,h]_{\mathrm{zar}} = \sum_{\sigma\in S_3} \epsilon(\sigma)\Delta_{\sigma_1} f \bullet_\theta \Delta_{\sigma_2} g \bullet_\theta \Delta_{\sigma_3} h$$

With following nice property.

$\bullet_\theta$: deformed product

commutative, associative and distributive.

Δ_{σ_a} : defomed derivative.

Leibniz rule and commutative.

↓

Fundamental identity is satisfied and efficient to let the algebra closed.

There are following **Problems**

1. The space with the new product $\bullet_\theta$ is
Completely new vector space V
different from the original functional space

If it is proper deformation, by the change of continuum parameter, we must be able recover the original NP structure.

→ It has not been clarified whether such recovering is possible or not.

2. How can we connect it to the VPD smoothly ?

Solving these problems is
One of the strategies.. !!

Quantization of NP has not been completed yet !!

summary

Motivation

＊We want to know what is the M-theory origin of the Non-commutative geometry ?

What we tried

＊We searched the M5-brane theory which can recover the Non-commutative geometry whose origin is string theory.

＊We tried to find out from the deformation of NP bracket, which partially recovers the NC geometry.

Result

- We found out the No-go theorem, which is the obstacle to recover the full Non-commutative geometry from VPD NP bracket.
- This theorem is meaningful to point out the proper way to recover.
- One possible alternative is the Zariski quantization. But the problems have been kept.

The End.

Although we can not define the gauge invariant inner product for the background, y^I

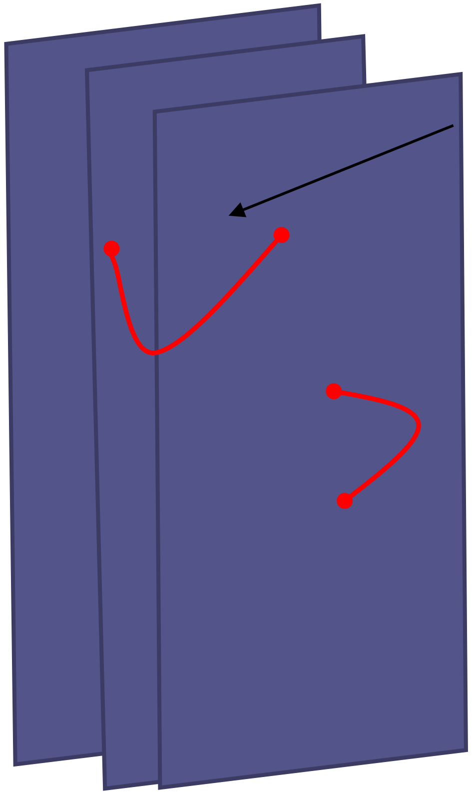
$$X^I(x,y) = y^I + \sum_a X^I_a(x)\chi^a(y)$$

The gauge symmetry of the action is kept as unbroken.

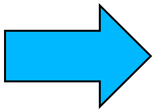
Because the background y^I shows up only through the NP-bracket,
After go through the NP-bracket
$\longrightarrow$ **it changed to trace element $\chi^a(y)$**

$$\begin{aligned}
\{e^{il_{\dot{\mu}}y^{\dot{\mu}}}, e^{im_{\dot{\mu}}y^{\dot{\mu}}}, e^{in_{\dot{\mu}}y^{\dot{\mu}}}\}_{NP} &= \epsilon_{\dot{\mu}\dot{\nu}\dot{\rho}} l^{\dot{\mu}} m^{\dot{\nu}} n^{\dot{\rho}} e^{i(l+m+n)_{\dot{\eta}}y^{\dot{\eta}}} \in \mathcal{V}_{tr} \\
\{e^{il_{\dot{\mu}}y^{\dot{\mu}}}, e^{im_{\dot{\mu}}y^{\dot{\mu}}}, y^{\dot{\sigma}}\}_{NP} &= \epsilon_{\dot{\mu}\dot{\nu}\dot{\rho}} \delta^{\dot{\sigma}\dot{\rho}} l^{\dot{\mu}} m^{\dot{\nu}} e^{i(l+m)_{\dot{\eta}}y^{\dot{\eta}}} \in \mathcal{V}_{tr} \\
\{e^{il_{\dot{\mu}}y^{\dot{\mu}}}, y^{\dot{\sigma}}, y^{\dot{\tau}}\}_{NP} &= \epsilon_{\dot{\mu}\dot{\nu}\dot{\rho}} \delta^{\dot{\sigma}\dot{\nu}} \delta^{\dot{\tau}\dot{\rho}} l^{\dot{\mu}} e^{il_{\dot{\eta}}y^{\dot{\eta}}} \in \mathcal{V}_{tr} \\
\{y^{\dot{\mu}}, y^{\dot{\nu}}, y^{\dot{\rho}}\}_{NP} &= \epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}} \cdot 1 \in \mathcal{V}_{tr}
\end{aligned}$$

N Dp-brane effective theory
= p-dimensional U(N) gauge theory



Open string



Behave as gauge fields or matter with adjoint representation