Holographic Confining Gauge theory and Response to Electric Field

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Gauge/String duality

Open strings

Closed strings

Dp-brane

the \((p+1)\) dimensional object
to which open strings can attach
Quark: strings connecting the N D3-branes and Nf D7-branes (N color, Nf flavor)

Meson: strings whose both end points are on the D7 branes

Gluon: strings whose both end points are on the D3 branes
The theory of open strings on $N$ D3 branes is regarded as 4d SU($N$) gauge theory.\[ S_{D3} = -\tau_3 \int d^4\xi \sqrt{-\det(G_{ab} + F_{ab})} \]
\[ \sim -\frac{1}{4g_{YM}^2} \int d^4\xi F^2 \]

On the other hand, the 10d space-time near the $N$ D3-branes becomes $AdS_5 \times S^5$.

Thus, 4d SU($N$) gauge theory is dual to the string theory in $AdS_5 \times S^5$ (gauge/string duality).
By considering D7-branes in $\text{AdS}_5 \times S^5$, we can examine mesons in 4d SU(N) gauge theory.

It is also known that by considering D7-brane in $\text{AdS}_5 \text{BH} \times S^5$, we can examine mesons in the SU(N) 4d gauge theory at high temperature.
Guage theory at finite E

In this research, we examine the gauge theory at the finite electric field and finite temperature by using gauge/string duality.

Electric field: U(1) gauge field on the D7 branes
Temperature: A radius of the Black Hole.
Electric Conductivity
We consider the $\text{AdS}_5 \times \text{S}^5$ background which is dual to the $4d$ SU(N) gauge theory at high temperature $T$ which corresponds to the radius of the BH horizon ($r_T$)

\[ ds^2 = \frac{r^2}{R^2} \left(-f^2(r) dt^2 + (dx^i)^2\right) + \frac{1}{f^2(r)} \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \]

\[ f(r) = \sqrt{1 - \left(\frac{r T}{r}\right)^4} \]

\[ R = (4\pi g_s \alpha'^2 N)^{1/4} \]
We consider mesons and quarks by introducing D7-branes as a probe in the $AdS_5 \ BH \times S^5$ Background.

Here, we consider 1 D7-brane (1-flavor).

$N$ D3 Branes become $AdS_5 \ BH \times S^5$ Background.
The induced metric on the D7-brane in the AdS$_5$ BH × S$^5$ becomes

$$ds_8^2 = \frac{r^2}{R^2} \left( -f^2(r) dt^2 + (dx^i)^2 \right) + \frac{R^2}{U^2} \left( (1 + (\partial_\rho \omega)^2) d\rho^2 + \rho^2 d\Omega_3^2 \right)$$

where $U$, $\rho$ and $\omega$ are defined as

$$X^8 = w(\rho) \quad X^9 = 0$$

$$U(r) = r \sqrt{\frac{1 + f(r)}{2}}$$

$$U^2 = \rho^2 + (X^8)^2 + (X^9)^2$$

We can set $X^9=0$ from the rotational symmetry on the $X^9$-$X^8$ plane.
We also consider gauge fields $A_x$ and $A_t$ on the D7-brane as

$$A_0(\rho), \quad A_x(\rho, t) = -Et + h(\rho)$$

where $E$ is an electric field and other gauge fields are assumed to be zero.
Then, the D7 brane DBI action becomes

\[
S_{D7} = -\tau_7 \int d^8 \xi \sqrt{g + F}
\]

\[
= -\tau_7 \int d^8 \xi \sqrt{\varepsilon_3 \rho^3 (r/U)^4 f \sqrt{(1 + (w')^2) \left(1 - \frac{R^4 E^2}{r^4 f^2}\right) + \frac{f^2 h(\rho)^2 - A_t(\rho)^2 U^2}{r^2 f^2}}}.
\]

We can get the conserved charges from D7-brane Lagrangian as

\[
n = \frac{\partial L_{D7}}{\partial A'_t}, \quad j_x = \frac{\partial L_{D7}}{\partial h'}
\]
$n$ and $j_x$ correspond to the baryon number density and electric current respectively.

$$n = \langle \psi^\dagger \psi \rangle \quad j_x = \langle \bar{\psi} \gamma^x \psi \rangle$$

By using the Legendre transformation, we can eliminate gauge fields $A_t$ and $A_x$ from the D7-brane action.

$$L_{D7} = \sqrt{1 + w'((\rho)^2)} F_A(r),$$

$$F_A(r) = \left( \frac{r}{U} \right)^2 \left( 1 - \frac{R^4 E^2}{r^4 f^2} \right) \left( f^2 n^2 + \left( \frac{r}{U} \right)^6 \rho^6 f^2 - j^2 \right)$$
For the positivity condition of $F_A(r)$, there is some $r_*$ which satisfies the following relations,

$$1 - \frac{R^4 E^2}{r_*^4 f_*^2} = 0$$

$$f_*^2 n_*^2 + \left(\frac{r_*}{U_*}\right)^6 \rho_*^6 f_*^2 - j^2 = 0$$

where $U_* = U(r_*)$, $f_* = f(r_*)$, $\rho_* = \rho(r_*)$.

From these relations, we can get the electrical conductivity $\sigma$. A. Karch and A. O'Bannon '08

$$j = \sigma(E)E \quad \sigma(E) = \sqrt{\frac{\rho_*^6}{U_*^4} + \frac{R^4 n^2}{r_*^4}}$$
$\dot{j}_x = \sigma(E) E \quad \sigma(E) = \sqrt{\frac{\rho^6}{U^4} + \frac{R^4 n^2}{r^4}}$

Even when the charge density $n=0$, electric conductivity $\sigma(E)$ is still finite because of the first term in the square root.

This first term corresponds to the effect of the pair creation of quark and anti-quark by the electric field $E$. 
The quark mass $m_q$ is given as $m_q = w(\rho = \infty)$.

D7 brane solutions $w(\rho)$ for various $m_q$ at zero density ($n=0$) and $E=3$ are given as follows. (Blue circle represents $r=r_*$)

There are two types of solutions,
- Solutions which don't intersect the $r=r_*$ circle
- Solutions which intersect the $r=r_*$ circle

J. Mas et al '09
The D7 brane solutions which don't intersect the $r = r_*$ circle satisfy
\[ 1 - \frac{R^4 E^2}{r^4 f^2} > 0 \]
for all $\rho$. ($0 \leq \rho \leq \infty$)
For the positivity of $F_A(r)$, the electric current $j$ must be zero. Thus, these solutions correspond to the insulator phase.
D7 brane solutions which intersect the $r = r_*$ circle correspond to the conductor phase ($j > 0$).
D7 brane solutions $w(\rho)$ for various $m_q$ at zero density $(n=0)$ and $E=3$ (Blue circle represents $r=r_*$)

Small $m_q$ solutions --- Conductor $(j>0)$
Large $m_q$ solutions --- Insulator $(j=0)$

This means that quark and antiquark of small mass are easier to be pair created with an electric field.

J. Mas et al '09
D7 brane solutions for finite density \((n=0.2)\) and \(E=3\)

There are no insulator phase solutions because there exists the charge density \(n\) explicitly.
Chiral symmetry and Electric field

We consider 10d background in which supersymmetry is broken and temperature is zero.

A. Kehagias and K. Sfetsos '99

\[ ds_{10}^2 = e^{\Phi/2} \left\{ \frac{r^2}{R^2} A^2(r)(-dt^2 + (dx^i)^2) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right\}. \]

\[ e^\Phi = \left( \frac{(r/r_0)^4 + 1}{(r/r_0)^4 - 1} \right)^{3/2} \]

\[ A(r) = \left( \left( 1 - \left( \frac{r_0^8}{r^8} \right) \right) \right)^{1/4}. \]

\( r = r_0 \) is the singularity where curvature radius becomes diverge. We only consider the \( r > r_0 \) region.
The gauge theory dual to this background is in confinement phase.

The potential between the quark and antiquark is given by the U-shaped string and it becomes the linear rising potential at large distance $L$. 
We consider the embedding D7-brane in this background with gauge fields $A_x$.

After the Legendre transformation, D7-brane Lagrangian becomes

$$L_{D7} = \sqrt{1 + w'(\rho)^2} F_A(r),$$

$$F_A(r) = \rho^6 e^{2\Phi} A^8 \left( 1 - \frac{R^4 E^2}{r^4 A^4 e^{\Phi}} \right) \left( 1 - \frac{R^2 j^2}{\rho^6 A^6 e^{\Phi}} \right).$$

Here, we consider the case of zero density ($n=0$).
From the positivity condition of $F_A(r)$, we find that there exists a critical electric field $E_c$ below which the electric current $j$ becomes zero for all D7-brane solutions.

$$ER^2 \leq A^2 e^{\Phi/2} r^2 \equiv T(r)$$

$E > E_c$

$E_c = 2.399$

$r^*_0 = 1$ and $R = 1$
Solutions \( w(\rho) \) for various mass \( m_q \) at \( E=1.5<E_c \)

Green circle is the singularity \((r=r_0)\).

In this case, there is no \( r=r_\ast \) circle because it becomes complex. Therefore, \( w(\rho) \) solutions for all \( m_q \) is in the insulator phase (electric current \( j=0 \)).
Solutions $w(\rho)$ for various mass $m_q$ at $E=2.3855$ which is slightly smaller than $E_c=2.39$

Solutions for all $m_q$ are still in the insulator phase.
Solutions $w(\rho)$ for $E=10>E_c$

The $r=r_*$ (blue) circle appears and there are solutions at conductor phase for small mass $m_q$.

Small $m_q$ solutions • • • Conductor phase

Large $m_q$ solutions • • • Insulator phase
In conductor phase, there are conical singular solutions which we are not sure about the physical meaning of it now.
From the value of the solutions $w(\rho)$ at large $\rho$, we can read the vev of chiral condensate $c$ as follows.

\[ w(\rho) = m_q + \frac{c}{\rho^2} \]

\[ c \sim \langle \bar{\psi}\psi \rangle \]

By noticing the solution $w(\rho)$ at $m_q=0$, we can find the chiral symmetry is restored at $E=E_c$ where the electric current $j$ becomes nonzero.

(K. Ghoroku, M. I. and T Taminato '10)
The similar result is obtained by NJL model.

*Klevansky and Lemmer '89*
Summary

- We consider the gauge theory at finite electric field by using the dual gravity background.
- We find that the chiral symmetry transition and conductor-insulator transition occur at the same electric field $E_c$.
- We don't know about the physical meaning of conical singular solutions.
- It is also interesting to consider gauge theory at finite magnetic field.