Holographic Confining Gauge theory and Response to Electric Field

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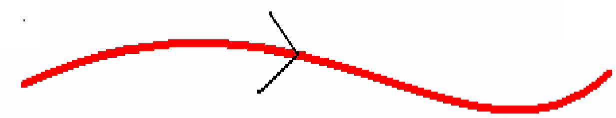
contents

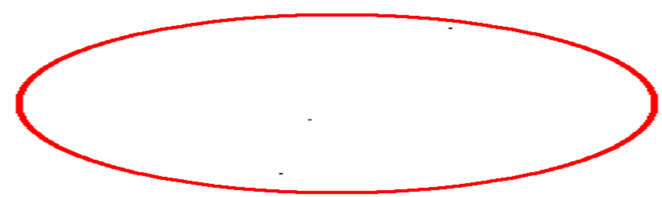
- Gauge/string duality
- Electric Conductivity
- Chiral symmetry and electric field.
- Summary

Gauge/String duality

Open strings



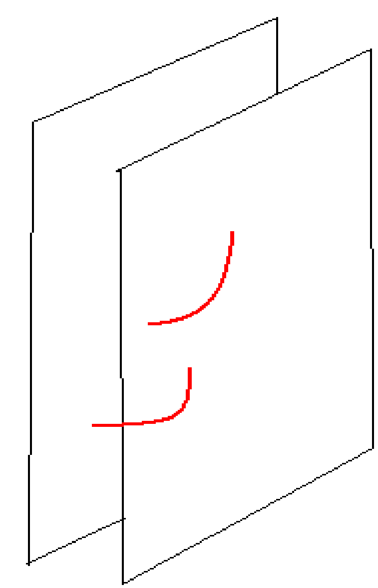
Closed strings



Dp-brane

the (p+1) dimensional object

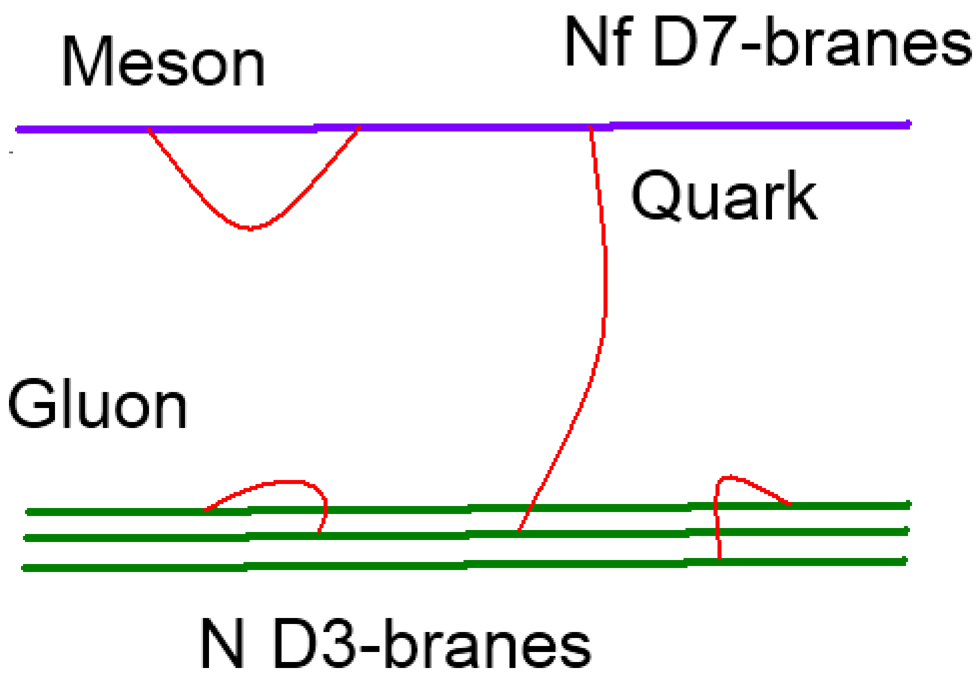
to which open strings can attach



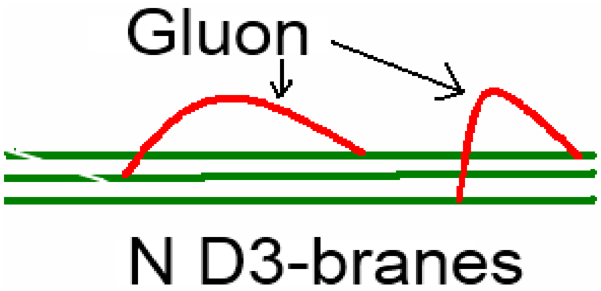
Quark: strings connecting the N D3-branes and Nf D7- branes (N color, Nf flavor)

Meson: strings whose both end points are on the D7 branes

Gluon: strings whose both end points are on the D3 branes

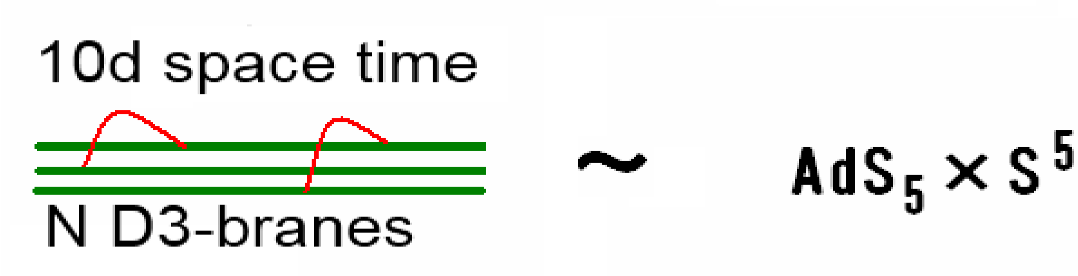


The theory of open strings on N D3 branes is regarded as 4d SU(N) gauge theory.



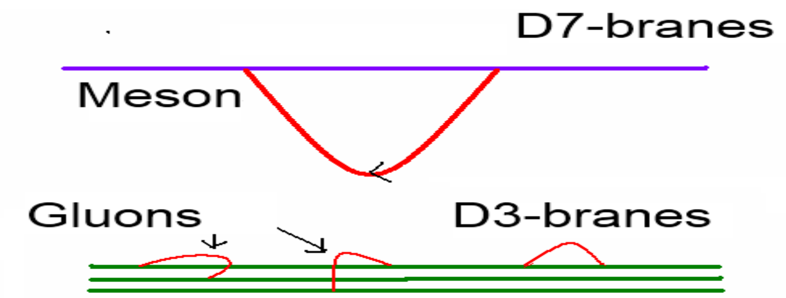
$$S_{D3} = -\tau_3 \int d^4\xi \sqrt{-\det(G_{ab} + F_{ab})}$$
$$\sim -\frac{1}{4g_{YM}^2} \int d^4\xi F^2$$

On the other hand, the 10d space-time near the N D3-branes becomes $AdS_5 \times S^5$



Thus, 4d SU(N) gauge theory is dual to the string theory in $AdS_5 \times S^5$ (gauge/string duality)

By considering D7-branes in $AdS_5 \times S^5$, we can examine mesons in 4d SU(N) gauge theory.



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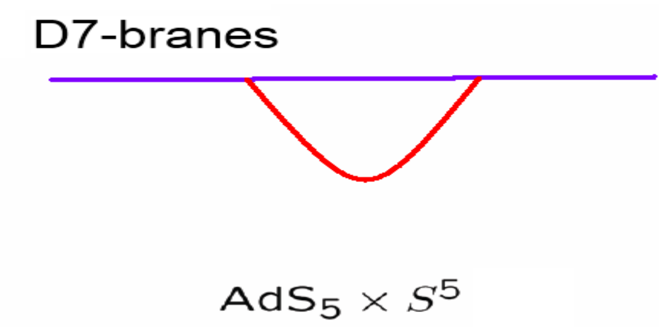
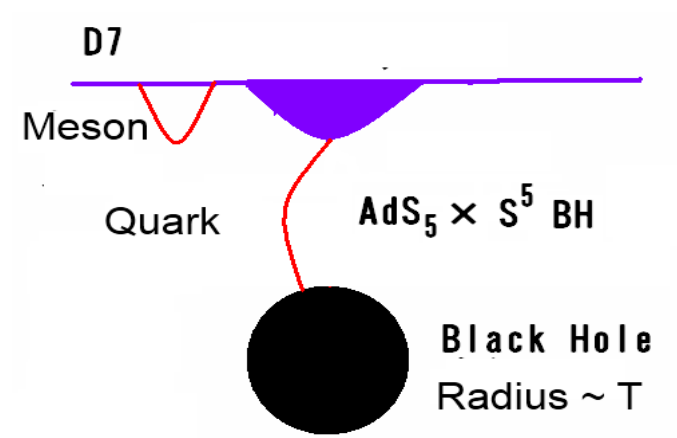


It is also known that by considering D7-brane in AdS_5 BH$\times S^5$, we can examine mesons in the SU(N) 4d gauge theory at high temperature.

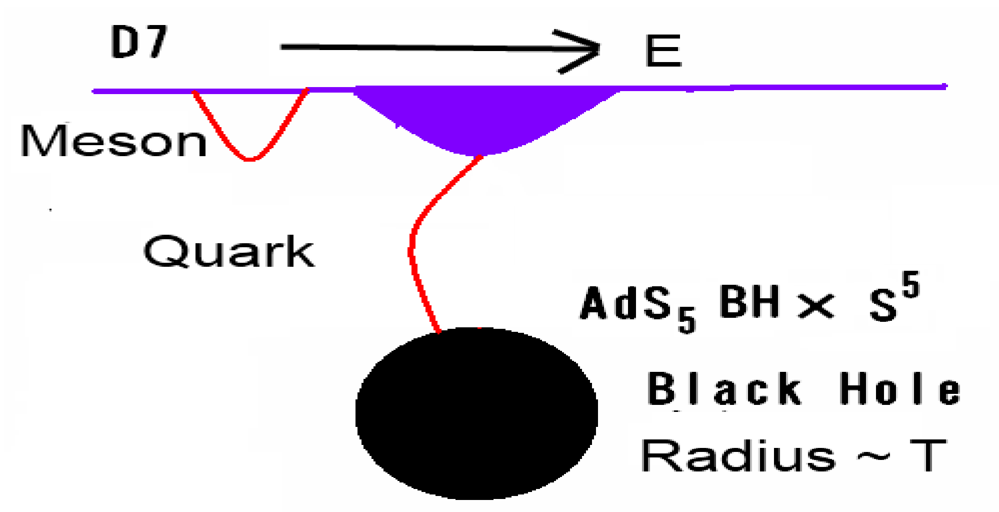


Guage theory at finite E

In this research, we examine the gauge theory at the finite electric field and finite temperature by using gauge/string duality.

Electric field: U(1) guage field on the D7 branes

Temperature: A radius of the Black Hole.



Electric Conductivity

We consider the $AdS_5\ BH \times S^5$ background which is dual to the *4d* SU(N) gauge theory at high temperature *T* which corresponds to the radius of the BH horizon(r_T)

$$ds^2 = \frac{r^2}{R^2}\left(-f^2(r)dt^2 + (dx^i)^2\right) + \frac{1}{f^2(r)}\frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

$$f(r) = \sqrt{1 - \left(\frac{r_T}{r}\right)^4}$$

$$R = (4\pi g_s \alpha'^2 N)^{1/4}$$

We consider mesons and quarks by introducing D7-branes as a probe in the AdS_5 $BH \times S^5$ Background.

	X^0	X^1	X^2	X^3	X^4	X^5	X^6	X^7	X^8	X^9
D3-brane :	○	○	○	○						
D7-brane :	○	○	○	○	○	○	○	○		

Here, we consider 1 D7-brane (1-flavor).

N D3 Branes become AdS_5 $BH \times S^5$ Background.

The induced metric on the D7-brane in the $AdS_5\,BH \times S^5$ becomes

$$ds_8^2 = \frac{r^2}{R^2}\left(-f^2(r)dt^2 + (dx^i)^2\right) + \frac{R^2}{U^2}\left((1+(\partial_\rho w)^2)d\rho^2 + \rho^2 d\Omega_3^2\right)$$

, where U, ρ and ω are defined as

$$X^8 = w(\rho) \quad X^9 = 0$$

$$U(r) = r\sqrt{\frac{1+f(r)}{2}}$$

$$U^2 = \rho^2 + (X^8)^2 + (X^9)^2$$

We can set $X^9=0$ from the rotational symmetry on the X^9-X^8 plane.

We also consider gauge fields A_x and A_t on the D7-brane as

$$A_0(\rho) \quad A_x(\rho, t) = -Et + h(\rho)$$

where E is an electric field and other gauge fields are assumed to be zero.

Then, the D7 brane DBI action becomes

$$S_{D7} = -\tau_7 \int d^8\xi \sqrt{g+F}$$
$$= -\tau_7 \int d^8\xi$$
$$\times \sqrt{\epsilon_3}\rho^3 (r/U)^4 f \sqrt{(1+(w')^2)\left(1-\frac{R^4E^2}{r^4f^2}\right)+\frac{f^2h(\rho)'^2 - A_t(\rho)'^2U^2}{r^2f^2}}$$

We can get the conserved charges from D7-brane Lagrangian as

$$n = \frac{\partial L_{D7}}{\partial A_t'} \quad j_x = \frac{\partial L_{D7}}{\partial h'}$$

n and j_x correspond to the baryon number density and electric current respectively.

$$n = \langle \psi^\dagger \psi \rangle \qquad j_x = \langle \bar{\psi} \gamma^x \psi \rangle$$

By using the Legendre transformation, we can eliminate gauge fields A_t and A_x from the D7-brane action.

$$I_{\mathrm{D7}} = -\tau_7 \int d^8\xi \sqrt{\epsilon_3}\; L_{\mathrm{D7}} \qquad L_{\mathrm{D7}} = \sqrt{(1 + w'(\rho)^2) F_A(r)},$$

$$F_A(r) = \left(\frac{r}{U}\right)^2 \left(1 - \frac{R^4 E^2}{r^4 f^2}\right) \left(f^2 n^2 + \left(\frac{r}{U}\right)^6 \rho^6 f^2 - j^2\right)$$

For the positivity condition of $F_A(r)$, there is some r_* which satisfies the following relations,

$$1 - \frac{R^4 E^2}{r_*^4 f_*^2} = 0 \qquad f_*^2 n_*^2 + \left(\frac{r_*}{U_*}\right)^6 \rho_*^6 f_*^2 - j^2 = 0$$

where $U_* = U(r_*)$, $f_* = f(r_*)$, $\rho_* = \rho(r_*)$.

From these relations, we can get the electrical conductivity σ. *A. Karch and A. O'Bannon '08*

$$j_x = \sigma(E)E \quad \sigma(E) = \sqrt{\frac{\rho_*^6}{U_*^4} + \frac{R^4 n^2}{r_*^4}}$$

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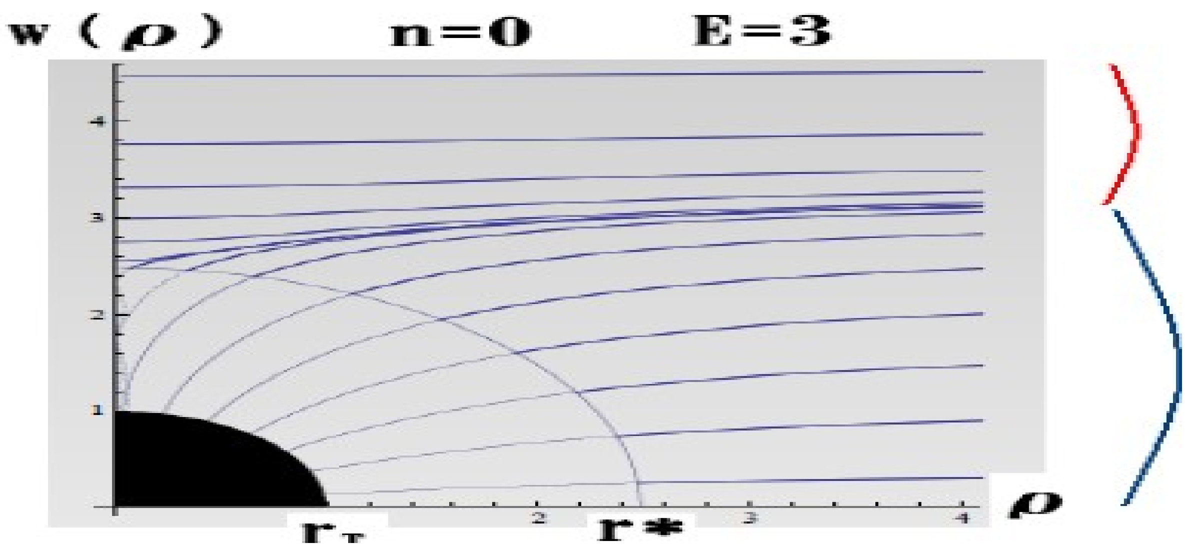
Even when the charge density *n=0*, electric conductivity *σ(E)* is still finite because of the first term in the square root.

This first term corresponds to the effect of the pair creation of quark and anti-quark by the electric field *E*.

The quark mass m_q is given as $m_q=w(\rho=\infty)$.

D7 brane solutions $w(\rho)$ for various m_q *at* zero density ($n=0$) and $E=3$ *are given as follows.* (Blue circle represents $r=r_*$)



J. Mas et al '09

There are two types of solutions,

･*Solutions which don't intersect the* $r=r_*$ *circle*

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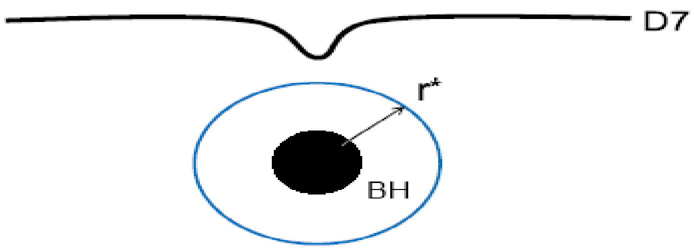
The D7 brane solutions which don't intersect the $r = r_*$ circle satisfy

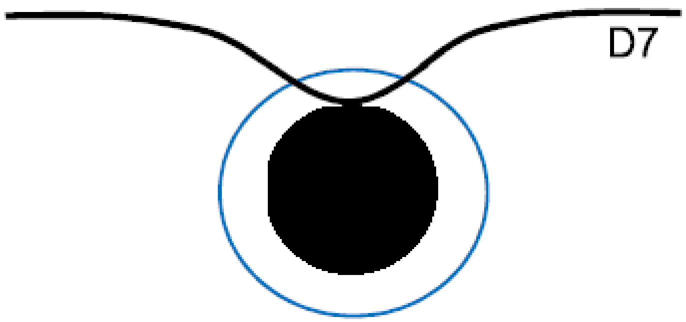
$$1 - \frac{R^4 E^2}{r^4 f^2} > 0$$



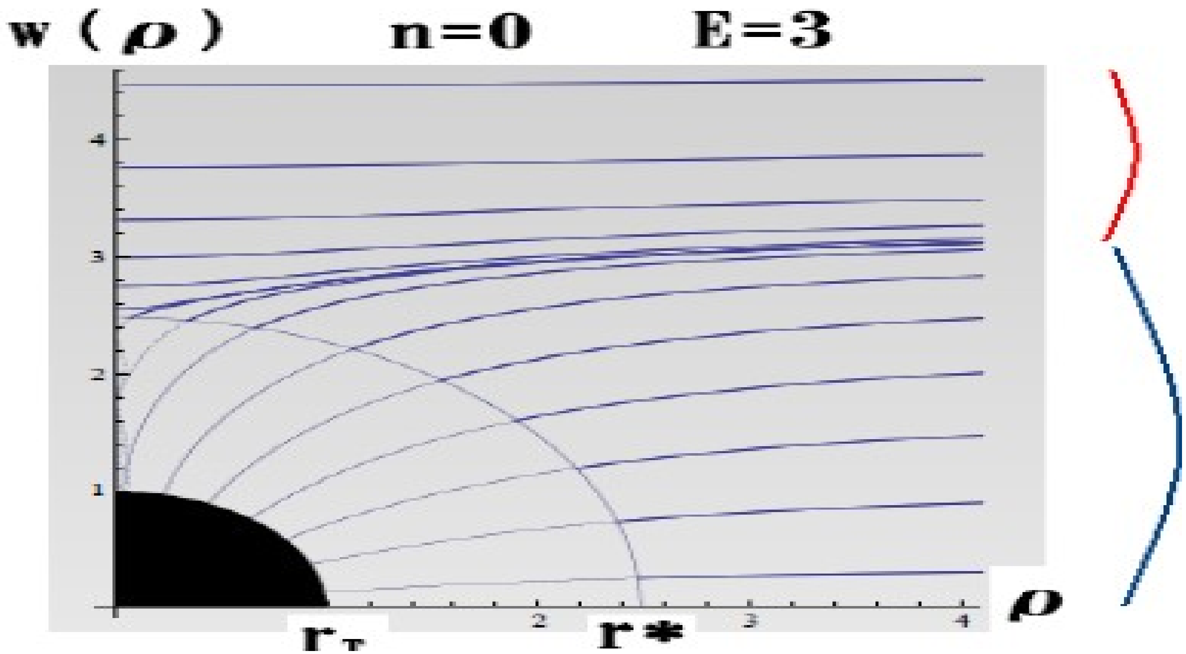
for all ρ. ($0 \leqq \rho \leqq \infty$)

For the positivity of $F_A(r)$, the electric current j must be zero.Thus, these solutions correspond to the insulator phase.

D7 brane solutions which intersect the $r=r_*$ cirle correspond to the conductor phase ($j>0$).



D7 brane solutions $w(\rho)$ for various m_q *at* zero density ($n=0$) and $E=3$ (Blue circle represents $r=r_*$)



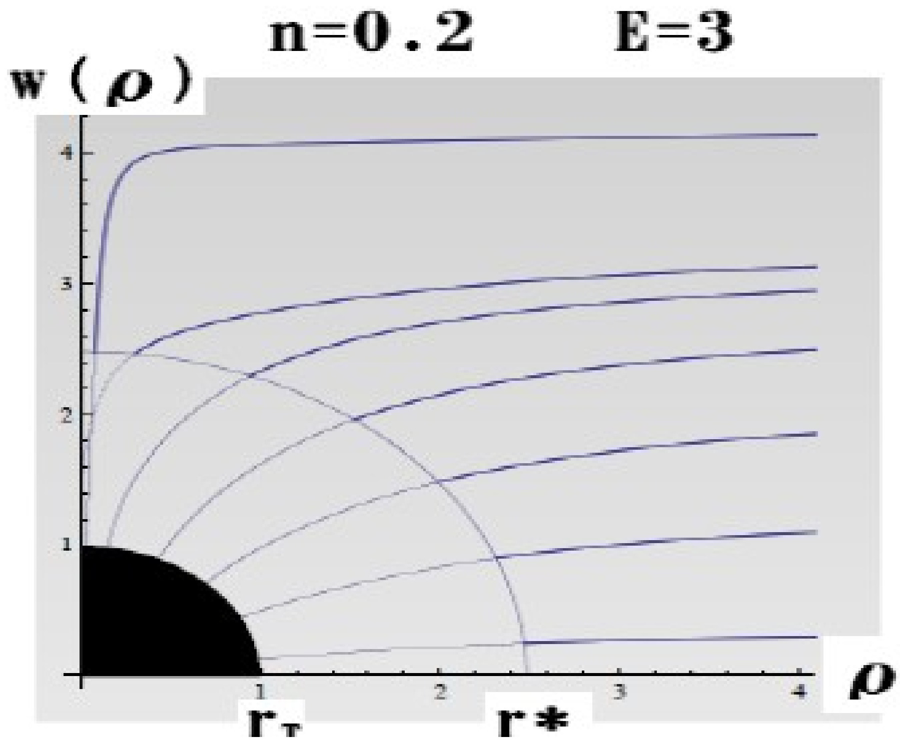
J. Mas et al '09

Small mq solutions・・・Conductor *(j>0)*

Large mq solutions・・・Insulator *(j=0)*

This means that quark and antiquark of small mass are easier to be pair created with an electric field.

D7 brane solutions for finite density (*n=0.2*) and *E=3*



There are no insulator phase solutions because there exists the charge density *n* explicitly.

Chiral symmetry and Electric field

We consider 10d background in which supersymmetry is broken and temperature is zero.

A. Kehagias and K.Sfetsos '99

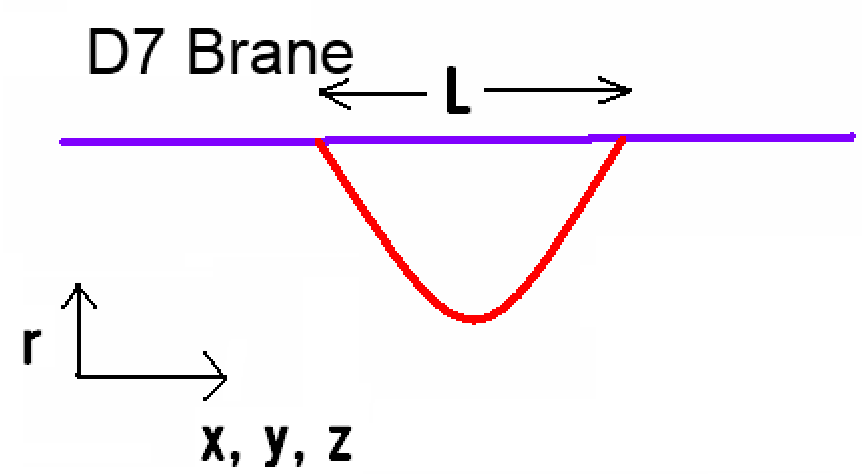
$$ds_{10}^2 = e^{\Phi/2}\left\{\frac{r^2}{R^2}A^2(r)(-dt^2 + (dx^i)^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right\}.$$

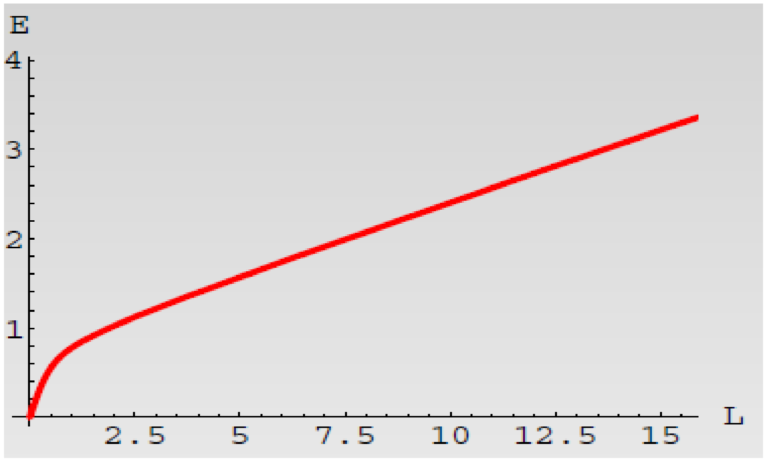
$$e^{\Phi} = \left(\frac{(r/r_0)^4 + 1}{(r/r_0)^4 - 1}\right)^{\sqrt{3/2}} \qquad A(r) = \left(\left(1 - \left(\frac{r_0}{r}\right)^8\right)\right)^{1/4}$$

$r = r_0$ is the singularity where curvature radius becomes diverge. We only consider the $r > r_0$ region.

The gauge theory dual to this background is in confinement phase.

The potential between the quark and antiquark is given by the U-shaped string and it becomes the linear rising potential at large distance L.





We consider the embedding D7-brane in this background with gauge fields *Ax*.

After the Legendre transformation, D7-brane Lagrangian becomes

$$L_{D7} = \sqrt{(1 + w'(\rho)^2)F_A(r)},$$

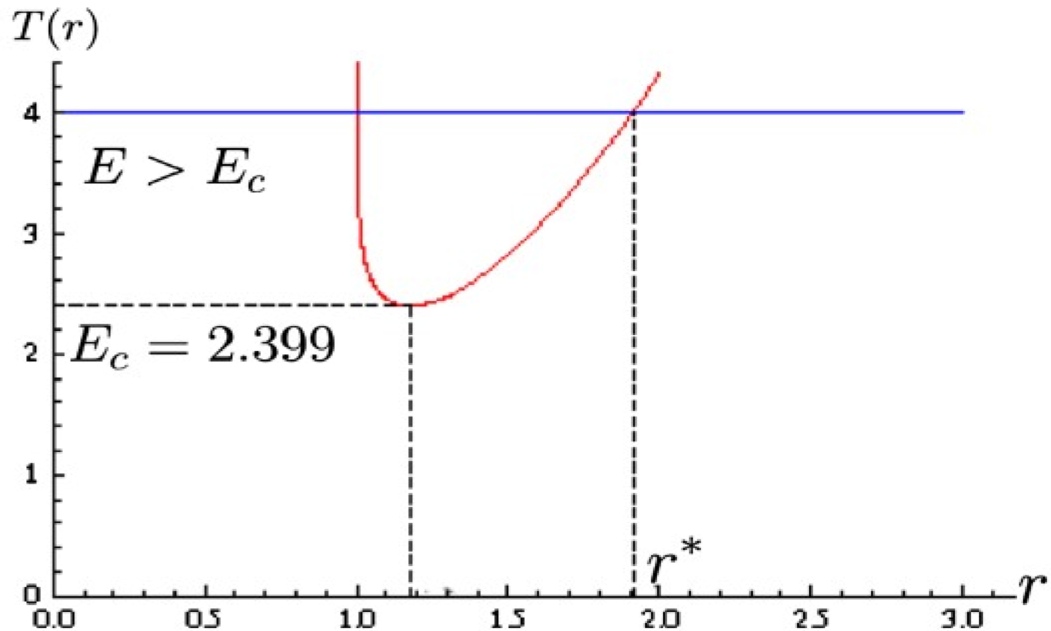
.

$$F_A(r) = \rho^6 e^{2\Phi} A^8 \left(1 - \frac{R^4 E^2}{r^4 A^4 e^{\Phi}}\right)\left(1 - \frac{R^2 j^2}{\rho^6 A^6 e^{\Phi}}\right).$$

Here, we consider the case of zero density (*n=0*).

From the positivity condition of $F_A(r)$, we find that there exists a critical electric field E_c below which the electric current j becomes zero for all D7-brane solutions.

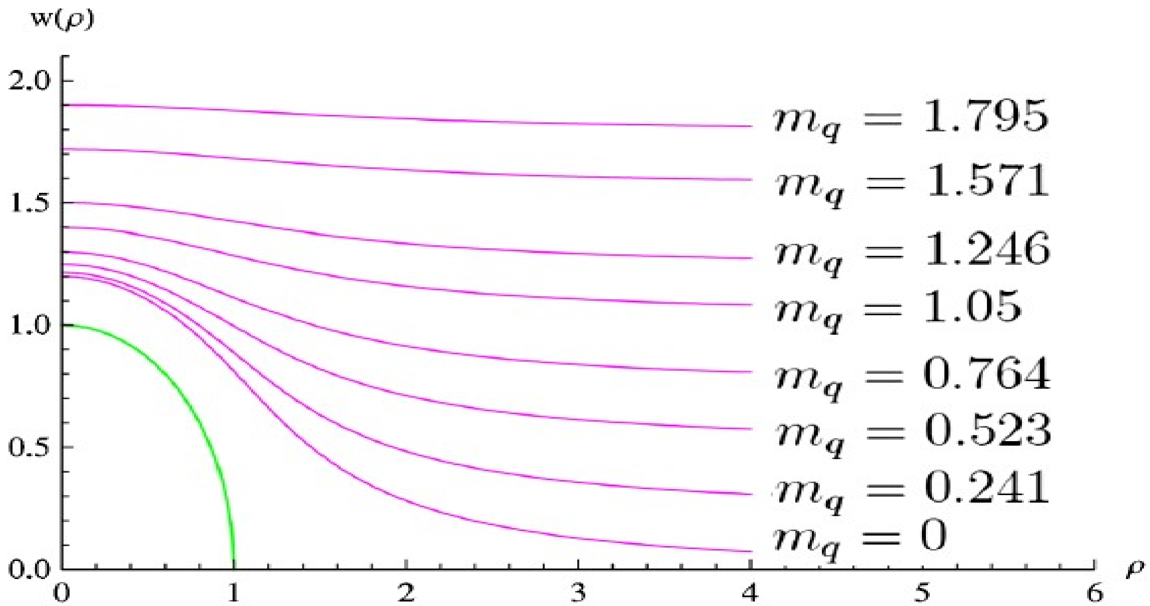
$$ER^2 \leq A^2 e^{\Phi/2} r^2 \equiv T(r)$$



$r_0 = 1$ and $R = 1$

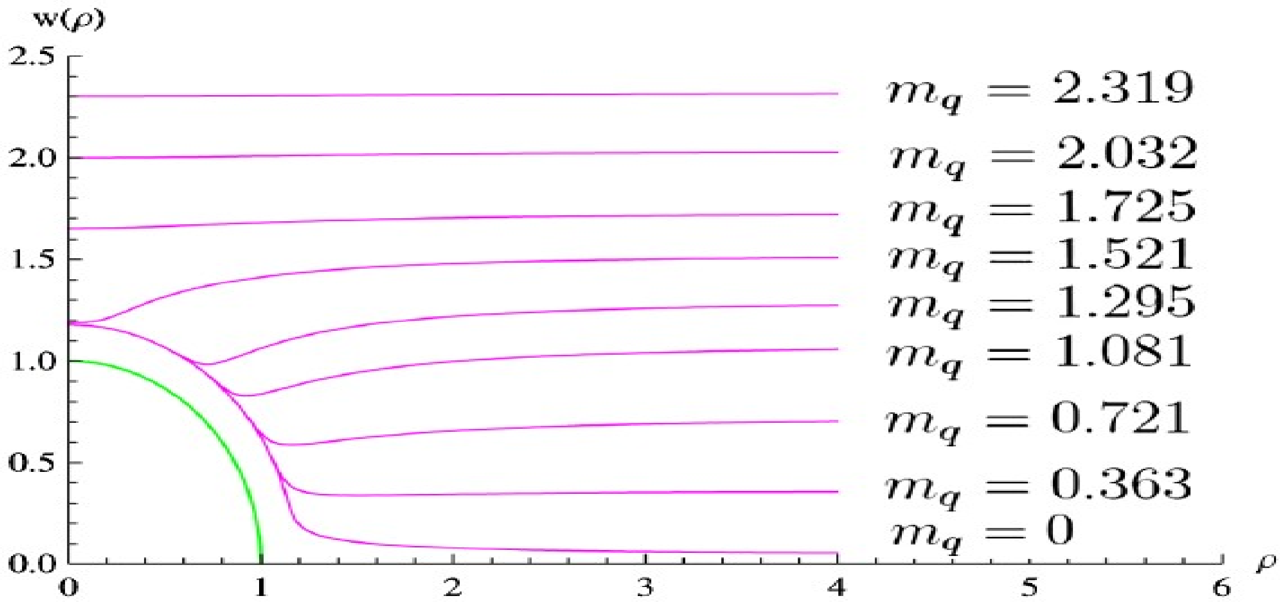
Solutions $w(\rho)$ for various mass m_q at $E=1.5<E_c$

Green circle is the singularity ($r=r_0$).



In this case, there is no $r=r_*$ circle because it becomes complex. Therefore, $w(\rho)$ solutions for all m_q is in the insulator phase (electric current $j=0$).

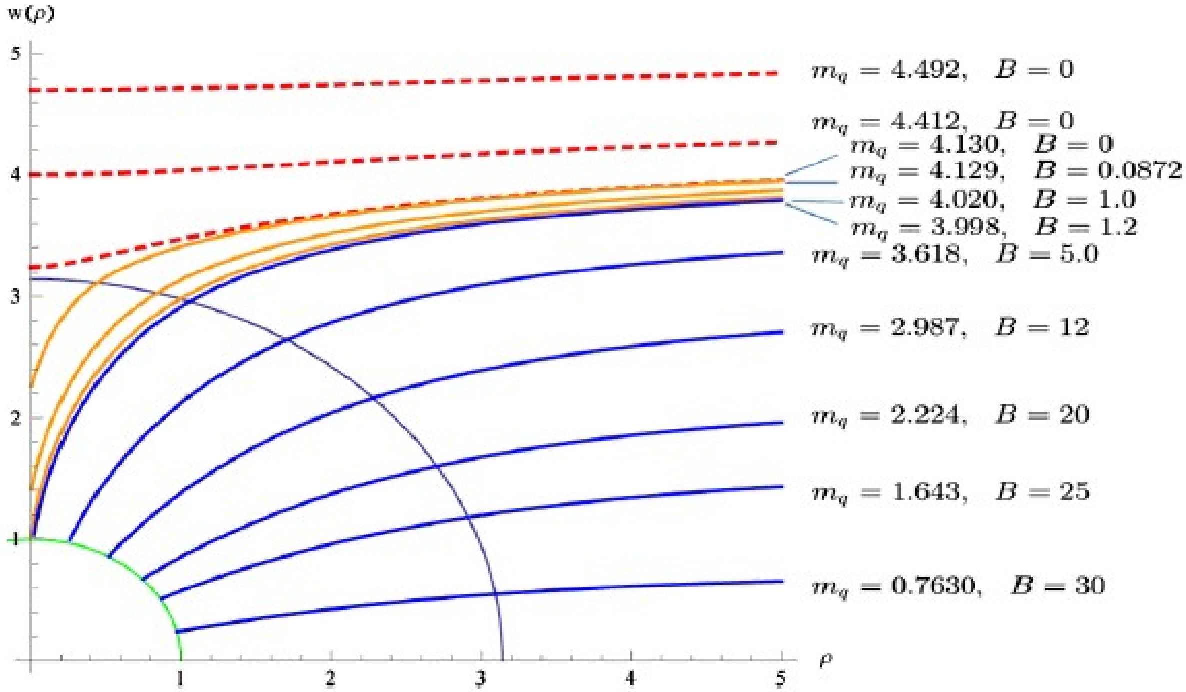
Solutions $w(\rho)$ for various mass m_q at $E=2.3855$ which is slightly smaller than $E_c=2.39$



Solutions for all m_q are still in the insulator phase.

Solutions w(ρ) for $E=10>E_c$

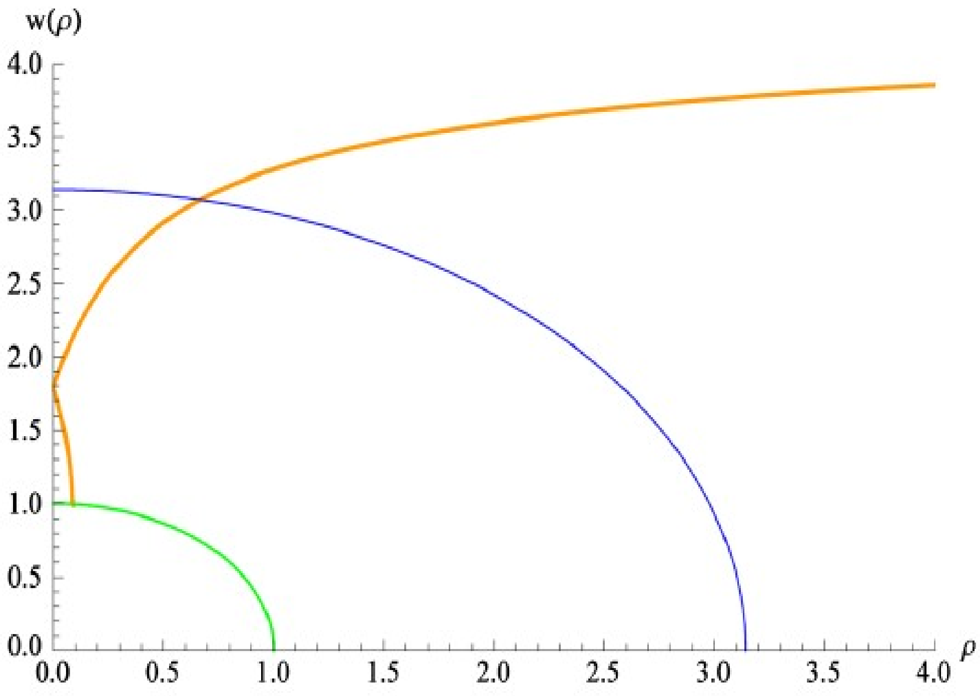
The $r=r_*$(blue) circle appears and there are solutions at conductor phase for small mass m_q



Small m_q solutions・・・Conductor phase

Large m_q solutions・・・Insulator phase

In conductor phase, there are conical singular solutions which we are not sure about the physical meaning of it now.

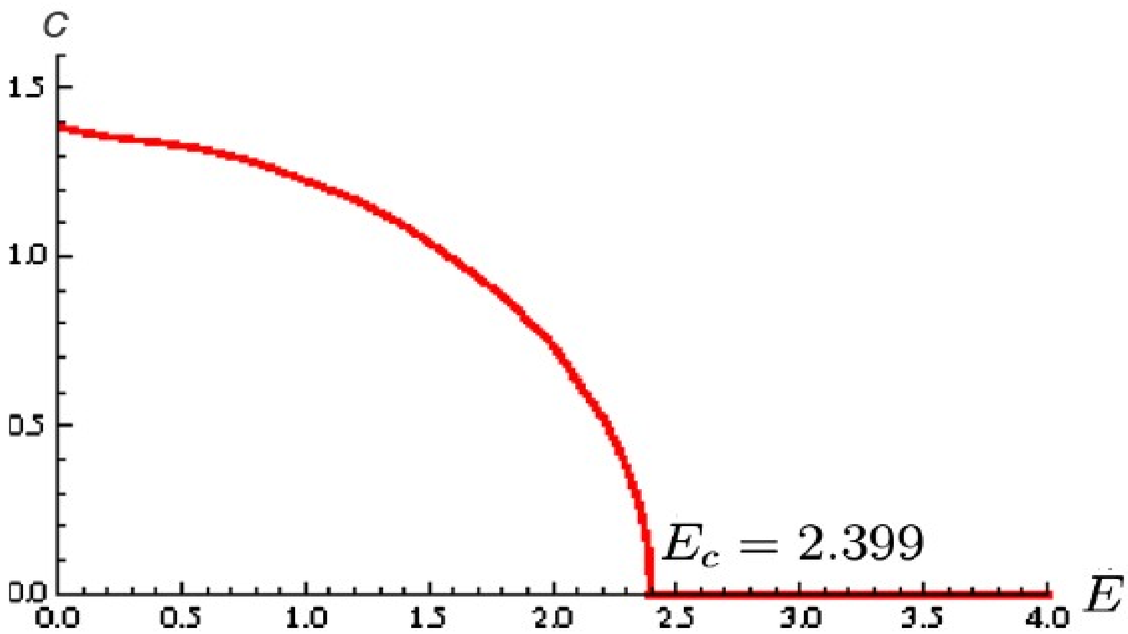


From the value of the solutions *w(ρ)* at large *ρ*, we can read the vev of chiral condensate c as follows.

$$w(\rho) = m_q + \frac{c}{\rho^2} \qquad c \sim \langle \bar{\psi}\psi \rangle$$

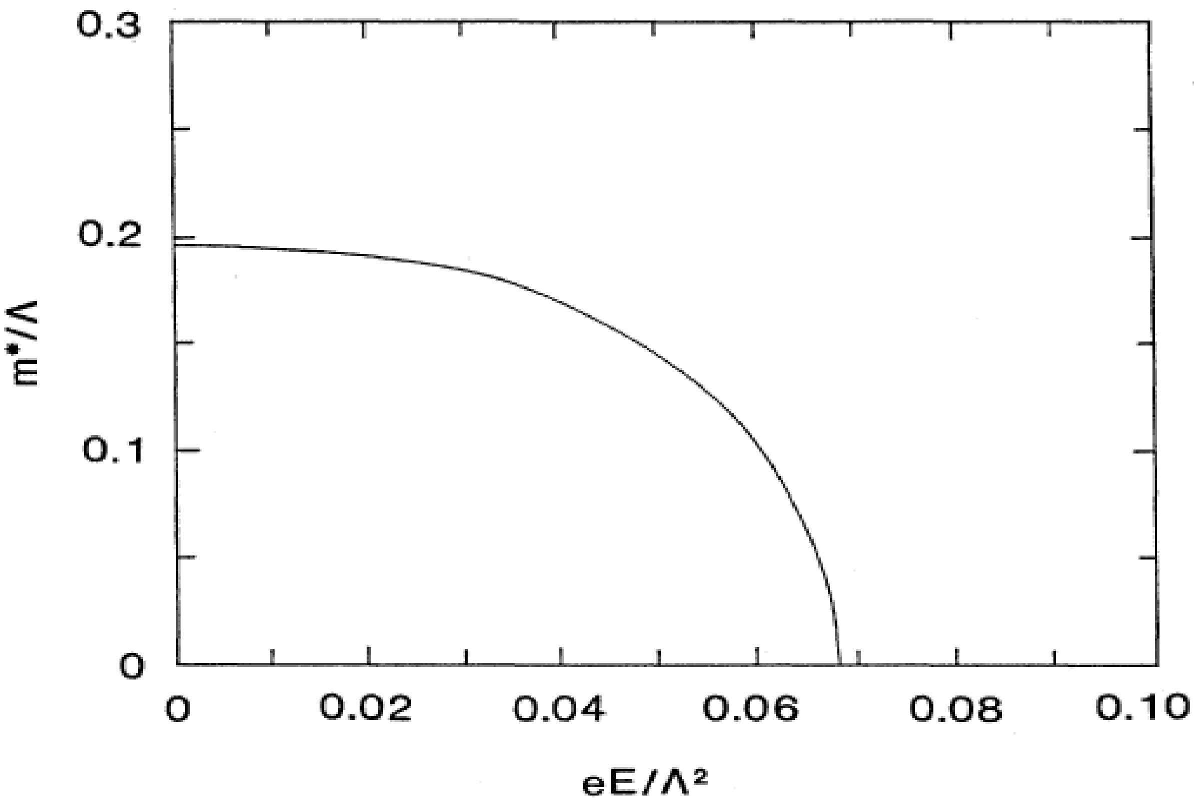
By noticing the solution *w(ρ)* at *mq=0*, we can find the chiral symmetry is restored at *E=Ec* where the electric current *j* becomes nonzero.

(K. Ghoroku, M. I. and T Taminato '10)



The similar result is obtained by NJL model.

Klevansky and Lemmer '89



Summary

- We consider the gauge theory at finite electric field by using the dual gravity background.
- We find that the chiral symmetry transition and conductor-insulator transition occur at the same electric field E_c.
- We don't know about the physical meaning of conical singular solutions.
- It is also interesting to consider gauge theory at finite magnetic field.