Development in theory of heavy quarkonia and precision determination of heavy quark masses

Y. Sumino
(Tohoku Univ.)

☆Plan of Talk

1. Before 1998: Theoretical problem

   IR renomalon

2. Around 1998: Drastic improvement

   Discovery of cancellation of renormalons

   Interpretation

3. After 1998: Theoretical development and applications

   EFT, higher-order calc.

   Spectroscopy

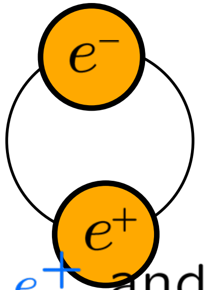
   Determinations of m_b, m_c (m_t)

   Determination of α_s

   Physical picture (gluon config.)

   ⋮

Computation of spectrum of Positronium (e^+e^- boundstate)



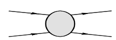
Free e^+ and e^-:

$$E_{\text{tot}} = 2m_{\text{pole}} - E_{\text{bin}}$$

Theoretical Framework (NRQCD)

Compute quark-antiquark scattering amplitude in ordinary perturbative QCD.
$\Longrightarrow$ Determine a quantum-mechanical Hamiltonian in expansion in $1/c$:

$$\hat{H} = \hat{H}_0 + \frac{1}{c}\,\hat{H}_1 + \frac{1}{c^2}\,\hat{H}_2 + \cdots .$$



Q

$\bar{Q}$

matching between pert. QCD and pNRQCD

$\Longrightarrow$ Solve the non-relativistic Schrödinger equation

$$\hat{H}\,\psi_X(r) = E_X\,\psi_X(r)$$

order by order in $1/c$ expansion. $\quad \frac{v}{c}, \qquad \alpha_s = \frac{g_s^2}{\hbar c}$

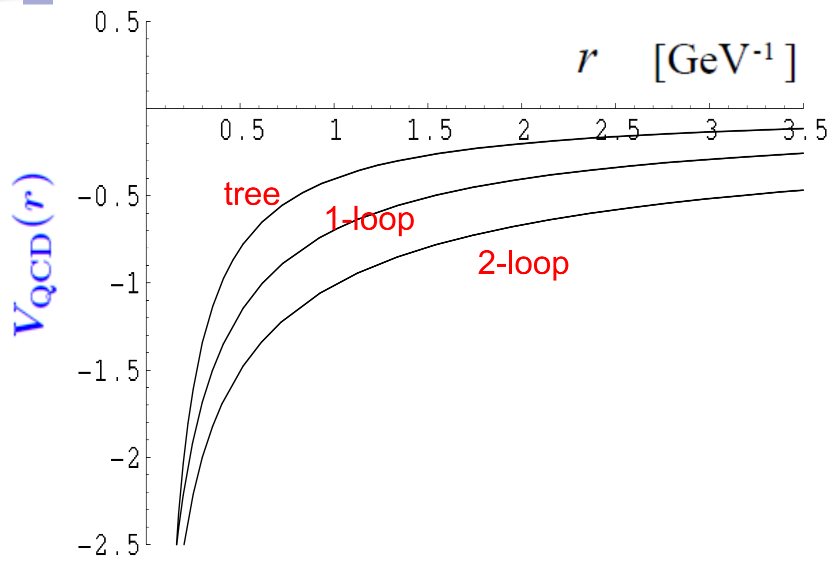
$$\hat{H}_0 = \frac{\vec{p}^{\,2}}{m} - C_F \frac{\alpha_S}{r},$$

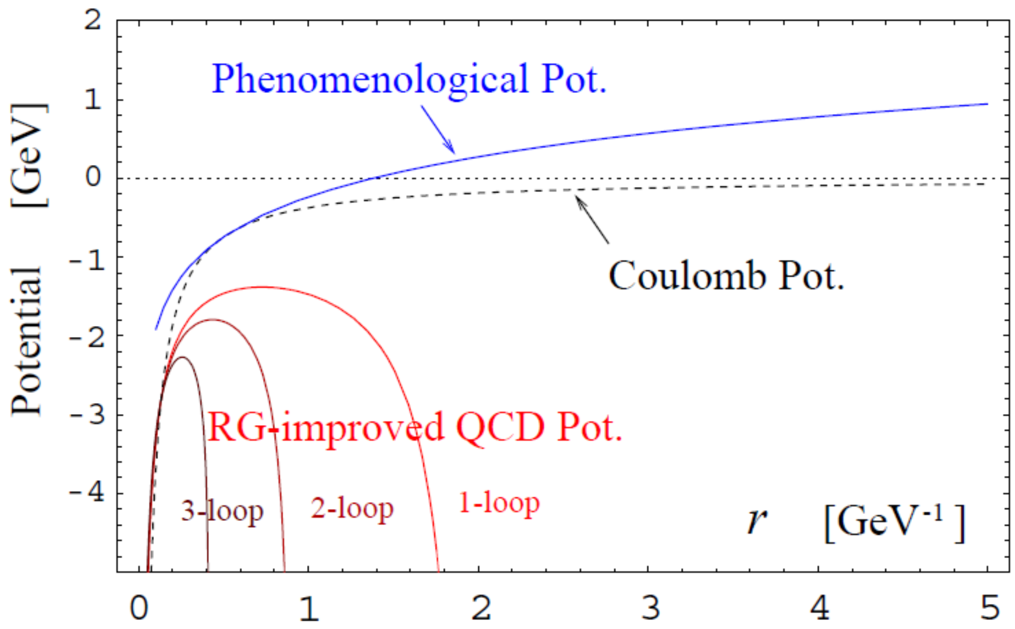
$$\hat{H}_1 = - C_F \frac{\alpha_S}{r} \cdot \left(\frac{\alpha_S}{4\pi}\right) \cdot \left\{\beta_0 \log(\mu'^2 r^2) + a_1\right\},$$

$$\hat{H}_2 = -\frac{\vec{p}^{\,4}}{4m^3} - C_F \frac{\alpha_S}{r} \cdot \left(\frac{\alpha_S}{4\pi}\right)^2 \cdot \left\{\beta_0^2 \left[\log^2(\mu'^2 r^2) + \frac{\pi^2}{3}\right] + (\beta_1 + 2\beta_0 a_1) \log(\mu'^2 r^2) + a_2\right\}$$
$$+\frac{\pi C_F \alpha_S}{m^2} \delta^3(\vec{r}) + \frac{3 C_F \alpha_S}{2m^2 r^3} \vec{L} \cdot \vec{S} - \frac{C_F \alpha_S}{2m^2 r} \left(\vec{p}^{\,2} + \frac{1}{r^2} r_i r_j p_j p_i\right) - \frac{C_A C_F \alpha_S^2}{2m r^2}$$
$$-\frac{C_F \alpha_S}{2m^2} \left\{\frac{S^2}{r^3} - 3\frac{(\vec{S} \cdot \vec{r})^2}{r^5} - \frac{4\pi}{3}(2S^2 - 3)\delta^3(\vec{r})\right\},$$

- $\Upsilon(1S)$: $M_{\Upsilon(1S)} = 9.94 - 0.17 - 0.20 - 0.30$ GeV
- $\Upsilon(2S)$: $M_{\Upsilon(2S)} = 9.94 - 0.10 - 0.19 - 0.45$ GeV

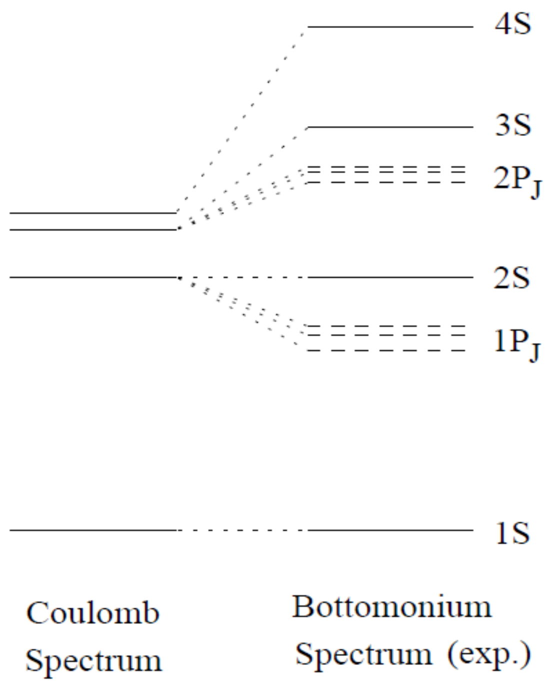
$\mathcal{O}(\alpha_s^0)$ $\mathcal{O}(\alpha_s^2)$ $\mathcal{O}(\alpha_s^3)$ $\mathcal{O}(\alpha_s^4)$





Folklore: pert. $\sim -C_F \frac{\alpha_s}{r}$, non-pert. $\sim K\, r$

$$V_{\rm QCD}(r) \simeq -C_F \frac{\alpha_s}{r} + K\, r$$



Inconsistency with OPE for $r \ll \Lambda_{\rm QCD}^{-1}$

non-pert. $\sim \langle G^a_{\mu\nu}(0)^2 \rangle\, r^3$

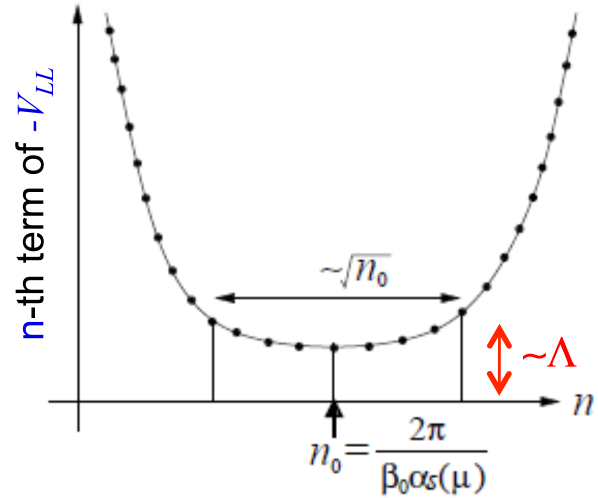
Brambilla, Pineda, Soto, Vairo

Renormalon in the QCD potential

Aglietti, Ligeti

$$\alpha_{1L}(q) = \frac{\alpha_S(\mu)}{1+\frac{\beta_0\alpha_S(\mu)}{4\pi}\log\left(\frac{q^2}{\mu^2}\right)} = \frac{4\pi/\beta_0}{\log\left(\frac{q^2}{\Lambda^2}\right)}$$

$$\Lambda \equiv \mu\,\exp\left[-\frac{2\pi}{\beta_0\alpha_S(\mu)}\right]$$



$$V_{LL}(r) = -\int\frac{d^3\vec{q}}{(2\pi)^3}\,e^{i\vec{q}\cdot\vec{r}}\,C_F\,\frac{4\pi\alpha_{1L}(q)}{q^2}$$

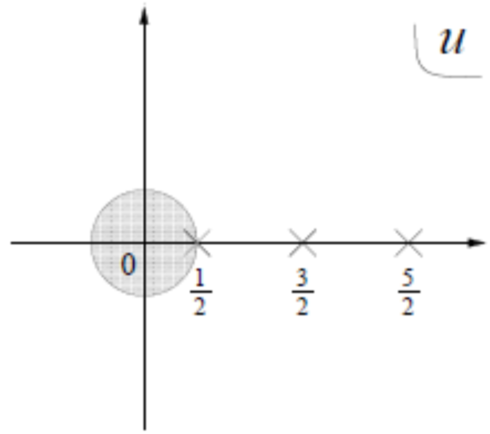
ill defined

$$= -C_F\,4\pi\alpha_S(\mu)\sum_{n=0}^{\infty}\int\frac{d^3\vec{q}}{(2\pi)^3}\,\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2}\left\{-\frac{\beta_0\alpha_S(\mu)}{4\pi}\log\left(\frac{q^2}{\mu^2}\right)\right\}^n$$

$$= -C_F\,4\pi\alpha_S(\mu)\sum_{n=0}^{\infty}\left\{\frac{\beta_0\alpha_S(\mu)}{4\pi}\right\}^n f_n(r,\mu)\times n!$$

$$F(r,\mu;u) \equiv \int\frac{d^3\vec{q}}{(2\pi)^3}\,\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2}\left(\frac{\mu^2}{q^2}\right)^u = \frac{(\mu r/2)^{2u}}{4\pi^{3/2}r}\,\frac{\Gamma(\frac{1}{2}-u)}{\Gamma(1+u)}$$

$$= \int\frac{d^3\vec{q}}{(2\pi)^3}\,\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2}\,\exp\left[-u\,\log\left(\frac{q^2}{\mu^2}\right)\right] = \sum_n f_n(r,\mu)\,u^n$$



Asymptotically $\quad f_n(r,\mu) \sim \frac{1}{2\pi^2}\,\mu\times 2^n$

$$\frac{1}{2\pi^2} = -2\,\mathrm{Res}\left[F;\,u=\frac{1}{2}\right]$$

Most dominant part is indep. of r !

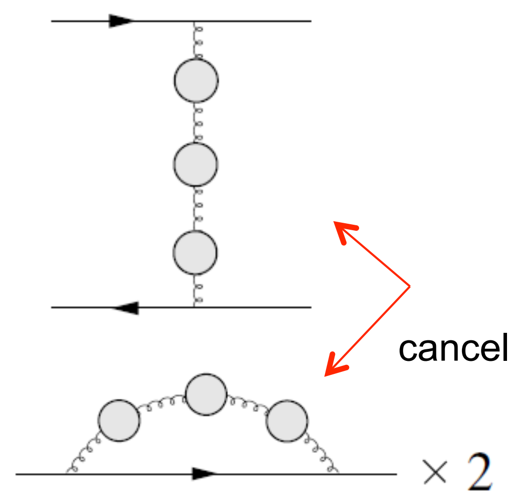
Accuracy of perturbative predictions for the QCD potential improved drastically around year 1998.

Pineda
Hoang,Smith,Stelzer,Willenbrock
Beneke

If we re-express the quark pole mass ($m_{\rm pole}$)
by the $\overline{\rm MS}$ mass ($m_{\overline{\rm MS}}$), IR renormalons cancel
in $E_{\rm tot}(r) = 2m_{\rm pole} + V_{\rm QCD}(r)$.

$$V_{\rm LL}(r) = -\int \frac{d^3\vec{q}}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{r}}\, C_F\, \frac{4\pi\alpha_{\rm 1L}(q)}{q^2}$$

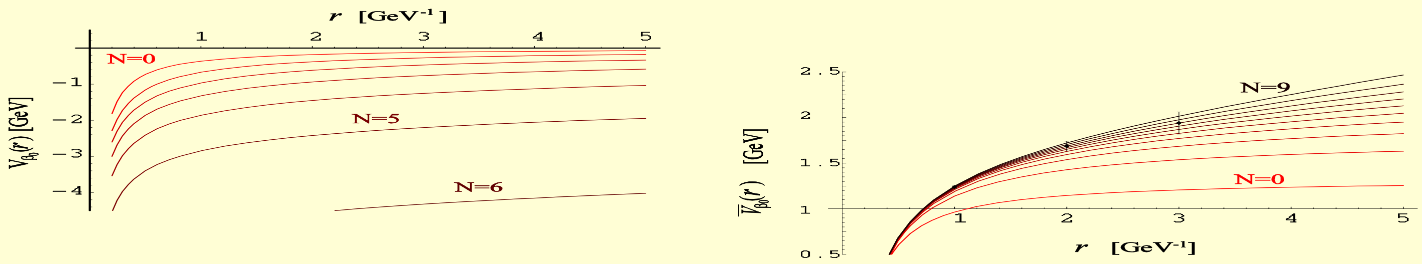
$$m_{\rm pole} \simeq m_{\overline{\rm MS}}(\mu) + \frac{1}{2}\int_{q<\mu} \frac{d^3\vec{q}}{(2\pi)^3}\, C_F\, \frac{4\pi\alpha_{\rm 1L}(q)}{q^2}$$



Expanding $e^{i\vec{q}\cdot\vec{r}} = \underline{1} + i\vec{q}\cdot\vec{r} + \frac{1}{2}(i\vec{q}\cdot\vec{r})^2 + \cdots$ for small q

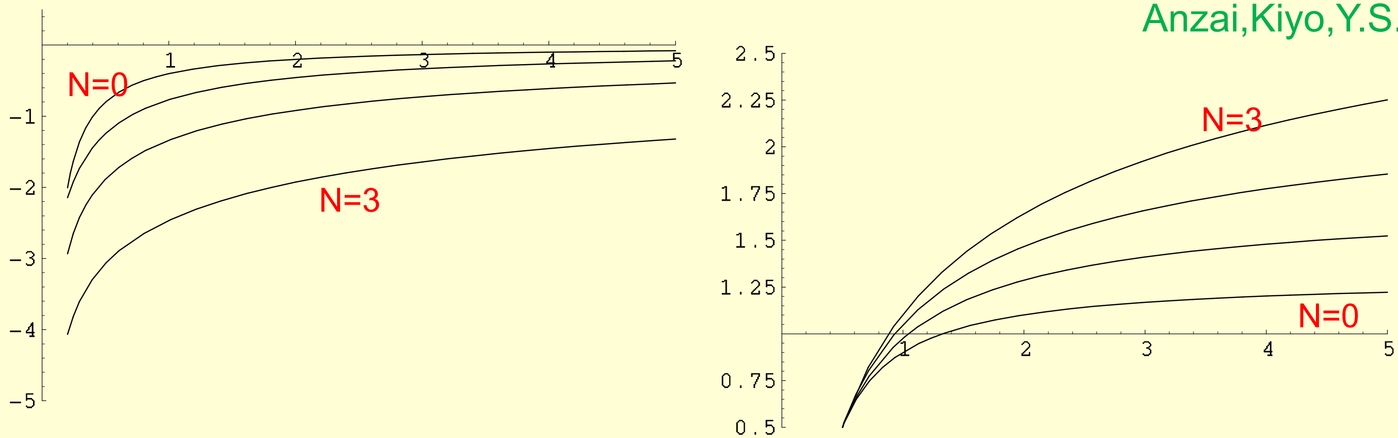
the leading renormalons cancel . ⇨ much more convergent series

Residual renormalon: $\Lambda \times \left\langle (\vec{q}\cdot\vec{r})^2 \right\rangle \sim \Lambda \times \left(\Lambda\, r\right)^2 \ll \Lambda$

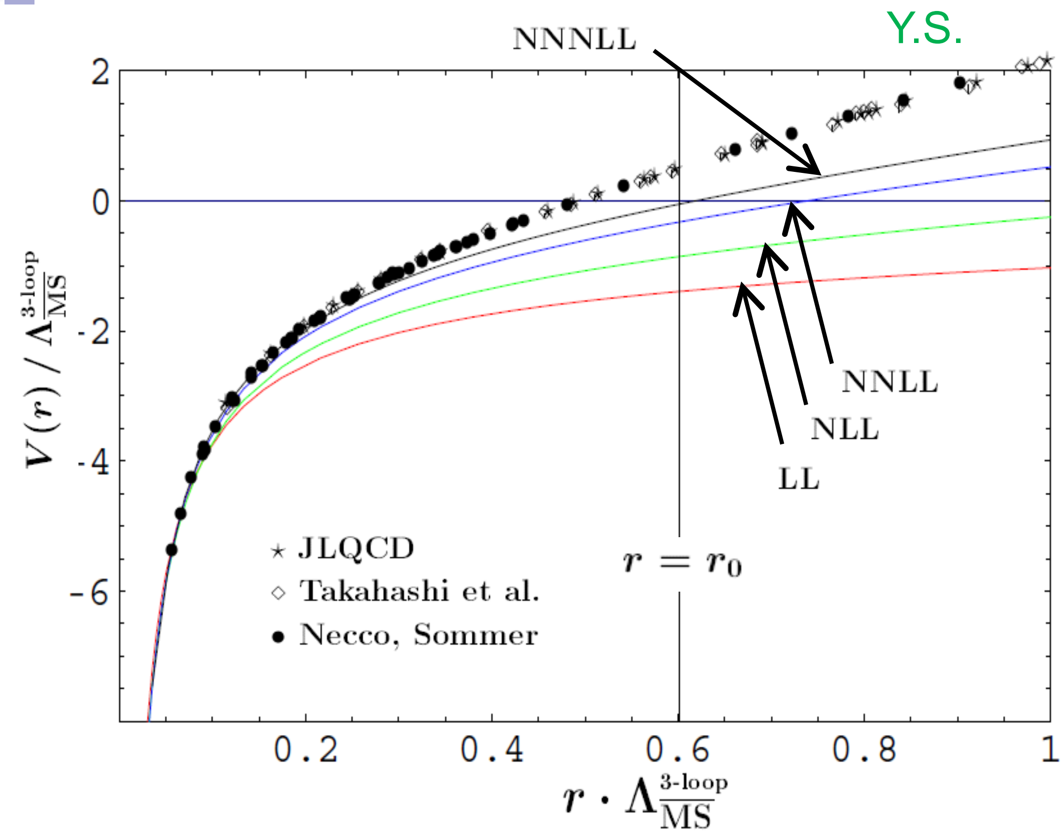


Leading log approximation

Anzai,Kiyo,Y.S.



Exact pert. potential up to 3 loops



Folklore ruled out

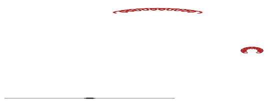
pert. $\sim -C_F \frac{\alpha_s}{r}$, non-pert. $\sim K\,r$

$$V_{\rm QCD}(r) \simeq -C_F \frac{\alpha_s}{r} + K\,r$$

- $\Upsilon(1S)$: $M_{\Upsilon(1S)} = 9.94 - 0.17 - 0.20 - 0.30$ GeV (Pole-mass scheme)
  $= 8.41 + 0.72 + 0.15 + 0.015 - 0.008$ GeV ($\overline{\rm MS}$-scheme)
- $\Upsilon(2S)$: $M_{\Upsilon(2S)} = 9.94 - 0.10 - 0.19 - 0.45$ GeV (Pole-mass scheme)
  $= 8.41 + 1.46 + 0.093 + 0.009$ GeV ($\overline{\rm MS}$-scheme)

Pole mass m_{pole}

$0 < \lambda_g < \infty$

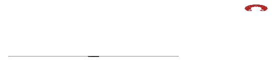


Q

m_{bare}

$\overline{\text{MS}}$ mass $\overline{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$

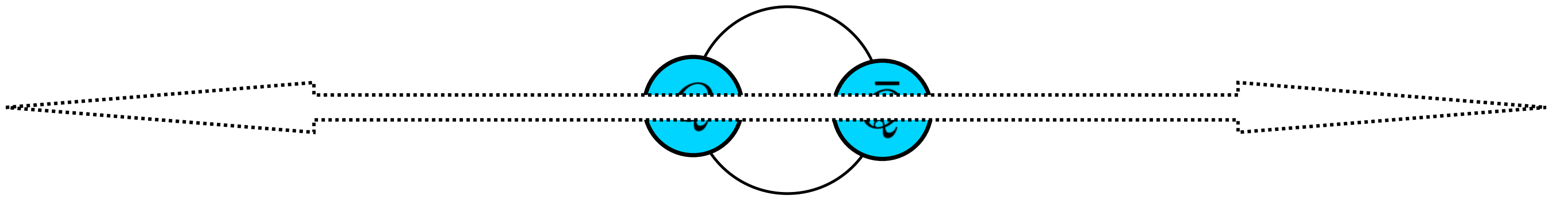
$0 < \lambda_g < 1/\overline{m}$



Q

m_{bare}

Computation of spectrum of Heavy Quarkonium ($Q\bar{Q}$ boundstate)



Free Q and $\bar{Q}$: } not well-defined

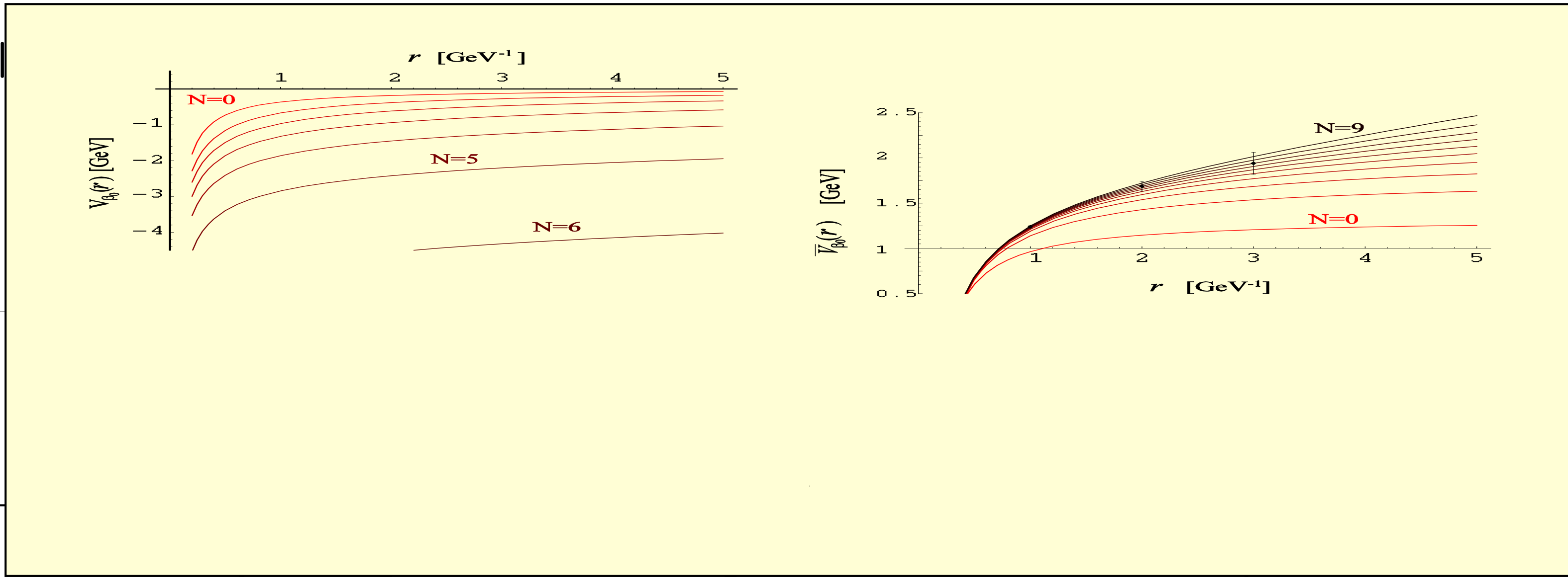
$E_{\text{tot}} = 2m_{\text{pole}} - E_{\text{bin}}$

Poorly convergent perturbative series

Pol

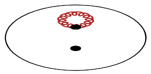
$\overline{m}$

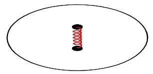
Q



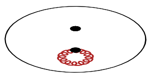
Computation of spectrum of Heavy Quarkonium ($Q\bar{Q}$ boundstate)

Using $\overline{\text{MS}}$ mass $2m_{\text{pole}} = 2\overline{m}\,(1 + c_1\,\alpha_S + c_2\,\alpha_S^2 + c_3\,\alpha_S^3 + \cdots)$





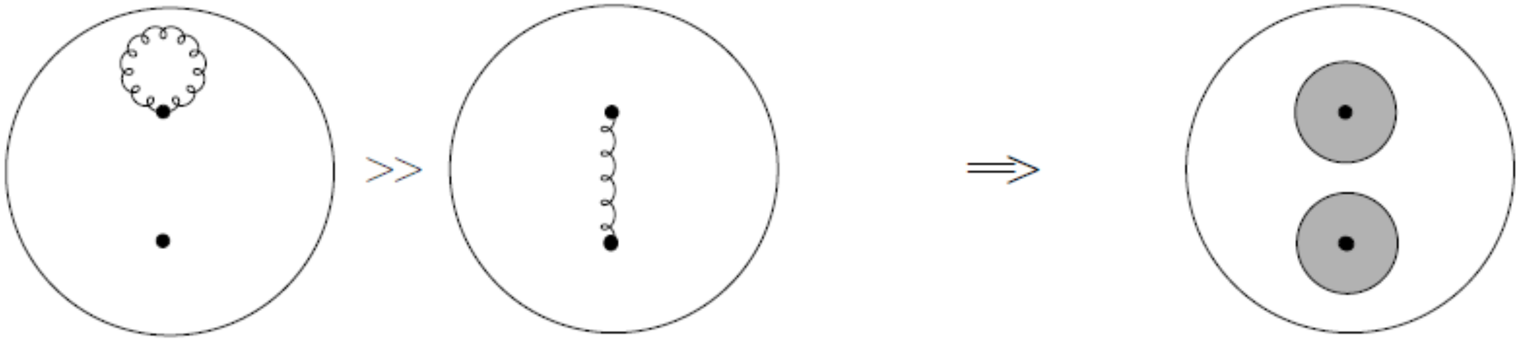
$E_{\text{tot}} =$ $+$ $+$ r

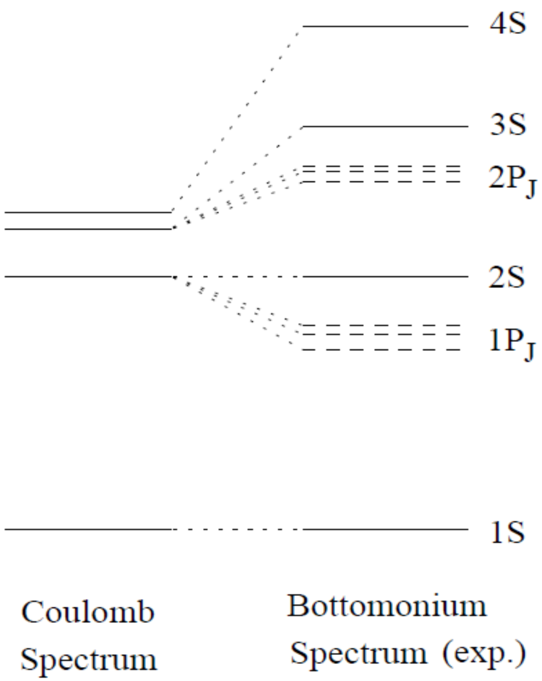


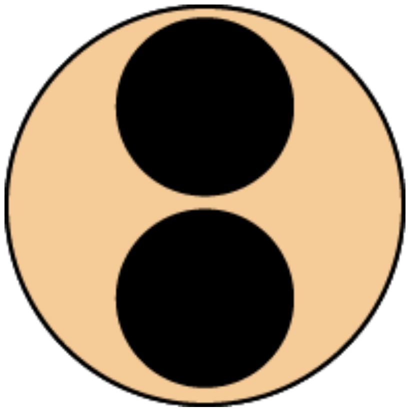
IR gluons $\lambda_g \gg r$ decouple ➡ much more convergent series

Rapid growth of masses of excited states originates from rapid growth of self-energies of Q & $\overline{Q}$ due to IR gluons.

Brambilla, Y.S., Vairo







3. After 1998: Theoretical development and Applications

EFT, higher-order calc.

Spectroscopy

Determinations of m_b, m_c (m_t)

Determination of α_s

Physical picture (gluon config.)

⋮

Application to quarkonium spectroscopy and determination of $\alpha_s,\ m_b,\ m_c$.

- Global level structure of bottomonium is reproduced. Brambilla, Y.S., Vairo; Recksiegel, Y.S

- Fine and hyperfine splittings of charmonium/bottomonium reproduced.
  In particular, mass of $\eta_c(2S)$ is predicted correctly. Recksiegel, Y.S.
  However, mass of $\eta_b(1S)$ disagrees:

$$M_{\Upsilon(1S)} - M_{\eta_b(1S)} = 44 \pm 11\ \mathrm{MeV} \quad \text{(prediction)}$$

$$67 \pm 5\ \mathrm{MeV} \quad \text{(exp.09)}$$

- Determination of bottom and charm quark $\overline{\mathrm{MS}}$ masses:

$$\overline{m}_b(\overline{m}_b) = 4190 \pm 30\ \mathrm{MeV}$$

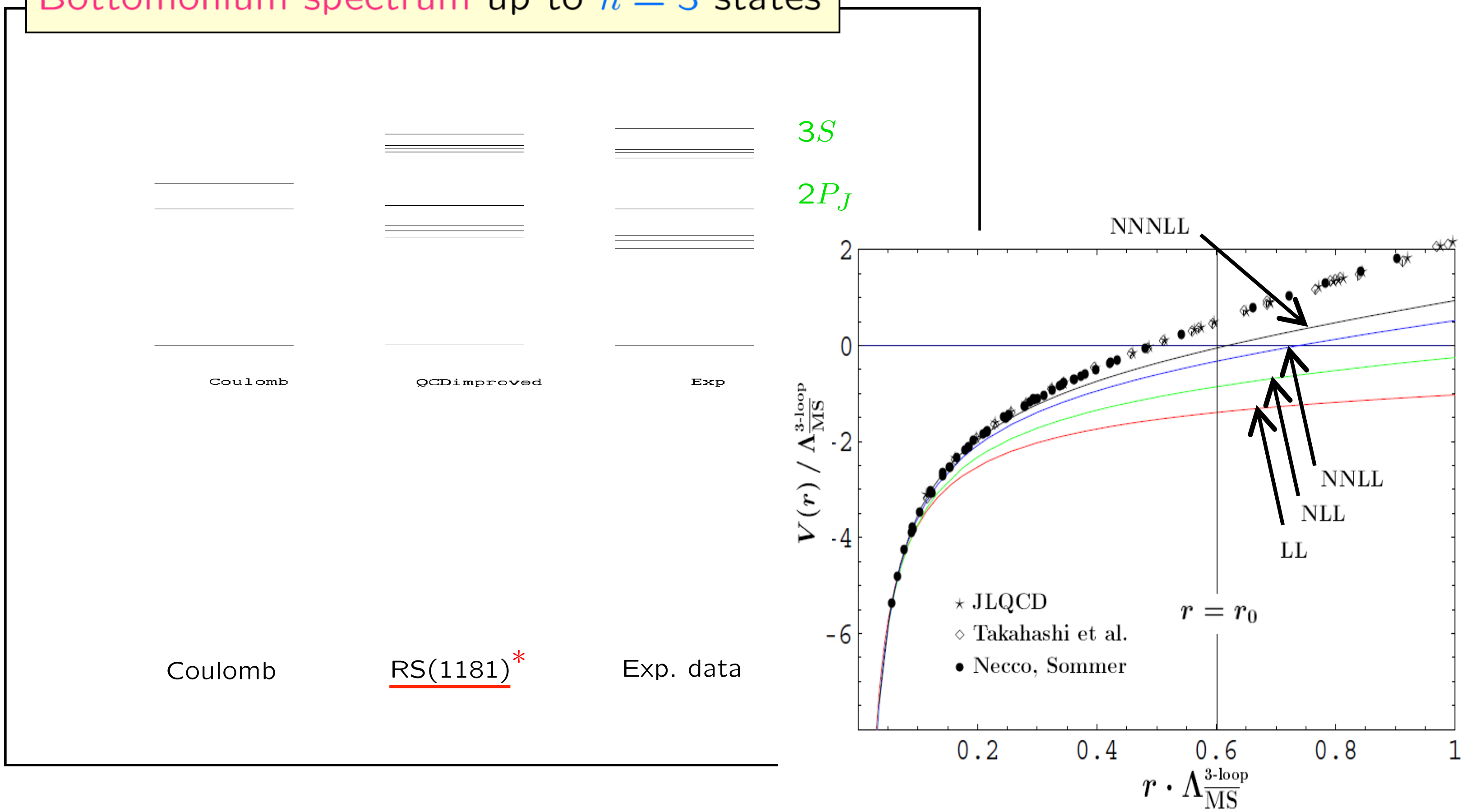
$$\overline{m}_c(\overline{m}_c) = 1243 \pm 100\ \mathrm{MeV}$$

Brambilla, Y.S., Vairo

- Relation between lattice α_s and $\overline{\mathrm{MS}}$ α_s is accurately measured (quenched approx.)

Y.S.

Bottomonium spectrum up to $n = 3$ states



* Pert. QCD prediction including full $\mathcal{O}(\alpha_S^4 m)$ corrections to individual energy levels, as well as full $\mathcal{O}(\alpha_S^5 m)$ corrections to fine structure [$\alpha_S(M_Z) = 0.1181$]. Recksiegel, Y.S.

Application to quarkonium spectroscopy and determination of $\alpha_s,\ m_b,\ m_c$.

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Motivation for precision determinations of heavy quark masses

- Bottom quark
  - Constraints on $b-\tau$ mass ratio of SU(5) GUT models
  - Input param. for $b-$physics (e.g. $\Gamma_b \propto m_b^5$) ⇨ LHC$_b$, Super-*B* factory

- Top quark
  - The only quark mass without $\overline{\rm MS}$ mass in current PDG data

$$m_t = 172.0 \pm 0.9 \pm 1.3 \text{ GeV}$$ ← What mass?

  - Test of MSSM prediction for Higgs mass at LHC

$$\delta_t M_H^{\rm MSSM} \propto m_t^4$$

cf. $\Delta M_H \sim 0.1\text{–}0.2 \text{ GeV}$ LHC

$\sim 0.05 \text{ GeV}$ ILC

More generally, tests of Yukawa couplings at LHC and beyond.

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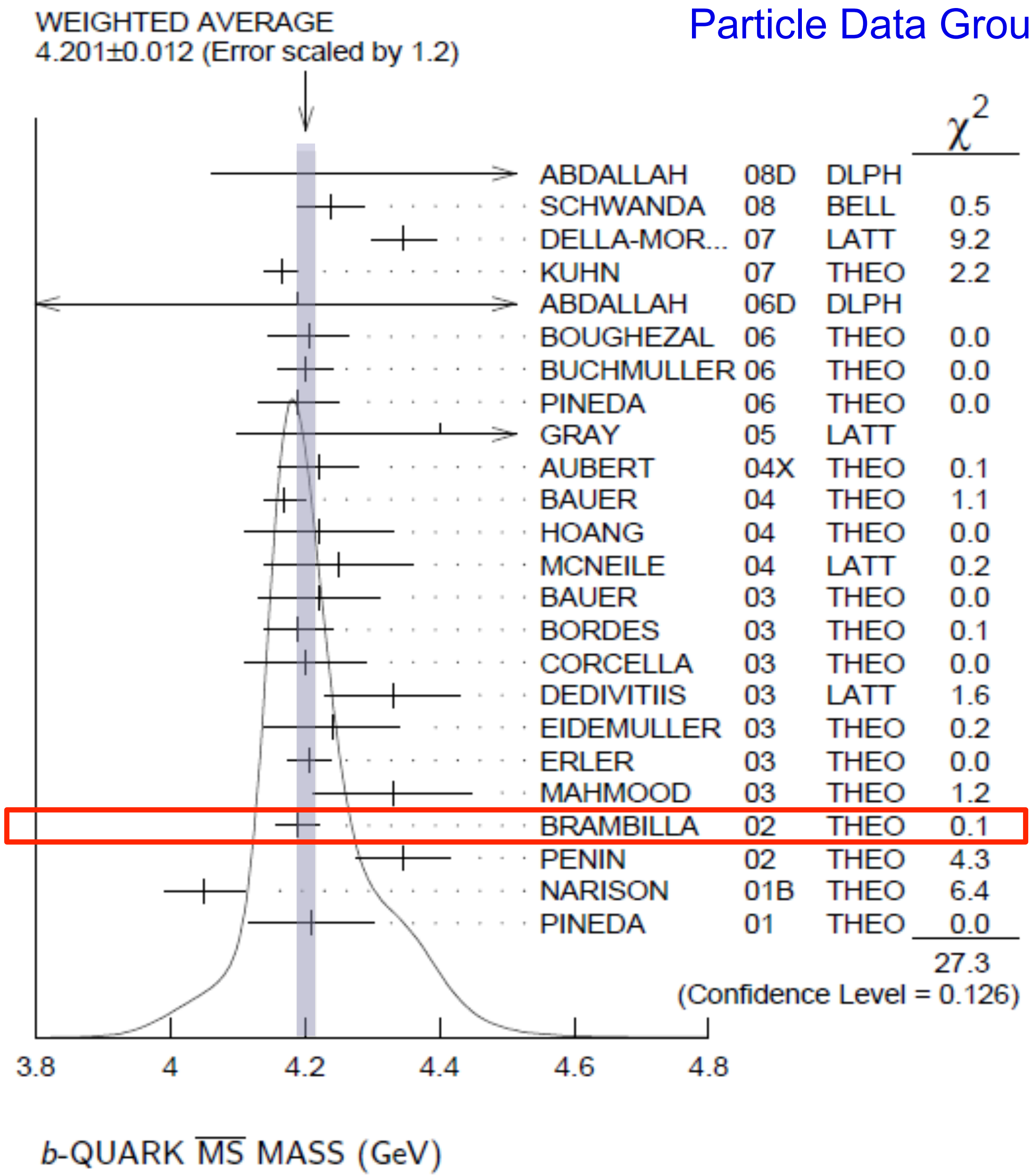
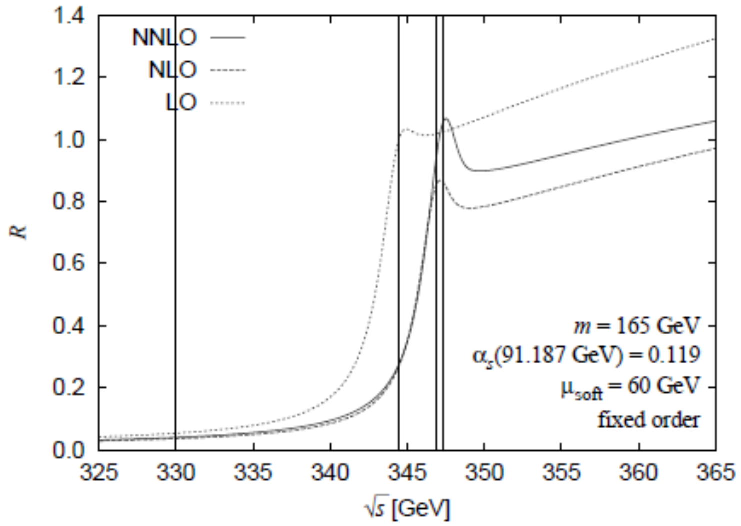
Particle Data Group 2010

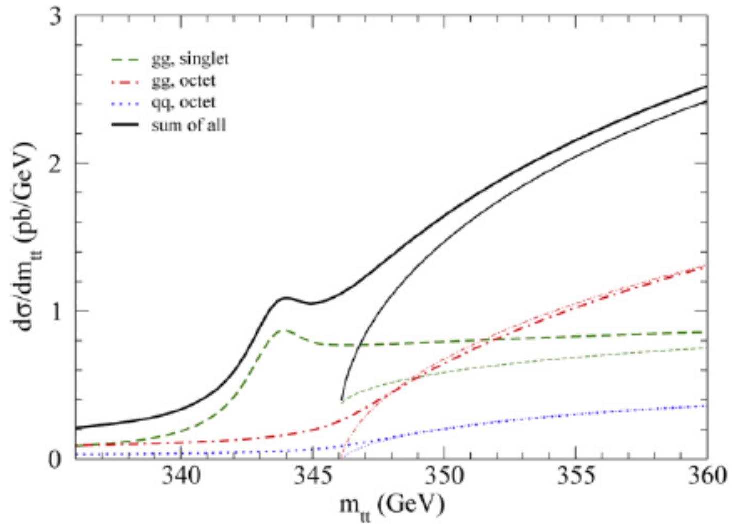


Prospects for precision determination of m_t from $M_{tt}(1S)$

Hoang, et al.

Hagiwara,Y.S.,Yokoya
Kiyo, et al.





$e^+e^- \to t\bar{t}$ in the threshold region
@ future Linear Collider

$p\,p \to t\bar{t}$ (threshold region)
@LHC

$\Delta\overline{m}_t \lesssim 100\ \mathrm{MeV}$

$\Delta\overline{m}_t$ significantly smaller than 1GeV?

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Brambilla,Tomo,Soto,Vairo

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   Physical picture (gluon config.)

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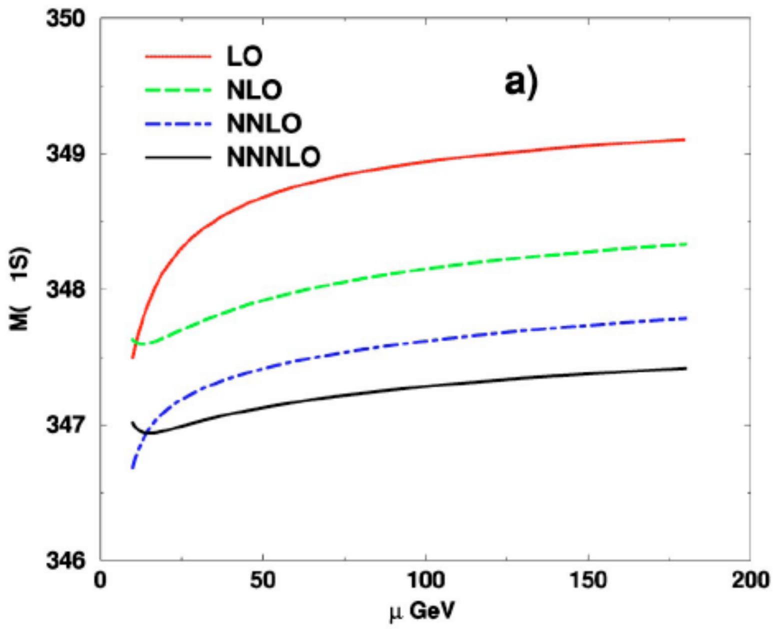
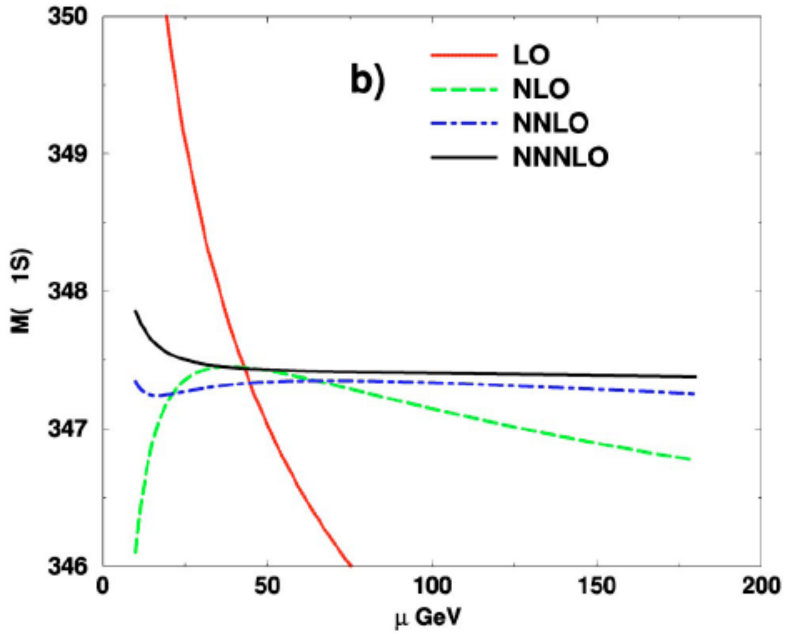
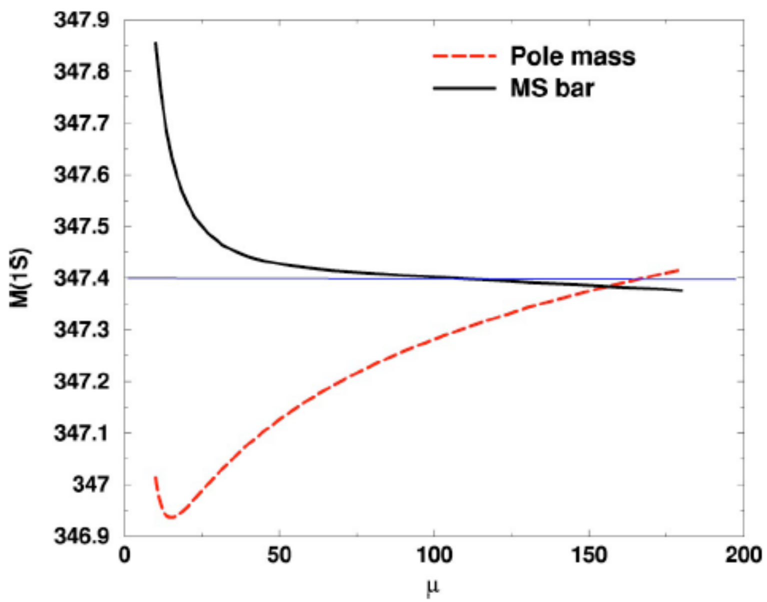
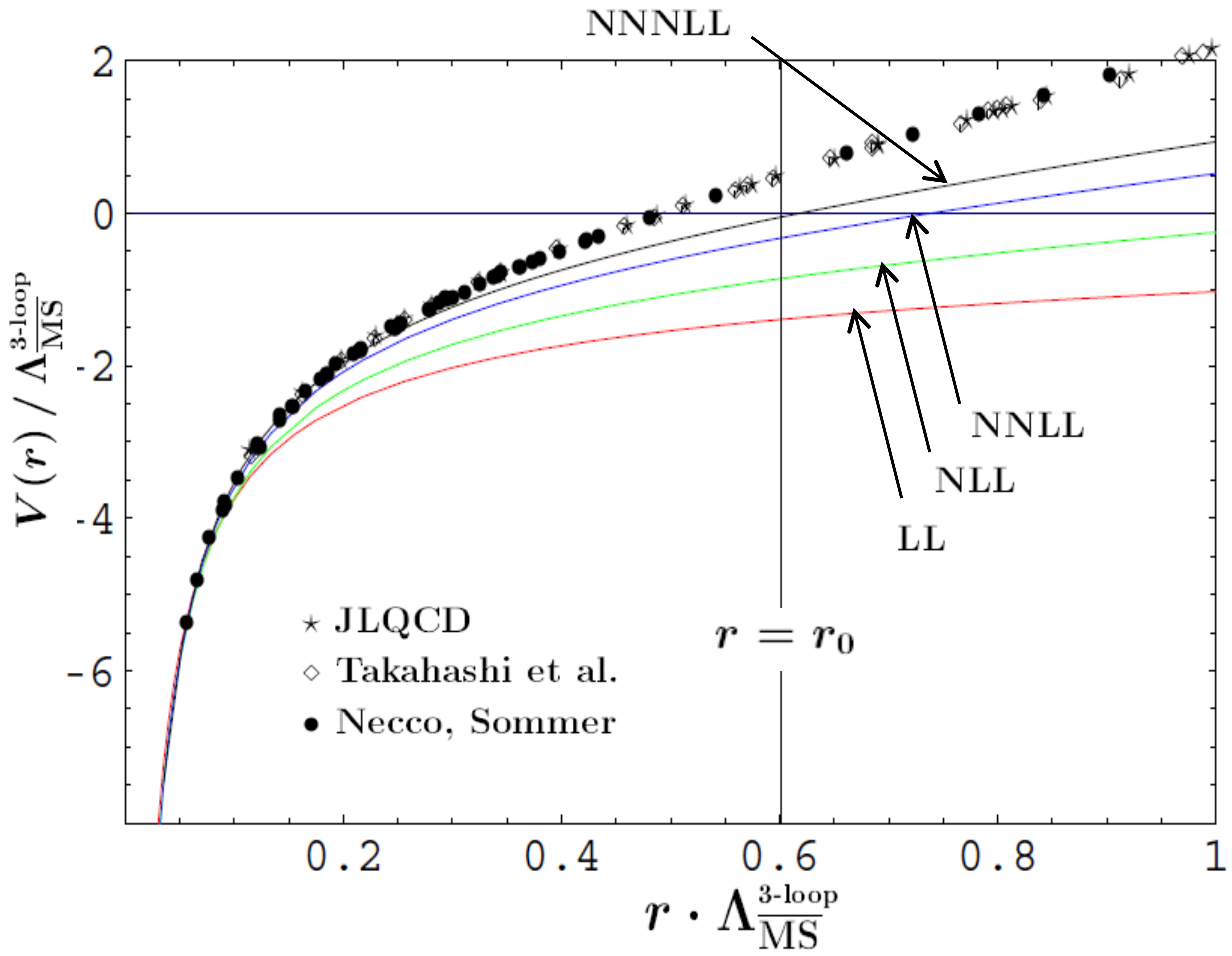
μ dependence and convergence of $M_{tt}(1S)$









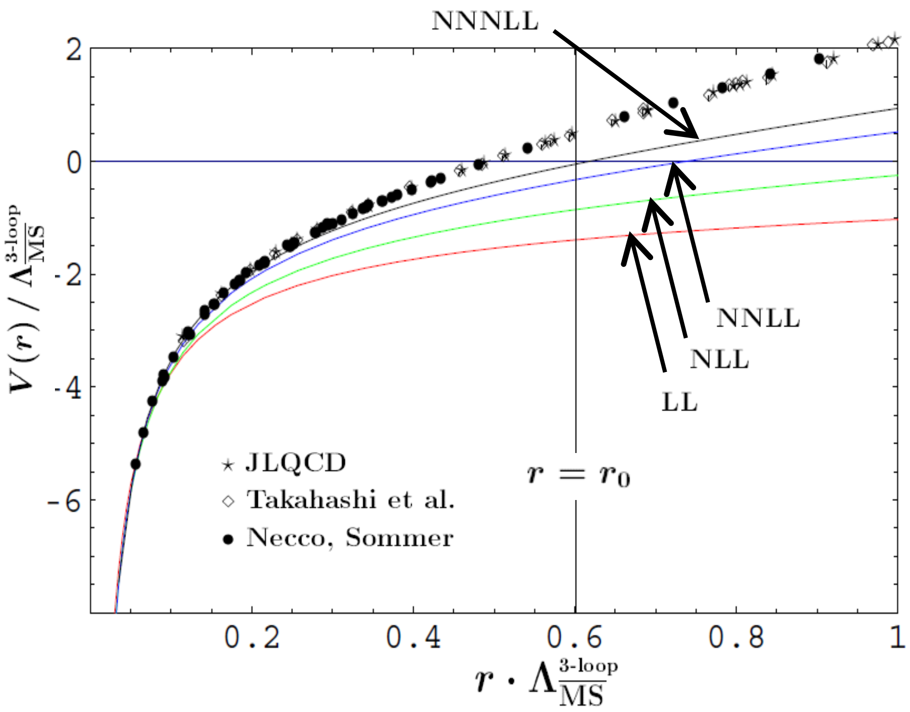
NNNLL
NNLL
NLL
LL
2
0
-2
-4
-6
0.2
0.4
0.6
0.8
1
$V(r)\,/\,\Lambda^{\text{3-loop}}_{\overline{\text{MS}}}$
$r \cdot \Lambda^{\text{3-loop}}_{\overline{\text{MS}}}$
$r = r_0$
JLQCD
Takahashi et al.
Necco, Sommer

Interquark force

$$F(r) \equiv -\frac{d}{dr}V_{\rm QCD}(r) \equiv -C_F\frac{\alpha_F(1/r)}{r^2}.$$

Renormalization-group equation: $\mu^2\frac{d}{d\mu^2}\alpha_F(\mu) = \beta_F(\alpha_F)$

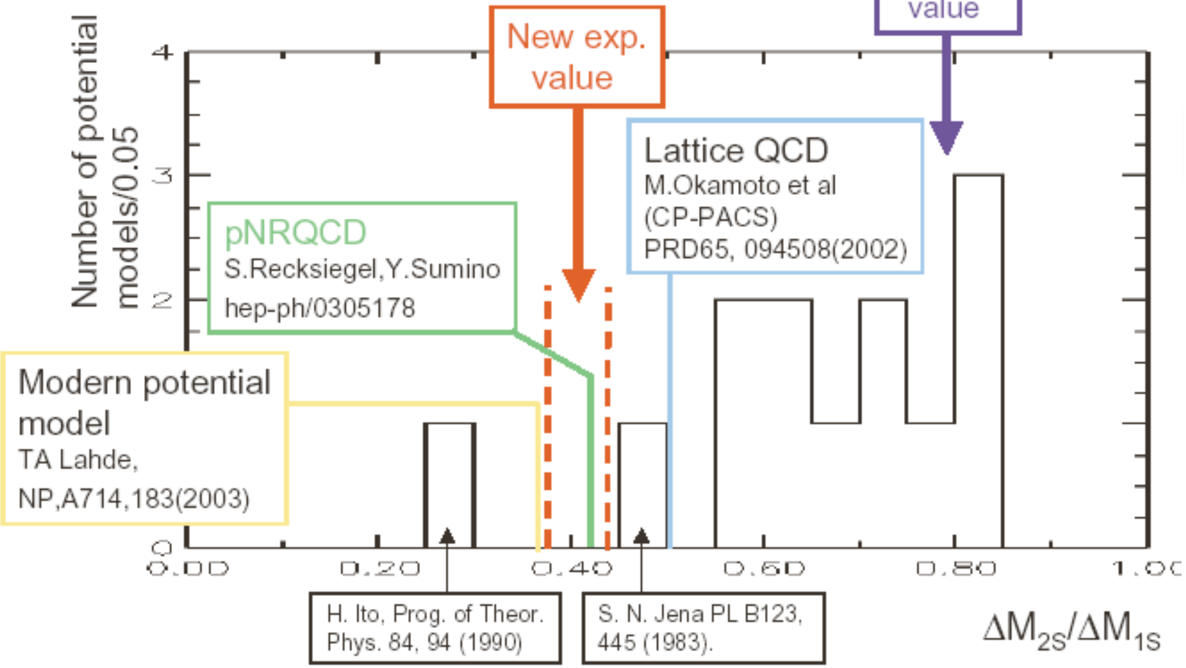
$\Longrightarrow$ Due to the running of $\alpha_F(1/r)$, the attractive force $|F(r)|$ increases at large r.

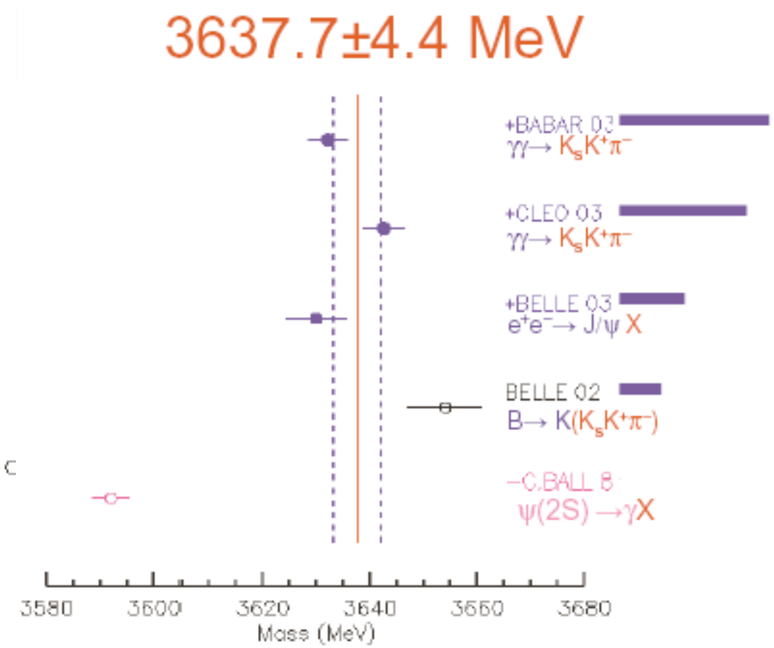


Slides from Skwarnicki's plenary talk at Lepton-Photon 2003

Predictions for hyperfine splitting ratio

- For 20 years theorists were exposed to the experimental value of ΔM_{2S}=M(ψ(2S))-M(η_c(2S)) which was wrong by a factor of 2
- Predictions for $\Delta M_{2S}/\Delta M_{1S}$





CL=14% scale factor=1.3

New measurements of mass are consistent

$$V_{\rm QCD}(r) = \underbrace{V_{\rm pert}(r;\mu_f)}_{\text{Wilson coeff.}} + \underbrace{\delta E(r;\mu_f)}_{\text{non-pert. contr.}}$$

$\delta E \to 0$ as $r \to 0$ $\sim \langle G^a_{\mu\nu}(0)^2 \rangle \, r^3$

Comparison of lattice $V_{\rm QCD}(r)$ and $V_{\rm pert.}(r;\mu_f)$ at short distances

$$V_{\rm latt}(r) - V_{\rm pert}(r;\mu_f) = \delta E(r;\mu_f)$$

$\frac{1}{r \log r}$ ↔ $\frac{1}{r \log r}$ (cancel) $\qquad r^3$

Sensitive to relation between r_0 and $\Lambda_{\overline{MS}}$ or $\alpha_S(M_Z)$

$r_0 \Lambda^{\text{3-loop}}_{\overline{\rm MS}} = 0.574 \pm 0.042$ Y.S.

c.f. Schrödinger functional method:

0.602 ± 0.048

Capitani, Lüscher, Sommer, Wittig

0.586 ± 0.048

Necco, Sommer

Including 3-loop QCD pot.

$r_0 \Lambda^{\text{3-loop}}_{\overline{\rm MS}} = 0.622^{+0.019}_{-0.015}$

Brambilla, Tomo, Soto, Vairo