Supersoft Goldstone bosons in vortex lattices

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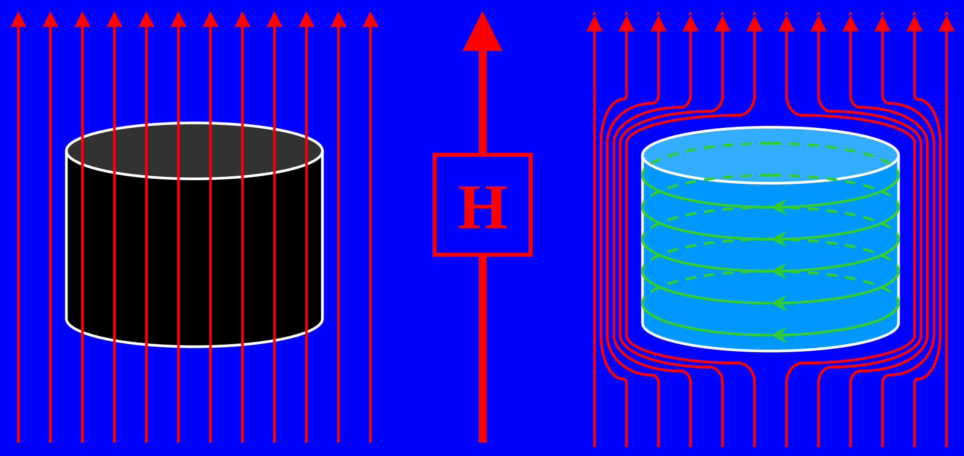
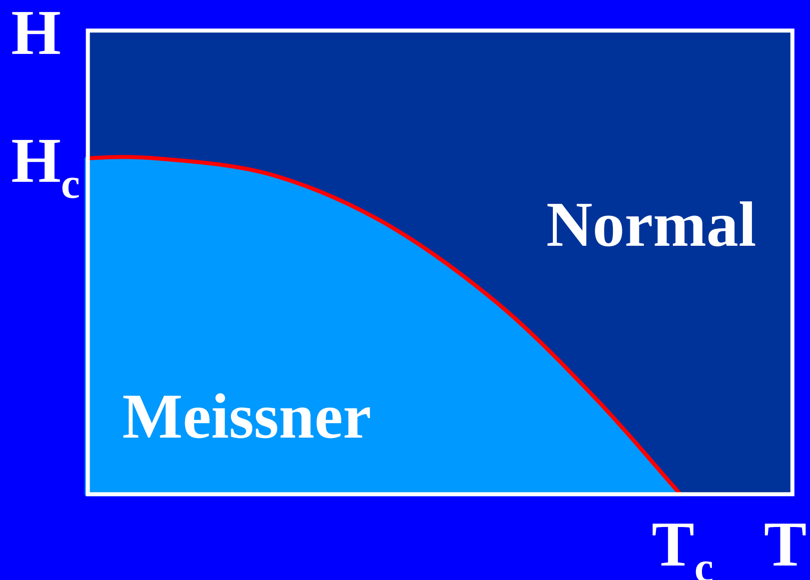
Superconductor under magnetic field

Magnetic flux is expelled from a Type I superconductor





However the situation in the Type II superconductors (including all the high Tc) is much more complex and interesting: magnetic field penetrated as an array of vortices.

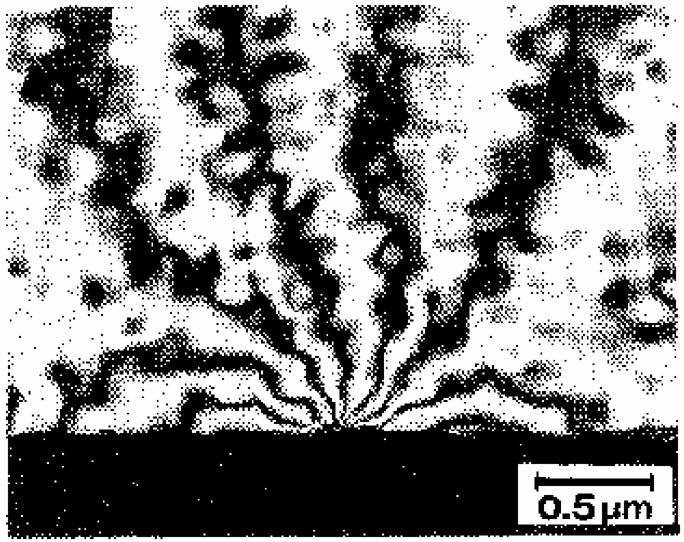
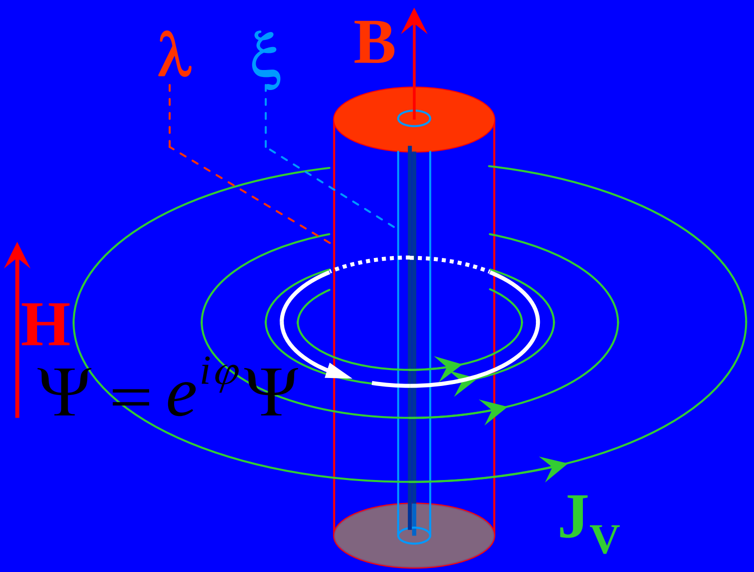


FIG. 2. 16-times phase-amplified interference micrograph of a single fluxon (film thickness =0.2 μm and sample temperature =4.5 K).

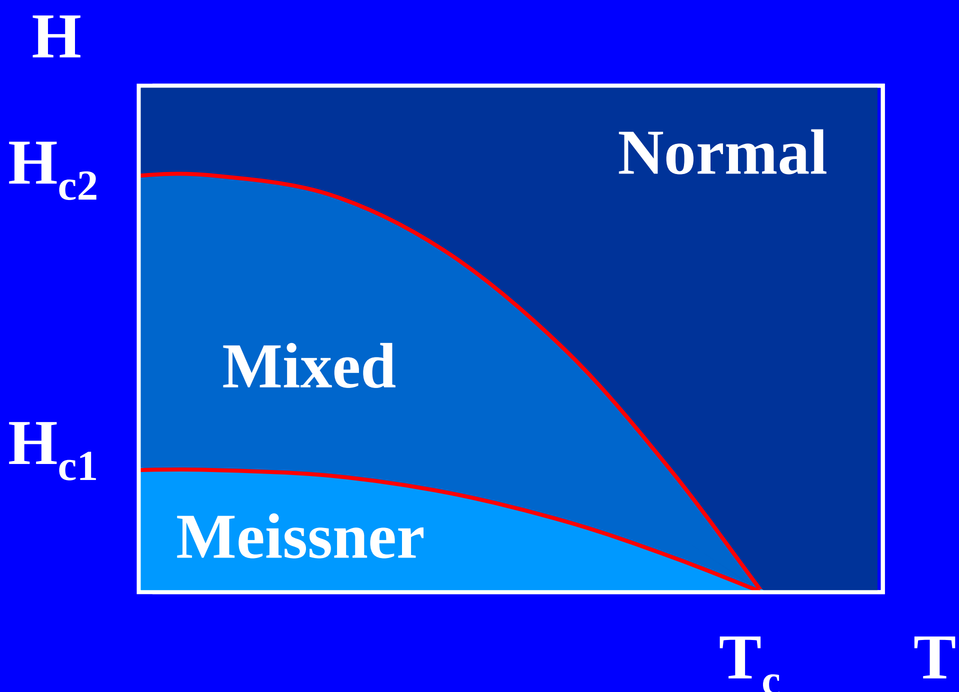
Electron tomography

Tonomura's group

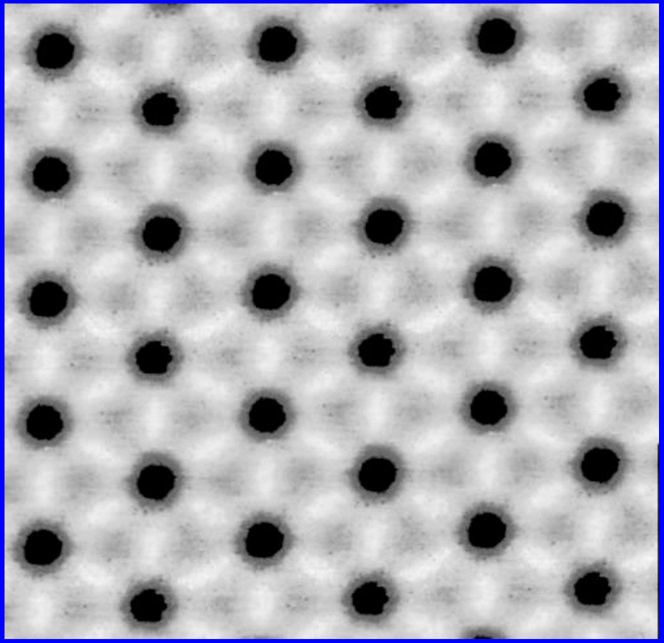
PRL66,2519 (1993)

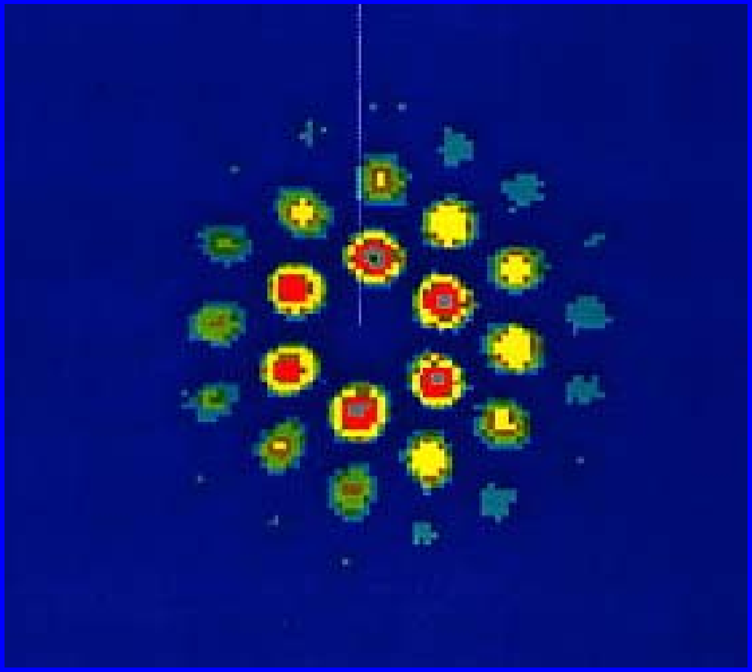


Phase diagram of the Type II superconductor



Vortex line repel each other forming highly ordered structures like flux line lattice (as seen by STM and neutron scattering)





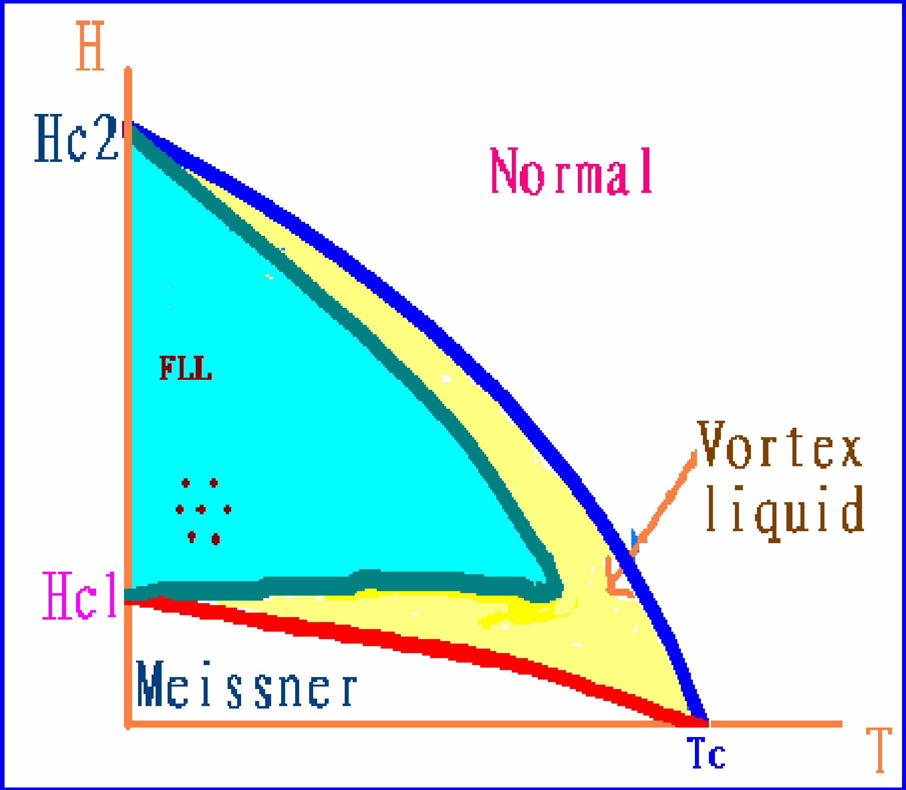
Pan et al
(2002)

S.R.Park et al
(2000)

Increased role of thermal fluctuations in high Tc

1. Ginzburg number is much larger

$$Gi \equiv \frac{1}{2}\left(\frac{Tc}{Hc^2\xi^3(0)}\right)^2$$



Metals:: $Gi \approx 10^{-6}$

High Tc: $Gi \approx .01 \rightarrow .5$

2. Magnetic field effectively reduces dimensionality of fluctuations from D to D-2

Plan

1. Ginzburg –Landau theory of vortex matter.
2. ”Supersoft” Goldstone modes in FLL
3. IR problems with thermal fluctuations of vortex lattice and their solution.
4. Quasi long range order in 2D XY model.
5. Why restoration of symmetry does not invalidates perturbative expansion starting with (quasi) ordered state?
6. Results for magnetization, specific heat, structure function, melting line....

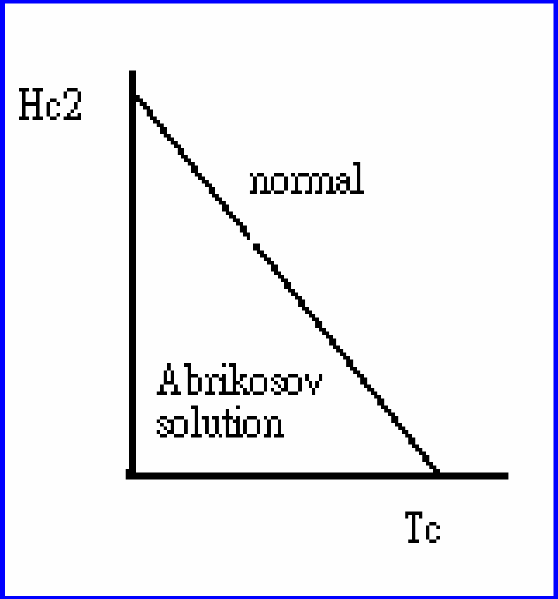
Ginzburg – Landau energy and the Abrikosov solution

$$F = \int d^3x \frac{h^2}{2m} \left| (\nabla - \frac{2ie}{hc}\mathbf{A})\psi \right|^2 + a(T)\left|\psi\right|^2 + \frac{b^{'}}{2}\left|\psi\right|^4 + \frac{(B-H)^2}{8\pi}$$

Near Hc2 neglecting fluctuations Abrikosov found a hexagonal lattice solution

$$\varphi(x,y) \propto \sum_{e=-\infty}^{\infty} \exp i[\frac{\pi}{2}(l-1)l + \frac{2\pi}{a} lx]$$

$$\exp[-\frac{1}{2}(y - \frac{2\pi l}{a})^2]$$



Second order transition

Supersoft phonons in vortex solid

There are two major modes in expansion around Abrikosov solution:

$$\psi(x,y,z) = \varphi(x,y) + \int_{k\in B.Z,k_z} d_k \varphi_k(x,y) e^{ik_z z}(O_k + iA_k)$$

Diagonalizing quadratic part of free energy one obtains:

$$F = F_{mf} + \int_k e_O(k) O_k^* O_k + e_A(k) A_k^* A_k + AAA + \ldots$$

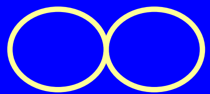
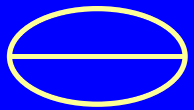
$$e_A(k) = a_T\left(1 - 2\frac{\beta_k}{\beta_A} + \frac{\gamma_k}{\beta_A}\right) + k_z^2 \approx 0.12\,|a_T|\,k^4 + k_z^2$$

Where the only parameter of the LLL model is scaled temperature:

$$a_T \equiv \frac{T - T_c(H)}{(TH)^{2/3}}$$

IR divergencies

Naively higher order contributions to energy are hopelessly divergent:

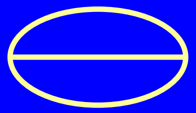
 $\to \log^2 L$  $\to L^4$

Since experimentally the corrections are small one can speculate that it is not analytic. The perturbation theory was abandoned.
Is this correct?

A question of principle

Is there a thermodynamic solid state for T>0 or experimentally observed vortex lattice is just a finite size effect or a quasi-long range solid?

The first surprise

The most divergent two loop diagram, the "setting sun" 

Is in fact convergent! Careful evaluation shows that vertices are also supersoft and all the divergences cancel.

B.R. ***PRB60, 4268 (1999)***

Other two loop diagrams are IR divergent, but only logarithmically divergent. Moreover the divergences look similar to "spurious divergences" in critical phenomena of models with broken continuous symmetry . It turns out indeed that all the divergences cancel. It is more instructive to consider a simple model.

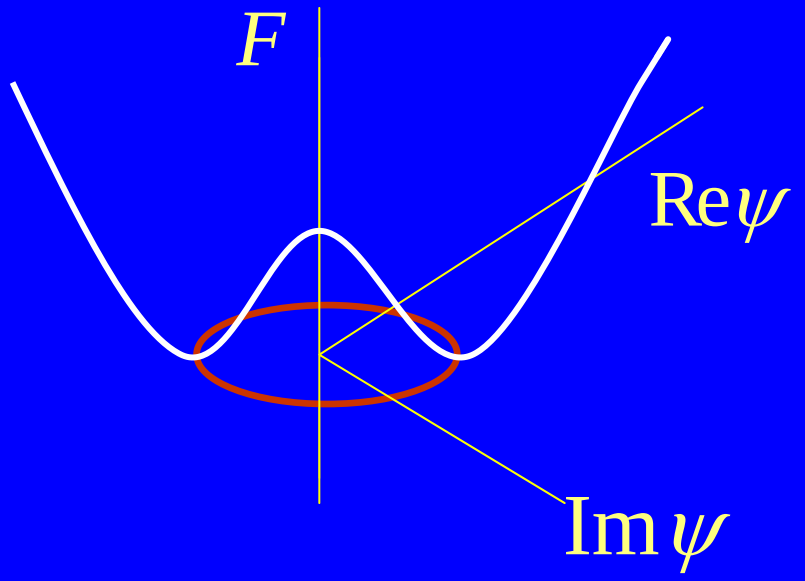
I will show in some detail what happens in D=2 O(2) symmetric model (which we completely understand) and then return to GL.

A simple U(1) symmetric model without magnetic field

$$F[\psi] = \frac{1}{\omega}\int d^D x[\psi^*(-\nabla^2)\psi - a|\psi|^2 + \frac{1}{2}|\psi|^4]$$

For positive a the energy is minimized by

$$\psi = \sqrt{a} \equiv v$$



To organize the pert. Theory around the ordered vacuum of "shifts" the field:

$$\psi(x) = v + [O(x) + iA(x)]$$

The energy in terms of two real fields O and A becomes:

$$F = \frac{1}{\omega}[-av^2 + \frac{1}{2}v^4] + \int [O(-\nabla^2 + e_O)O + A(-\nabla^2 + e_A)A]$$

where

$$e_O = -a + 3v^2 = 2v^2 > 0$$

$$e_A = -a + v^2 = 0$$ ⟹ **Goldstone mode**

$$e_A(k) = k^2$$

Dispersion relation is that of the acoustic phonons: $\omega_A \equiv \sqrt{e_A(k)} = |k|$

Correlator of the field

The massless propagator of the A mode is:

$$\left\langle A(\vec{x})A(0)\right\rangle \approx \int d^D k \, \frac{1}{k^2} e^{i\vec{k}\cdot\vec{x}} =$$

$$\begin{cases} \dfrac{1}{|x|} \xrightarrow[|x|\to\infty]{} 0 & \text{in 3D} \\ \log\dfrac{L}{|x|} \xrightarrow[|x|\to L]{} 0 & \text{in 2D} \end{cases}$$

The field A itself is not the order parameter. The order parameter $e^{i\Theta(x)}$ transforms linearly under the symmetry transformation. The phase of ψ is:

$$\Theta(x) = arctg(A/O)\;;\;\; A/v$$

The order parameter correlator

$$\left\langle e^{i\Theta(x)} e^{i\Theta(0)} \right\rangle \approx e^{-\langle \Theta(x)\Theta(0)\rangle} \approx e^{-\langle A(x)A(0)\rangle} =$$

$$\begin{cases} e^{\frac{1}{|x|}} \to 1 & \text{in 3D} \\ e^{-\alpha \log \frac{L}{|x|}} = \dfrac{1}{|x|^{\alpha}} \to 0 & \text{in 2D} \end{cases}$$

The 2D correlator in the "ordered" phase decays albeit slowly (as a power rather than exponential in the disordered phase $\left\langle e^{i\Theta(x)} e^{i\Theta(0)} \right\rangle \approx e^{-\frac{|x|}{\xi}}$). Fluctuations due to Goldstone bosons "destroy" perfect order. Such a phase is called the quasi – long range order phase (or the Berezinski-Kosterlitz-Thouless phase).

We started from the assumption of nonzero VEV. It seems that fluctuations destroy this assumption!

Destructions caused by IR divergences and MWC theorem.

The energy to the one loop level is:

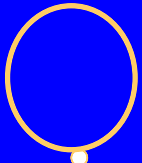
$$F = \frac{1}{\omega}[-av^2 + \frac{1}{2}v^4] + \frac{1}{2}[Tr\log(-\nabla^2 + e_O) + Tr\log(-\nabla^2 + e_A)]$$

The corrected value of v is found by minimizing it perturbatively in “loops”:

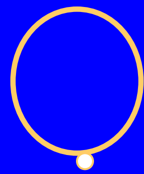
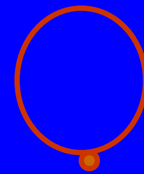
$$v^2 = a^2 + \omega \Delta v^2$$

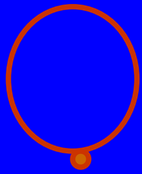
Noting that

$$\frac{\partial}{\partial v^2} Tr\log(-\nabla^2 + e) = \int_k \frac{1}{k^2 + e} \equiv$$



one obtains a logarithmically divergent correction to VEV:

$$\Delta v^2 = -3$$  $$-$$ 

 $$: \int d^2k \, \frac{1}{k^2} \approx \log L$$

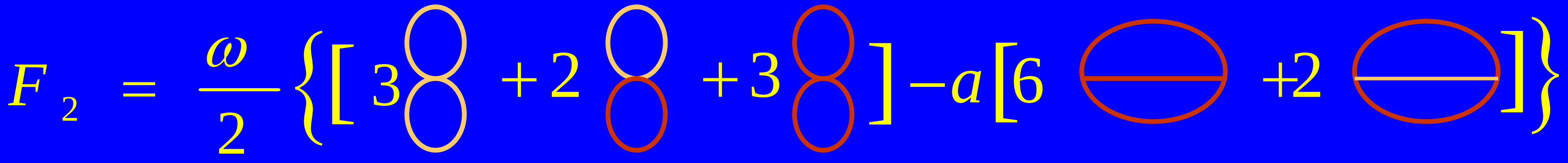
To higher orders the logs can be resummed:

$$v^2 = a\,(1 - \omega \log L + \omega^2 \log^2 L + \ldots) = a\, e^{-\omega \log L} \approx \frac{1}{L^{\omega}} \to 0$$

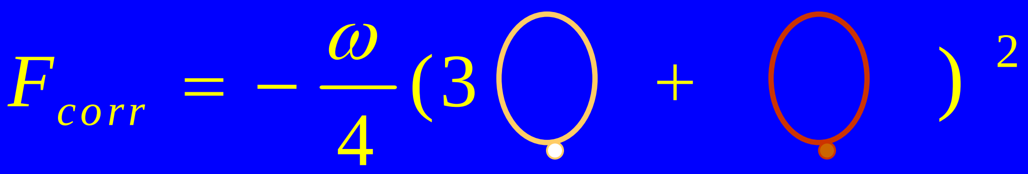
The VEV decays – do not diverges, indicating that order is "slowly" restored. This is Hohenberg-Mermin-Wagner-Coleman theorem: in 2D continuous symmetry is not broken. More importantly this does not mean the perturbation theory is useless.

O(2) invariant quantities

For such quantities the “collective coordinates” method simplifies into perturbation theory around “broken” vacuum. All the IR divergencies cancel. Let us see this for the energy to two loops order (Jevicki, **PLB, 1987**)



There are also there is correction due to change in v:



The leading $\log^2 L$ IR divergences are easy to evaluate:

$$\frac{3}{2}\log^2 L-\frac{a}{2}\cdot\frac{2}{a}\log^2 L-\frac{1}{4}\log^2 L=0$$

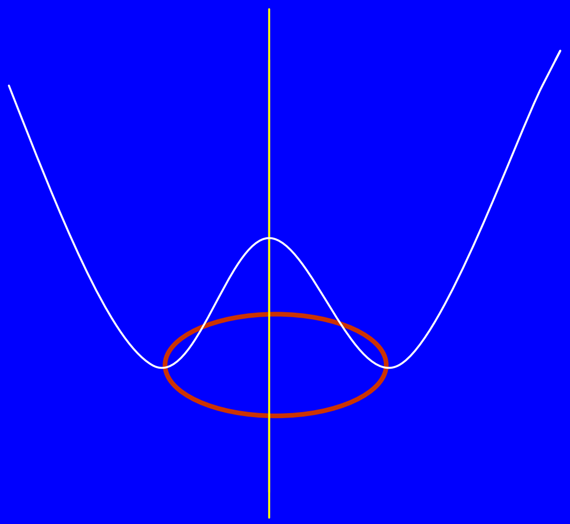
Subleading $\log L$ divergences also cancel although it is much less obvious. Cancellations occur to all orders (F.David, **CMP, 1990) in loop expansion.**

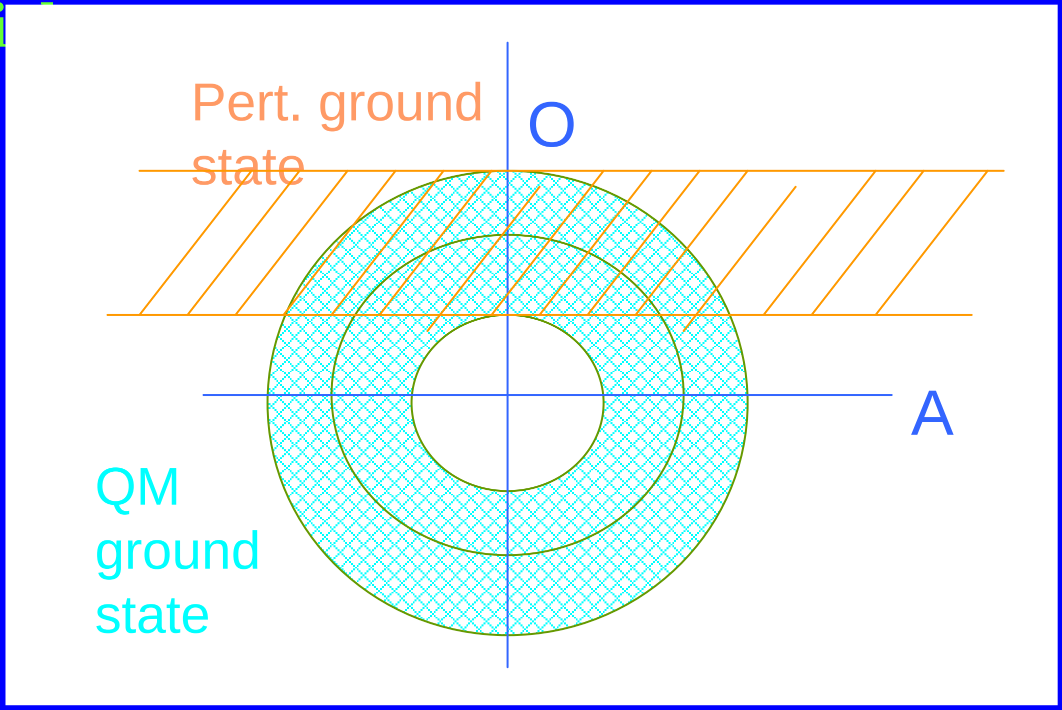
What is the mechanism behind this cancellation of "spurious divergences?

It is hard to say generally, but at least in extreme case of 1D the answer is clear.

Physics below lower critical dimension

For D=1 the model is equivalent to QM of particle on a plane with the “Mexican hat” potenti





Ground state is O(2) invariant but is very far from the origin (0,0): pert. ground state is bad, but theory “corrects” it using IR divergent matrix elements **Kao,B.R.,Lee PRB61, 12652 (2000)**

Back to Ginzburg – Landau theory

Analogous events take place in GL up to two loops. Since the correction to the order parameter VEV

$$v^2 = -\frac{a_T}{\beta_A} + \frac{1}{2\pi\sqrt{2}} \log\frac{\pi}{L}$$

is divergent, which means after resummation that it slowly vanishes. Therefore translation noninvariant solid is "destroyed" by thermal fluctuations and becomes a "quasi – solid" with quasi long rage order.

However the perturbation theory for translation invariant quantities remains valid: no nonanalyticity. B.R. **PRB60, 4268 (1999)**
I believe cancellations occur beyond two loops, but mathematical proof is not available up to now.

Free energy

For energy, which is invariant under translation, we get to the two loop order:

$$f_{sol} = -\frac{a_T^2}{2\beta_A} + 2.848|a_T|^{1/2} + \frac{2.4}{a_T}$$

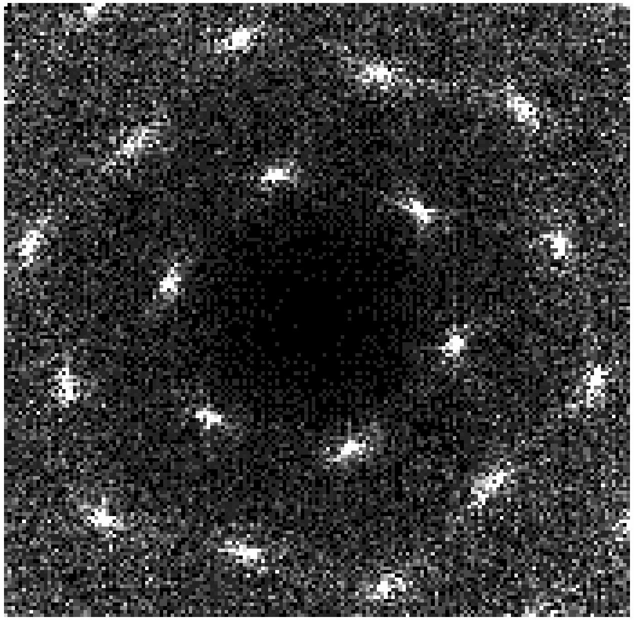
B.R. ***PRB60,4268 (1999),***
D.P. Li and B.R. ***PRB65,024514(2001)***

Even at melt ($a_T^m = -9.5$ **) the precision is 0.1%. From this one calculates magnetization, specific heat. Structure function**

$$S(\vec{q},0) = \left\langle \int_{\vec{r},\vec{r}'} |\psi(r)|^2 |\psi(r')|^2 e^{i\vec{q}(\vec{r}-\vec{r}')} \right\rangle$$

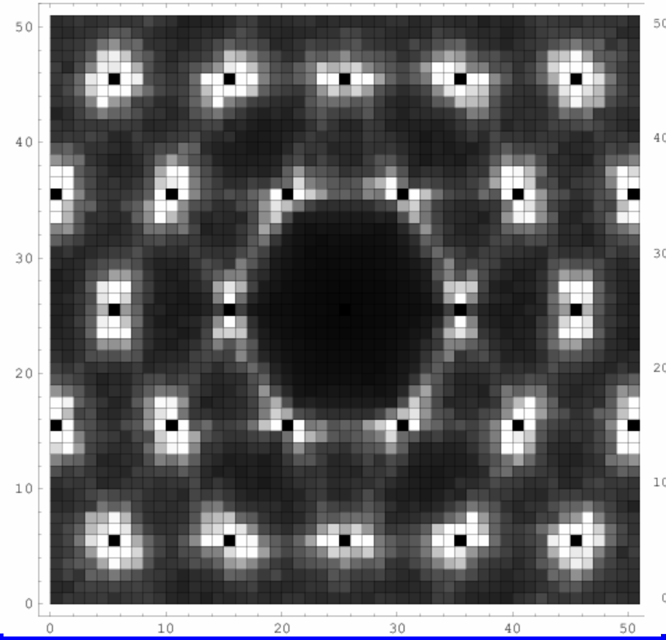
and other physical quantities are also calculated perturbatively.

Bragg peaks



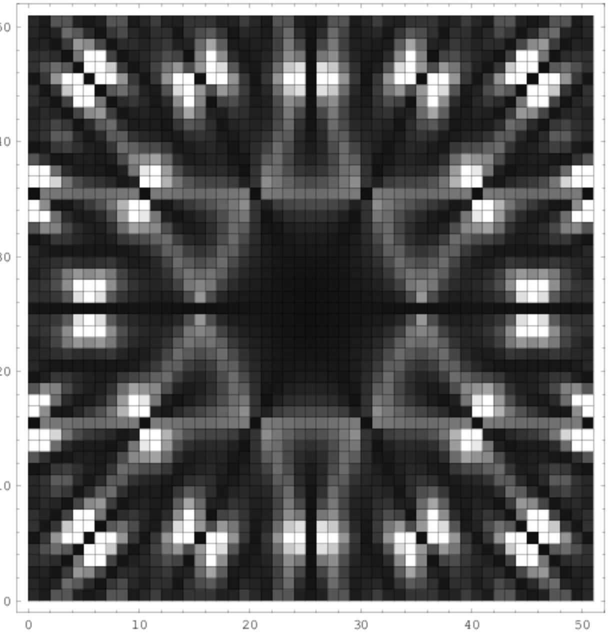
Kim et.al.

PRB60, R12589 (1999)



Sasik, Stroud

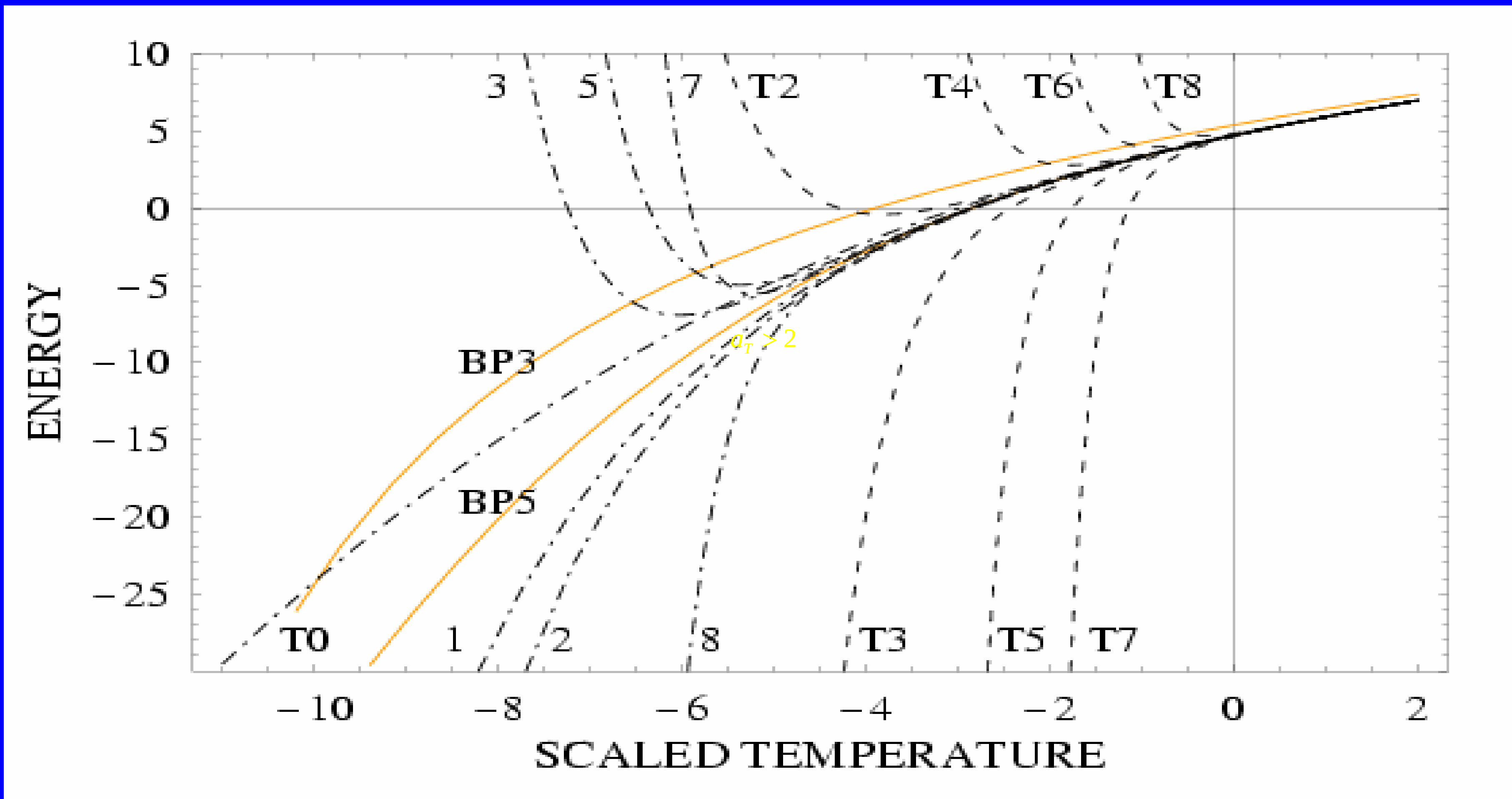
PRL75,2582 (1995)



Li, B.R.

PRB60,9704 (1999)

Theory of vortex liquid: a tougher challenge



Standard high temperature pert. theory works only for $a_T > -2$

Recent improvements

1. **We constructed the Optimized gaussian series which are convergent rather than asymptotic.**

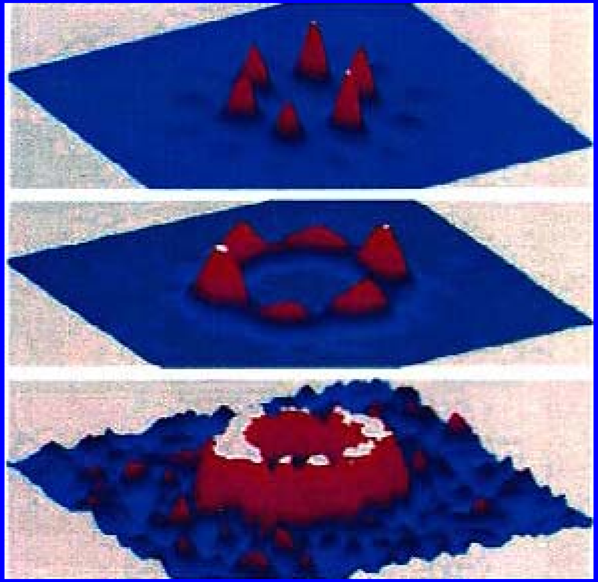
Li, B.R. ***PRL86,3618 (2001)***

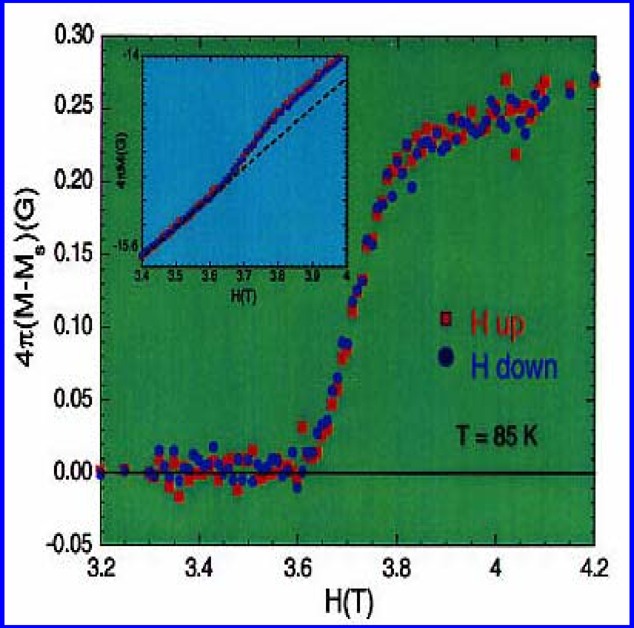
Radius of convergence was found to be $a_T = -4.5$ **still a bit short.**

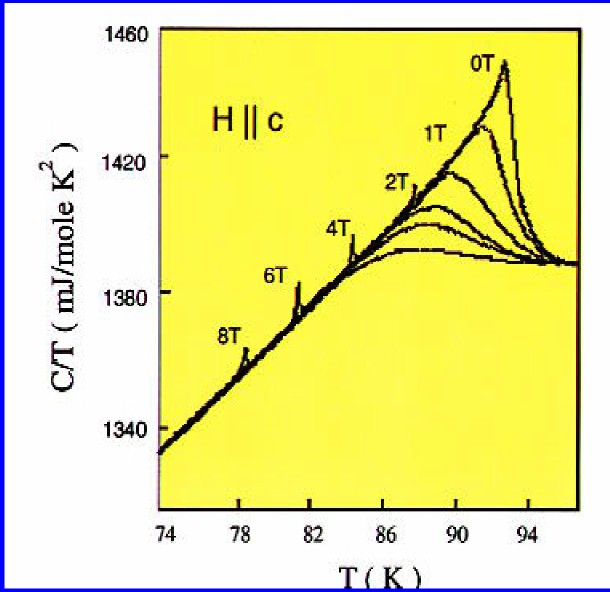
2. **However it allowed us to check the validity of Borel-Pade method which provided a convergent scheme everywhere down to T=0.**

Precision was finally good enough (0.1%) to study melting quantitatively

First order melting of the Abrikosov lattice into a “vortex liquid”.





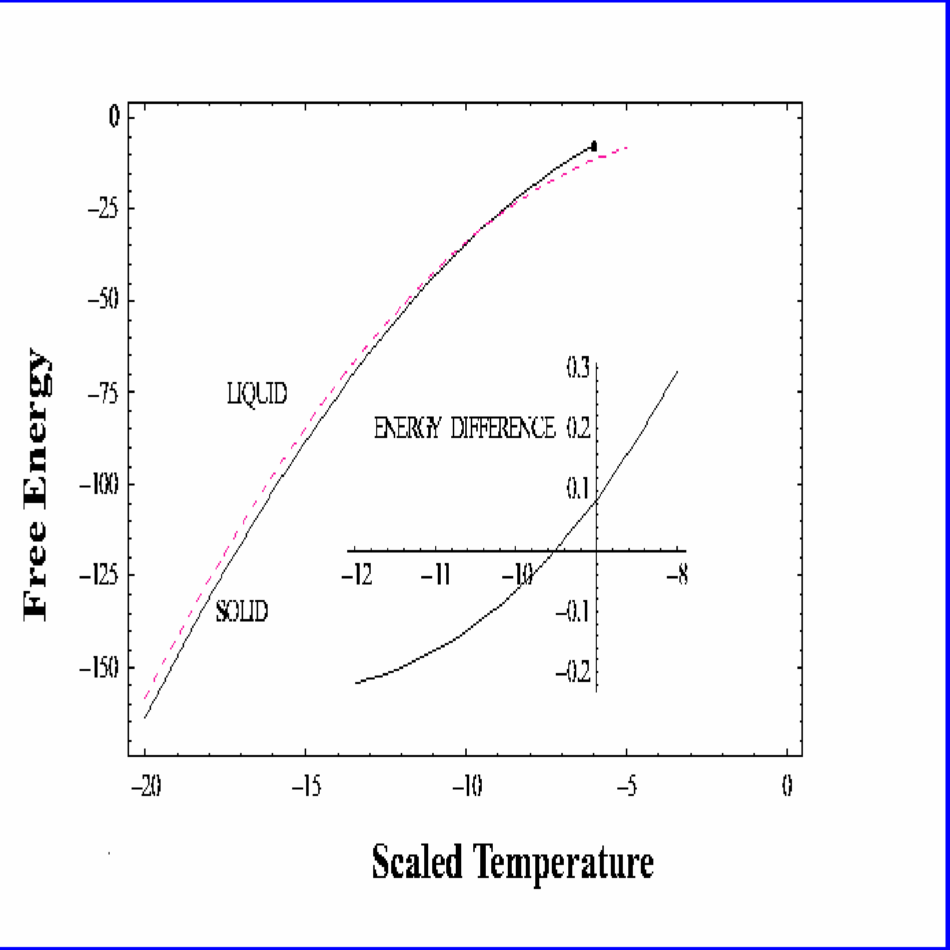


Gammel et al
PRL80,833 (1998)

Welp et al
PRL76,4809 (1996)

Schilling et al
Nature 382,791 (1996)

Melting line and discontinuities at melting



The melting point is:

$$a_T^m = -9.5$$

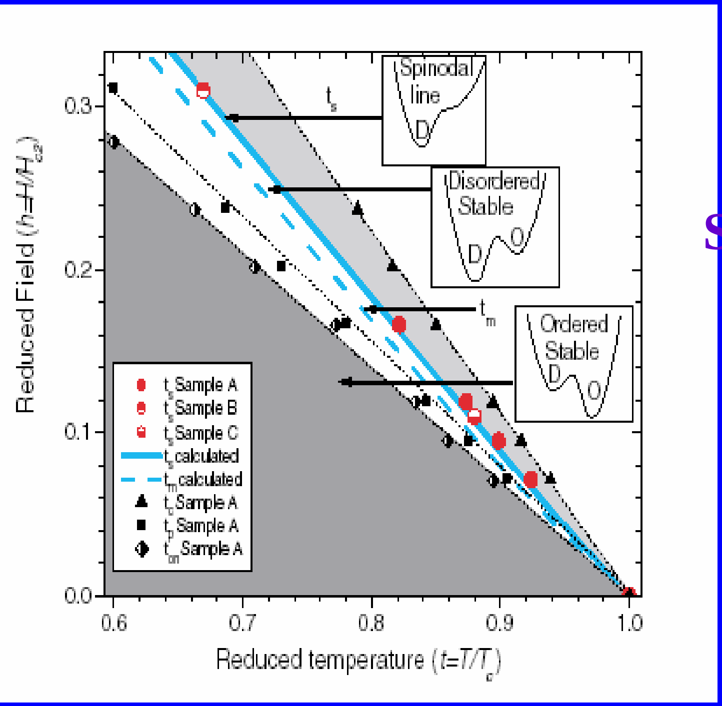
The magnetization jump:

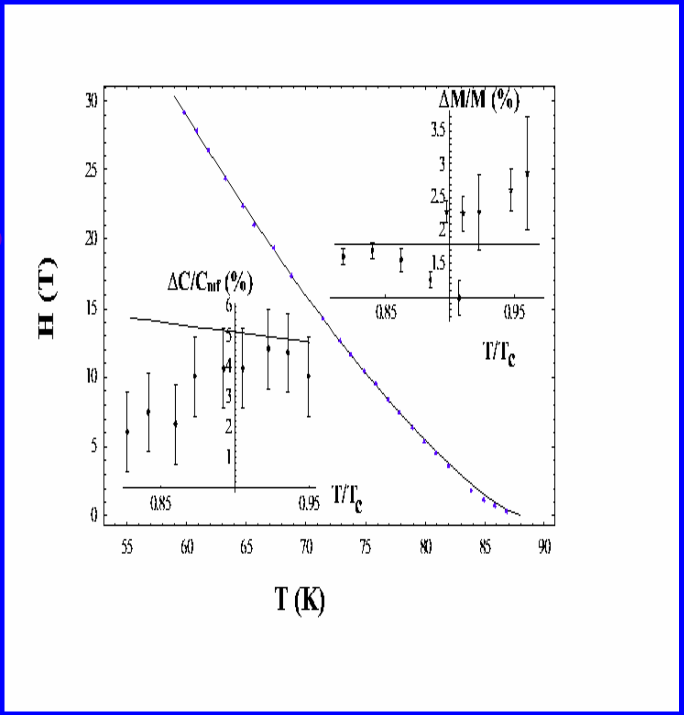
$$\frac{\Delta M}{M_{sol}} = 1.8\ \%$$

Specific heat jump:

$$\frac{\Delta C}{C_{mf}} = 0.75\left(\frac{2-2b+t}{t}\right)^2\ \%$$

Comparison with experiments





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Xiao et al *PRL92, 227004 (04)*

Conclusions

1. **Due to the "supersoft" Goldstone mode vortex lattice in 3D exhibits quasi-long range order only.**
2. **Nevertheless all the IR divergencies in perturbation theory cancel enabling precise calculation of magnetization, structure functions and other quantities.**
3. **Results for melting temperature, magnetization, specific heat… are in good agreement with experiments**

Open questions

1. **What is the symmetry reason for supersoft Goldstone bosons. Can Goldstone theorem and “soft pion” theorems be generalized? Some nontrivial group theory involved.**
2. **Is there an order parameter for the transition of the BKT type? Obviously the usual field is not (quasilong order).**