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Oct. 2 2007.

* Special relative

  * 4-momentum, Lorentz transformation
  * tensor
  * proper velocity.

HW problem : ① Mandelstam variable (3-22), (3-23)

③ GZK cut-off.

④ (3-16), $A \to B + C$

⑤ $\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$ → part of the Maxwell's Equas.

HW: 3; { 16, 22, 23,

plan : * H.M.'s lecture note

* E&M
  - $F^{\mu\nu}$ transformation.
  - $\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$

* Gallilean space & time

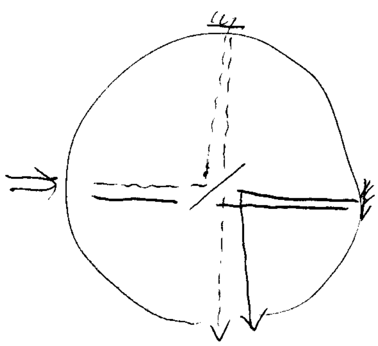
An abstract universal time for every "inertial" frame:

$$\vec{x}'(t) = \vec{x}(t) - \vec{V}t \quad , \quad t' = t$$

$$\vec{v}' = \dot{\vec{x}}' = \dot{\vec{x}} - \vec{V} = \vec{u} - \vec{V}$$

* Michaelson–Morley exp.

They tried to measure the "ether" –



Found no interference

only possible explaination

$\Rightarrow$ speed of light remains the same in every inertial frame.

consider small time & space intervals dt and $d\vec{x} = (dx, dy, dz)$

$$(d\vec{x})^2 = (dx)^2 + (dy)^2 + (dz)^2 = (c\,dt)^2 \quad \text{in the frame } A$$

for frame B, it must satisfies that

$$(d\vec{x}')^2 = (dx')^2 + (dy')^2 + (dz')^2 = (c\,dt')^2$$

In Gallilean tranf. $dt' = dt$, $d\vec{x}' = d\vec{x} - \vec{V}dt$

$$\Rightarrow (d\vec{x}')^2 = (d\vec{x})^2 + \vec{V}^2 dt^2 - 2(\vec{V}\cdot d\vec{x})\,dt \neq (dx)^2$$

$$= (c^2 + \vec{V}^2)\,dt^2 - 2(\vec{V}\cdot d\vec{x})\,dt$$

we are forced to look for a more complicated transf.

Assume that the transformation is <u>Linear</u> (GR is nonlinear)

$$\begin{pmatrix} c\,dt' \\ dz' \end{pmatrix} = \underset{2\times 2 \text{ matrix}}{\underset{\sim}{A}} \begin{pmatrix} c\,dt \\ dz \end{pmatrix} = \begin{pmatrix} A_{tt}(v) & A_{tz}(v) \\ A_{zt}(v) & A_{zz}(v) \end{pmatrix} \begin{pmatrix} c\,dt \\ dz \end{pmatrix}$$

In the Gallilean transf. $A = \begin{pmatrix} 1 & 0 \\ -\frac{v}{c} & 1 \end{pmatrix}$

☆ requirement. $(v \Leftrightarrow -v) \sim (dz \leftrightarrow -dz)$

so $A(-v) = P\,A(v)\,P$, $P = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

☆ $A(-v)$ after $A(v)$ should return to $\mathbb{1}$

$$P A(v) P\, A(v) = \mathbb{1} = A(v)\, P A(v) P$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} A_{tt} & A_{tz} \\ A_{zt} & A_{zz} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} A_{tt} & A_{tz} \\ A_{zt} & A_{zz} \end{pmatrix} = \begin{pmatrix} A_{tt} & A_{tz} \\ -A_{zt} & A_{zz} \end{pmatrix}\begin{pmatrix} A_{tt} & A_{tz} \\ -A_{zt} & -A_{zz} \end{pmatrix}$$

$$= \begin{pmatrix} A_{tt}^2 - A_{tz}A_{zt} & A_{tt}A_{tz} - A_{tz}A_{zz} \\ A_{zz}A_{zt} - A_{zt}A_{tt} & -A_{tz}A_{zt} + A_{zz}^2 \end{pmatrix} = \begin{pmatrix} 1 & \\ 0 & \end{pmatrix}$$

$\Rightarrow$ $A_{tt} = A_{zz}$, & $A_{tt}^2 - A_{tz}A_{zt} = 1$

☆ consider. The frame A, $z = 0$, @ rest,

$$\begin{pmatrix} c\,dt' \\ dz' \end{pmatrix} = \begin{pmatrix} A_{tt}(v), & A_{tz}(v) \\ A_{zt}(v) & A_{tt}(v) \end{pmatrix}\begin{pmatrix} c\,dt \\ 0 \end{pmatrix} = \begin{pmatrix} cA_{tt}(v)\,dt \\ c\,A_{zt}(v)\,dt \end{pmatrix}$$

we know that $\frac{dz'}{c\,dt'} = \frac{-v}{c}$ $= \frac{A_{zt}(v)}{A_{tt}(v)}$,

$A_{tt}^2 + A_{tz}\frac{v}{c}A_{tt} = 1$

or $A_{tz} = \frac{c}{v}\,\frac{1 - A_{tt}^2}{A_{tt}}$

$$\Rightarrow A(v) = A_{tt}(v)\begin{pmatrix} 1 & \frac{c}{v}\frac{(1-A_{tt}^2)}{A_{tt}^2} \\ -\frac{v}{c} & 1 \end{pmatrix}$$

Const speed of light.

In frame A, $dz = c\,dt$.

In frame B

$$\begin{pmatrix} c\,dt' \\ dz' \end{pmatrix} = A_{tt}(v)\begin{pmatrix} 1 & \frac{c}{v}\left(\frac{1-A_{tt}^2}{A_{tt}^2}\right) \\ -\frac{v}{c} & 1 \end{pmatrix}\begin{pmatrix} c\,dt \\ dz = c\,dt \end{pmatrix}$$

$$= A_{tt}(v)\; c\,dt \times \begin{pmatrix} 1 + \frac{c}{v}\left(\frac{1-A_{tt}^2}{A_{tt}^2}\right) \\ 1 - \frac{v}{c} \end{pmatrix}$$

$$\cancel{1} + \frac{c}{v}\left(\frac{1-A_{tt}^2}{A_{tt}^2}\right) = \cancel{1} - \frac{v}{c}$$

$$\frac{1-A_{tt}^2}{A_{tt}^2} = -\frac{v^2}{c^2} \Rightarrow A_{tt} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow A(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\begin{pmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$$

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \quad \beta \equiv \frac{v}{c}$$

This is the Lorentz transf.

~~A~~ subsequential transf.

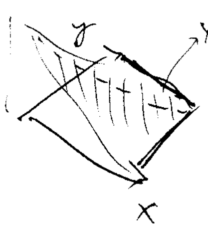
$$A(\beta_2)\,A(\beta_1) = \begin{pmatrix} \gamma_2 & -\gamma_2\beta_2 \\ -\gamma_2\beta_2 & \gamma_2 \end{pmatrix}\begin{pmatrix} \gamma_1 & -\gamma_1\beta_1 \\ -\gamma_1\beta_1 & \gamma_1 \end{pmatrix}$$

$$\frac{1}{\sqrt{1-\left(\frac{\beta_1+\beta_2}{1+\beta_1\beta_2}\right)^2}} = \frac{1+\beta_1\beta_2}{\sqrt{(1+\beta_1\beta_2)^2-(\beta_1+\beta_2)^2}}$$

$$(1+\beta_1\beta_2+\beta_1+\beta_2)(1+\beta_1\beta_2-\beta_1-\beta_2)$$

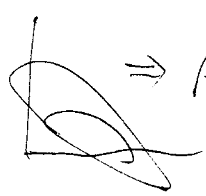
for $x, y \in [0,1]$

$1 + xy - x - y \geq 0$



$$= \begin{pmatrix} \frac{1}{\sqrt{1-\beta_2^2}} & \frac{-\beta_2}{\sqrt{1-\beta_2}} \\ \frac{-\beta_2}{\sqrt{1-\beta_2}} & \frac{1}{\sqrt{1-\beta_2^2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{1-\beta_1^2}} & \frac{-\beta_1}{\sqrt{1-\beta_1^2}} \\ \frac{-\beta_1}{\sqrt{1-\beta_1^2}} & \frac{1}{\sqrt{1-\beta_1^2}} \end{pmatrix} = \begin{pmatrix} \frac{1+\beta_1\beta_2}{\sqrt{(1-\beta_1^2)(1-\beta_2^2)}} & \frac{-(\beta_1+\beta_2)}{\sqrt{(1-\beta_1^2)(1-\beta_2^2)}} \\ \cdot\cdot & \cdot\cdot \end{pmatrix}$$

$$\Rightarrow \beta_{eff} \equiv \frac{\beta_1+\beta_2}{1+\beta_1\beta_2} \quad \text{for } \beta_1 = 1 \Rightarrow \beta_{eff} = 1$$



$\Rightarrow 0 \leq \beta \leq 1 \quad \Rightarrow$ speed of light is the limit!!

$$\Rightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} \geq 1$$

For a general ~~tra~~ relative velocity, the transformation looks like:

$$A(\vec{\beta}) = \left(\begin{array}{c|c} \gamma & -\gamma\vec{\beta}^T \\ \hline -\gamma\vec{\beta} & \underbrace{\left(I - \frac{\vec{\beta}\vec{\beta}^T}{\vec{\beta}^2}\right)}_{\text{orthogonal to}} + \gamma \underbrace{\frac{\vec{\beta}\vec{\beta}^T}{\vec{\beta}^2}}_{\text{parallel to } \vec{\beta}} \end{array} \right)$$

Implications of Lorentz transformation

(1) Time is no more universal.

suppose a time interval dt in the rest frame of an object $dz = 0$

$$\Rightarrow dt' = \gamma dt$$

$\tau_{\mu} = 2.19703(4) \times 10^{-6}$ sec at rest.

It survives $\gamma\tau$ or traveling a distance $\gamma\beta c\tau$

such that the μ produced at the ceiling of atm. by cosmic rays can reach the surface.

(2). Lorentz contraction.

In the rest frame of ~~muon~~, atm, the thickness $dz' = L$

and the muon takes, $dt' = \frac{L}{v} = \frac{L}{c\beta}$, to pass through it.

In the rest frame of muon:

$$\begin{pmatrix} c\,dt \\ dz \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} c\,dt' = \frac{L}{\beta} \\ L \end{pmatrix} = \begin{pmatrix} \frac{\gamma}{\beta}L - \gamma\beta L \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{L}{\gamma\beta} \\ 0 \end{pmatrix}$$

muon takes $\frac{L}{c\gamma\beta}$ time to travel the thickness of atm.

in other word $L \Rightarrow \frac{L}{\gamma}$

when $v \ll c$, or $\beta \ll 1$ $\quad \frac{1}{\sqrt{1-\beta^2}} \sim 1 + \frac{\beta^2}{2}$

$$A(\beta) \sim \begin{pmatrix} 1+\frac{\beta^2}{2} & -\beta \\ -\beta & 1+\frac{\beta^2}{2} \end{pmatrix} \sim \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}$$

$$\begin{pmatrix} c\,dt' \\ dz' \end{pmatrix} = \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}\begin{pmatrix} c\,dt \\ dz \end{pmatrix} = \begin{pmatrix} c\,dt - \frac{v}{c}\,dz \\ -v\,dt + dz \end{pmatrix} \sim \begin{pmatrix} c\,dt \\ dz - v\,dt \end{pmatrix}$$

✡ Another important consequence of Lorentz transformation is that

$(c\,d\tau)^2 \equiv (c\,dt)^2 - (dx)^2 - (dy)^2 - (dz)^2$ is invariant in any reference frame

$$\because \; (c\,d\tau')^2 = (c\,dt')^2 - (dz')^2 - (dx)^2 - (dy)^2$$
$$= (c\gamma\,dt - \gamma\beta\,dz)^2 - (-\gamma\beta c\,dt + \gamma\,dz)^2 - (dx)^2 - (dy)^2$$
$$= (c\,dt)^2(\gamma^2 - \gamma^2\beta^2) + dz^2(-\gamma^2 + \gamma^2\beta^2) - (dx)^2 - (dy)^2$$
$$+ (c\,dt\,dz)\underbrace{(-2\gamma^2\beta + 2\gamma^2\beta)}_{0}$$
$$= (c\,dt)^2 - (dz)^2 - (dx)^2 - (dy)^2$$

* A quantity that does not change from one reference frame to another is called a "Lorentz invariant".

In this case, $d\tau$ = proper time is an invariant.

(is the time interval in the rest frame of an object)

can be thought as the length of a vector

$$|\vec{V}| = |\vec{V}'|$$

* Four - vector notation.

$\Rightarrow$ time & space get "mixed up" under Lorentz transformations.

$\Rightarrow$ they should be consider different components of a single object

$\Rightarrow$ a 4 - component space - time vector.

$\Rightarrow$ $dx^\mu = (c\,dt, dx, dy, dz)$, $\mu = 0, 1, 2, 3$

$$(dx')^\mu = A^\mu{}_\nu \, dx^\nu$$

Einstein convention: any repeated index is always summed over from $0 \to 3$

$\Rightarrow$ Any 4-vector transforms as the space - time vector

is said to be "contravariant"!

Since proper time is an invariant, it's useful to introduce a "covariant" vector which has a lower index

$$dx_\mu = (c\,dt, -dx, -dy, -dz)$$

$\Rightarrow$ $(c\,d\tau)^2 = dx_\mu dx^\mu = \sum_{\mu=0}^{3} dx_\mu dx^\mu = g_{\mu\nu} dx^\mu dx^\nu$
$= g^{\mu\nu} dx_\mu dx_\nu$

* upper & lower indices get cancelled. $\Rightarrow$ invariant.

$$(dx')^\mu = A^\mu{}_\nu \, dx^\nu$$

effectively 1 - index on r.h.s

$\Rightarrow$ metric tensor $g_{\mu\nu}$, such that $dx_\mu = g_{\mu\nu} dx^\nu$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

, just a convention

east side $\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

west side $\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ if define $(icdt$

the inverse of $g_{\mu\nu}$, $g_{\mu\nu} g^{\nu\lambda} = \delta^\lambda_\mu$

$$g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad dx^\mu = g^{\mu\nu} dx_\nu$$

$$dx^{\mu'} = \Lambda^{\mu}{}_{\nu}\, dx^{\nu}$$

$$dx_{\mu}' = \Lambda_{\mu}{}^{\nu}\, dx_{\nu} = \Lambda_{\mu}{}^{\nu} g_{\nu\lambda}\, dx^{\lambda}$$

$$= g_{\mu\sigma}\, dx^{\sigma'} = g_{\mu\sigma} \Lambda^{\sigma}{}_{\lambda}\, dx^{\lambda} \qquad \Rightarrow \Lambda_{\mu}{}^{\nu} = g_{\mu\sigma} g_{\nu\lambda} \Lambda^{\sigma}{}_{\lambda}$$

$$\begin{pmatrix} \gamma & +\beta\gamma \\ +\beta\gamma & \gamma \end{pmatrix} \qquad \begin{matrix} (\Lambda^{\sigma}{}_{\lambda})_{00} & = & g_{0\sigma} g_{0\lambda} \\ -(\Lambda^{\sigma}{}_{\lambda})_{10} & & g_{1\sigma} g_{0\lambda} \\ -(\Lambda^{\sigma}{}_{\lambda})_{01} & & g_{0\sigma} g_{1\lambda} \\ (-)^2(\Lambda^{\sigma}{}_{\lambda})_{11} & & g_{1\sigma} g_{1\lambda} \end{matrix} \qquad \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$$

$$dx_{\mu}'\, dx^{\mu'} = dx_{\mu}\, \Lambda^{T} \Lambda\, dx^{\mu}$$

$$\begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Energy & momentum 4-vector.

- The action of a system must be Lorentz invariant.
  The only invariant for a point particle is its proper time $d\tau$

$\Rightarrow$ Action for a point particle is

$$S = A \int d\tau$$

$$S = \frac{A}{c} \int \sqrt{(c\,dt)^2 - (d\vec{x})^2} = A \int dt \sqrt{1 - \frac{1}{c^2}\left(\frac{d\vec{x}}{dt}\right)^2}$$

$\Rightarrow$ Lagrangian is given by $\quad L = A \sqrt{1 - \frac{v^2}{c^2}} = A \sqrt{1 - \frac{\dot{x}^2}{c^2}}$

if $v \ll c$, $\Rightarrow L \simeq A \left(1 - \frac{1}{2}\frac{\dot{x}^2}{c^2}\right) = \frac{A}{mc^2}\left(mc^2 - \frac{1}{2}mv^2\right)$

$$L = T - V, \Rightarrow A = -mc^2$$

$$\vec{p} = \frac{\partial L}{\partial \dot{\vec{x}}} = \frac{\partial}{\partial \dot{\vec{x}}}\left(-mc^2 \sqrt{1 - \frac{\dot{x}^2}{c^2}}\right) = \frac{m\dot{\vec{x}}}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}} = \boxed{mc\gamma\vec{\beta}}$$

$$\vec{p} = \frac{\cancel{m} m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \underset{v<<c}{\sim} m\vec{v}$$

the energy is given by the Hamiltonian $H = p\cdot\dot{q} - L$

$$\Rightarrow E = H = \frac{m\dot{\vec{x}}\,\dot{\vec{x}}}{\sqrt{1-\frac{\dot{x}^2}{c^2}}} + mc^2\sqrt{1-\frac{\dot{x}^2}{c^2}} = mc^2\beta^2\gamma + \frac{mc^2}{\gamma}$$

$$\boxed{= mc^2\gamma}$$

$\beta << 1$

$$E = mc^2\gamma \simeq \frac{mc^2}{\sqrt{1-\beta^2}} \sim mc^2\left(1+\frac{1}{2}\beta^2\right) = mc^2 + \frac{1}{2}mv^2 \;-$$

comparing the expression of momentum & energy, one sees.

$$\left(\frac{E}{c}\right)^2 - \vec{p}^{\,2} = \left(\frac{mc}{\sqrt{1-\beta^2}}\right)^2 - \left(\frac{mc\vec{\beta}}{\sqrt{1-\vec{\beta}^2}}\right)^2 = m^2c^2\,\frac{(1-\vec{\beta}^2)}{(1-\vec{\beta}^2)} = m^2c^2$$

or $E^2 = m^2c^4 + \vec{p}^{\,2}\vec{c}^{\,2}$

or $\underline{E^2 - \vec{p}^{\,2}c^2 = m^2c^4}$ is an invariant in any frame.

$$\Rightarrow \quad p^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right)$$

$$\& \quad p_\mu = \left(\frac{E}{c}, -p_x, -p_y, -p_z\right) \quad \Rightarrow \quad p_\mu p^\mu = m^2c^2$$

e.g. at rest frame

$$p^\mu = (mc, 0.0.0)$$

moving with v along x-direction

$$\Rightarrow (p')^\mu = (\gamma mc, +\beta\gamma mc, 0, 0)$$

$$= \left(\frac{E'}{c}, p_x', p_y', p_z'\right)$$ $\Rightarrow$ exactly the expression we just got.

Energy-momentum

$\Rightarrow$ 4-vector conservation if L is invariant under $x^\mu \to x^\mu + a^\mu$

$$\mathbb{P}_\mu = \sum_{i=1}^{N} P_{i\mu}.$$

$(\mathbb{P}_\mu \mathbb{P}^\mu)$ is an invariant.

$$\Rightarrow (\mathbb{P}_\mu \mathbb{P}^\mu) = \sum_{i,j=1}^{N} P_{i\mu} P_j^\mu = \sum_{i=1}^{N} m_i^2 c^2 + \sum_{i\neq j} P_{i\mu} P_j^\mu$$

$$= \sum_{i=1}^{N} m_i^2 c^2 + \sum_{i\neq j} (m_i c \gamma_i, - m_i c \gamma_i \vec{\beta}_i) \cdot (m_j c \gamma_j, m_j c \gamma_j \ldots$$

$$= \sum_{i=1}^{N} m_i^2 c^2 + \sum_{i\neq j} m_i m_j c^2 \underbrace{\gamma_i \gamma_j}_{\geq 1} \underbrace{(1 - \vec{\beta}_i \cdot \vec{\beta}_j)}_{\geq 0} \geq 0$$

$\Rightarrow$ One can always find a frame such that

$$\mathbb{P}_\mu = (E_{total}, \vec{0}) \Rightarrow \text{CM frame}.$$

☆ Tensor transformation.

$$t'^{\mu\nu\alpha\cdots\lambda} = \Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta}\cdots\Lambda^{\lambda}{}_{\sigma}\, t^{\alpha\beta\cdots\sigma}$$

☆ Doppler shift.

for photon, the plane wave solution to the maxwell eq. $\sim \cancel{e^{i(\omega t)}}\; e^{i(\vec{k}\vec{x}-\omega t)}$

$$k^{\mu} = \left(\frac{\omega}{c}, \vec{k}\right) \qquad \omega = c|\vec{k}| \qquad = e^{-i(k^{\mu}x_{\mu})}$$

In one frame $k^{\mu} = \left(\frac{\omega}{c}\right)(1, 0, 0, 1)$

$$\begin{pmatrix} \frac{\omega'}{c} \\ k' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}\begin{pmatrix} \frac{\omega}{c} \\ k \end{pmatrix} = \begin{pmatrix} \frac{\gamma}{c}\omega - \gamma\beta k \\ -\frac{\gamma\beta\omega}{c} + \gamma k \end{pmatrix} = \begin{pmatrix} \frac{\omega}{c}(\gamma - \gamma\beta) \\ k(\gamma - \gamma\beta) \end{pmatrix}$$

$$\Rightarrow \omega' = \omega(\gamma - \gamma\beta) = \omega\sqrt{\frac{1-\beta}{1+\beta}}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y, & -E_z \\ & & -B_z & +B_y \\ & & & -B_x \end{pmatrix} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

$A^{\mu} = (\phi, \vec{A})$, $j^{\mu} = (\rho, \vec{j}$

D.O.F. # =

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + j^{\mu}A_{\mu}$$

$$E = -\nabla\phi - \frac{\partial A}{\partial t}$$

$$B = \nabla \times A$$

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}}, \quad \partial^{\mu} \equiv \frac{\partial}{\partial x_{\mu}}$$

E, B = 3+3 = 6

must be an anti-symmetry

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