Oct. 21, 2007 Sun. 2-5 pm ①

Int to HEP

* Plan for this course: { E & M. Lagrangian, example. estimation of synchrotron radiation; accelerator { History, working principle

* Maxwell eqs. (in SI-unit) (Heaviside-Lorentz) $c=1$, natural unit

$$\nabla\cdot E = \frac{1}{\varepsilon_0}\rho = \rho \qquad E = -\nabla\phi - \frac{\partial A}{\partial t}$$

$$\nabla\times E = -\frac{\partial B}{\partial t} \qquad B = \nabla\times A$$

$$\nabla\cdot B = 0 \qquad F = q(E + v\times B)$$

$$\nabla\times B = \mu_0 j + \mu_0\varepsilon_0\frac{\partial E}{\partial t} = j + \frac{\partial E}{\partial t}$$

gauge-transformation

$$A' = A + \vec\nabla\lambda$$

$$\phi' = \phi - \frac{\partial\lambda}{\partial t}$$

or $A^\mu(\phi, \vec A) \Rightarrow A^\mu - \partial^\mu\lambda$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}, \quad \partial^\mu \equiv \frac{\partial}{\partial x_\mu}$$

$$E' = \left(-\vec\nabla\phi + \cancel{\frac{\partial}{\partial t}\vec\nabla\lambda}\right) - \left(\frac{\partial}{\partial t}A + \cancel{\frac{\partial}{\partial t}\vec\nabla\lambda}\right) = E$$

$$B' = \nabla\times(A + \vec\nabla\lambda) = \nabla\times A + \begin{vmatrix} \hat i & \hat j & \hat k \\ \partial_x & \partial_y & \partial_z \\ \partial_x\lambda & \partial_y\lambda & \partial_z\lambda \end{vmatrix}^{\,0} = B$$

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* The transformation of $\partial_\mu / \partial^\mu$

$$\Lambda(\beta) = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}, \quad \Lambda(-\beta) = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix}. \quad \Lambda(\beta)\Lambda(-\beta) = \Lambda(-\beta)\Lambda(\beta) = \mathbb{1}$$

$$x^\mu = \begin{pmatrix} t \\ x \end{pmatrix} \qquad x^{\mu'} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} \gamma(t-\beta x) \\ \gamma(x-\beta t) \end{pmatrix}$$

$$x_\mu = \begin{pmatrix} t \\ -x \end{pmatrix} \qquad x_\mu' = \begin{pmatrix} \gamma(t-\beta x) \\ -\gamma(x-\beta t) \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & +\gamma\beta \\ +\gamma\beta & \gamma \end{pmatrix}}_{=\,\Lambda(-\beta) = \Lambda^{-1}}\begin{pmatrix} t \\ -x \end{pmatrix}$$

$$\frac{\partial f}{\partial x^\mu} \to \left(\frac{\partial f}{\partial x^\mu}\right)' = \frac{\partial f}{\partial x^{\mu'}} = \frac{\partial x^\alpha}{\partial x^{\mu'}}\frac{\partial f}{\partial x^\alpha}$$

$$x^\alpha = \begin{pmatrix} \gamma & +\gamma\beta \\ +\gamma\beta & \gamma \end{pmatrix}^\alpha{}_\mu x^{\mu'} = \begin{pmatrix} \gamma(t'+\beta x') \\ \gamma(x'+\beta t') \end{pmatrix}$$

$x^\alpha = (\Lambda^{-1})^\alpha{}_\mu x^{\mu'}$ is contravariant

similarly $\left(\frac{\partial}{\partial x_\mu}\right)' = \Lambda\left(\frac{\partial}{\partial x_\mu}\right)$

$$\Rightarrow \left(\frac{\partial}{\partial x^\mu}\right)' = \Lambda^{-1}\left(\frac{\partial}{\partial x^\mu}\right)$$

$$\left(\frac{\partial f}{\partial x^\mu}\right)' = \begin{pmatrix} \frac{\partial f}{\partial t} \\ \frac{\partial f}{\partial x} \end{pmatrix}' = \begin{pmatrix} \frac{\partial t}{\partial t'}\frac{\partial f}{\partial t} + \frac{\partial x}{\partial t'}\frac{\partial f}{\partial x} \\ \frac{\partial t}{\partial x'}\frac{\partial f}{\partial t} + \frac{\partial x}{\partial x'}\frac{\partial f}{\partial x} \end{pmatrix} = \begin{pmatrix} \gamma\frac{\partial f}{\partial t} + \gamma\beta\frac{\partial f}{\partial x} \\ \gamma\beta\frac{\partial f}{\partial t} + \gamma\frac{\partial f}{\partial x} \end{pmatrix} = \Lambda^{-1}\frac{\partial f}{\partial x^\mu}$$

$\Leftrightarrow$ is covariant.

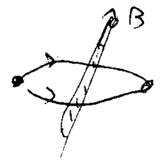
DOF

$$\#\phi = 1,\ \#\vec{A} = 3 \ \} \Rightarrow 4 \ \longrightarrow$$ more fundamental than E & B,

$$\#E = \#B = 3 \ \} \Rightarrow 6$$

$$\#\rho = 1,\ \#\vec{j} = 3 \ \} \Rightarrow 4$$

Aharonov-Bohm effect



Thus: the only two possible 4-vectors in E&M we can construct are

$$A^\mu = (\phi, \vec{A}) \quad \& \quad j^\mu = (\rho, \vec{j})$$ → treated as background without dynamics.

⇒ To form a Lorentz invariant.

$A^\mu A_\mu \rightarrow$ photon mass, gauge non invariant

$j^\mu j_\mu \rightarrow$ totally ~~redundant~~ non dynamical.

$A^\mu j_\mu$ ✓ ok.

$$\left.\begin{aligned} A^\mu &\rightarrow A^\mu - \partial^\mu \lambda \\ A_\mu &\rightarrow A_\mu - \partial_\mu \lambda \end{aligned}\right\} \quad A^\mu A_\mu \rightarrow (A^\mu - \partial^\mu\lambda)(A_\mu - \partial_\mu \lambda)$$

$$= A^\mu A_\mu - \partial^\mu\lambda A_\mu - \partial_\mu \lambda A^\mu + \partial^\mu\lambda\, \partial_\mu \lambda$$

$$\neq A^\mu A_\mu \quad \Rightarrow \text{gauge non invariant.}$$

How to see it gives photon mass?

physical photon. transversed. or L-R ⇒ only 2 components.

⇒ you can't find a frame such that it has no chirality.

⇒ traveling at speed of light ⇒ mass is zero;

extra 2 DOF are unphysical. ⇒ redundent DOF

$2 \sim e^{i\alpha} \rightarrow U(1)$

Therefore, with 2 4-vectors, we can only write down

$$\mathcal{L} \supset 1\, A^\mu j_\mu$$

↳ minimum coupling. can always absorb the normalization to j_μ

* so, let's try higher rank tensor, constructed from A^μ

① $A^\mu A^\nu \longrightarrow$ ~~most~~ $(A^\mu A^\nu_{\ })(A_\mu A_\nu)$ no good -

② by derivatives $\partial^\mu A^\nu$

		gauge	DOF
symmetric	$(\partial^\mu A^\nu + \partial^\nu A^\mu)$	X	10
anti symmetric	$(\partial^\mu A^\nu - \partial^\nu A^\mu) \equiv F^{\mu\nu}$	V	6 $= \# E + \# B$

③ mix ? $\underbrace{F^{\mu\nu}}_{\text{anti}} \underbrace{A_\mu A_\nu}_{\text{sym}} \to 0$

$\cancel{F^{\mu\nu}}\ \underbrace{\cancel{(A_\mu A_\nu - A_\nu A_\mu)}}_{\text{anti}} \to 0$ A commute $\to$ This is no more true for nonabelian gauge group

* therefore, the only invariant term we can construct is
$\widehat{\text{Lorentz, gauge}}$

$F^{\mu\nu} F_{\mu\nu}$

* $\Rightarrow \mathcal{L} = \underbrace{a}_{\parallel} F^{\mu\nu} F_{\mu\nu} \oplus j^\mu A_\mu$ $\qquad T - V \qquad \underline{\underline{\rho\phi}}$

$(-\frac{1}{4})$ this will be clear later

* EOM of A^μ ← which is the only physical & dynamical variables.

(from $\delta S = 0$, the least action principle)

$$\partial^\alpha \left(\frac{\partial \mathcal{L}}{\partial(\partial^\alpha A^\mu)} \right) - \frac{\partial \mathcal{L}}{\partial A^\mu} = 0$$

$$\mathcal{L} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + j^\mu A_\mu$$
$$= -\frac{1}{4}(\partial^\mu A^\alpha - \partial^\alpha A^\mu)(\partial_\mu A_\alpha - \partial_\alpha A_\mu) + j^\mu A_\mu$$

$-\left(\frac{\partial \mathcal{L}}{\partial A^\mu}\right)$ is easy $\Rightarrow \boxed{+j_\mu}$

$F^2 \to$ factor 2,
$\mu \leftrightarrow \alpha$ factor 2 $\Rightarrow$ why $\frac{1}{4}$

$\mathcal{L} \ni g_{\mu\alpha} g_{\nu\beta} F^{\mu\nu} F^{\alpha\beta}$

$$\frac{\partial \mathcal{L}}{\partial(\partial^\alpha A^\mu)} = F_{\mu\alpha} \Rightarrow \text{EOM}$$

$$\boxed{\partial^\nu F_{\mu\nu} + j_\mu = 0}$$

How about the other 2 ?

They can be derived from the symmetry properties of $F_{\mu\nu}$!

A Matter 2D.

$$\boxed{\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0}$$

$$\partial_\mu(\partial_\nu A_\lambda - \partial_\lambda A_\nu) + \partial_\nu(\partial_\lambda A_\mu - \partial_\mu A_\lambda) + \partial_\lambda(\partial_\mu A_\nu - \partial_\nu A_\mu) = 0$$

$\mu = 0.$

$$\partial_0(\partial_\nu A_\lambda - \partial_\lambda A_\nu) + \partial_\nu(\partial_\lambda A_0 - \partial_0 A_\lambda) + \partial_\lambda(\partial_0 A_\nu - \partial_\nu A_0) = 0$$

$\nu = 2$ $$\partial_0(\partial_2 A_\lambda - \partial_\lambda A_2) + \partial_2(\partial_\lambda A_0 - \partial_0 A_\lambda) + \partial_\lambda(\partial_0 A_2 - \partial_2 A_0) = 0$$

If $\lambda = \nu \Rightarrow$ ~~$\partial_\mu F_{\lambda\mu} + \partial_\lambda F_{\mu\lambda} = 0$~~ trivial.

~~$\partial_\mu(\partial_\lambda A_\mu - \partial_\mu A_\lambda) + \partial_\lambda(\partial_\mu A_\lambda - \partial_\lambda A_\mu)$~~

only interesting result will be $\lambda \neq 2$

$\lambda = 1$ $$\partial_0(\partial_2 A_1 - \partial_1 A_2) + \partial_2(\partial_1 A_0 - \partial_0 A_1) + \partial_1(\partial_0 A_2 - \partial_2 A_0) = 0 \quad -(1)$$

$\lambda = 3$ $$\partial_0(\partial_2 A_3 - \partial_3 A_2) + \partial_2(\partial_3 A_0 - \partial_0 A_3) + \partial_3(\partial_0 A_2 - \partial_2 A_0) = 0 \quad -(2)$$

Eq(1) $\Rightarrow$ $$\frac{d}{dt}\left(\partial_y(-A_x) - \partial_x(-A_y)\right) + \partial_y\left(\partial_x(\phi) - \partial_t(-A_x)\right) + \partial_x\left(\partial_t(-A_y) - \partial_y\phi\right) = 0$$

$$= \frac{\partial}{\partial t}(B_z) + \partial_y(-E_x) + \partial_x(E_y) = \frac{\partial}{\partial t}(B_z) + (\nabla\times E)_z = 0$$

$$\Rightarrow \boxed{(\nabla\times E) = -\frac{\partial}{\partial t}B} \Rightarrow \text{2nd Maxwell eqn.}$$

overall derivative ∂^μ on the EOM

$$\underbrace{\partial^\mu \partial^\nu}_{\text{sym}} \underbrace{F_{\mu\nu}}_{\text{anti}} + \partial^\mu j_\mu = 0 \quad \Rightarrow \quad \partial^\mu j_\mu = 0$$

(sym anti $= 0$)

$$\partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z} \right)$$

$$j_\mu = (\rho, -\vec{j})$$

$$\Rightarrow \partial^\mu j_\mu = \frac{\partial}{\partial t}\rho + \nabla \cdot \vec{j} = 0 \quad \Rightarrow \text{charge conservation.}$$

* $\partial^\nu F_{\mu\nu} = -j_\mu$

① $\mu = 0$, $\nu \neq 0$, $\quad i = 1, 2, 3 \quad (x, y, z)$

$$-\rho = \partial^\nu F_{0\nu} = \partial^\nu (\partial_0 A_\nu - \partial_\nu A_0) = \frac{\partial}{-\partial x_i}\left(\partial_t(-A_i) - \partial_i(\phi) \right]$$

$$= \nabla_i \cdot \left(-\vec{\nabla}\phi - \frac{\partial}{\partial t}\vec{A}\right) = -\nabla \cdot \vec{E}$$

$$\Rightarrow \boxed{\nabla \cdot \vec{E} = \rho} \quad \text{1st Maxwell eq.}$$

② $\mu \neq 0$. R.H.S $\quad -j_\mu \quad \vec{j}_i$ $\qquad \partial^\mu F_{\mu\nu} = -j_\mu$

→ denoted as i

~~$\nu = 0, 1, 2, 3$~~

~~$\nu = 3 \quad \partial^3 F_{\mu 3} = -\frac{\partial}{\partial z}(\partial_\mu A_3 - \partial_3 A_\mu) = -\frac{\partial}{\partial z}(\partial_i(-A_z) - (\partial_z)(-A_i))$~~

~~$\mu = x, y \quad = \frac{\partial}{\partial z}(\partial_i A_z - \partial_z A_i)$~~

~~$= \frac{\partial}{\partial z}(\partial_x A_z - \partial_z A_x)$~~

$\nu = 1, 2, 3$

$$\partial^\nu(\partial_\mu A_\nu - \partial_\nu A_\mu) + \partial^0(\partial_\mu A_0 - \partial_0 A_\mu) = -j_\mu$$

$$(\nu = 3) \quad -\frac{\partial}{\partial z}\left(\frac{\partial}{\partial x_i}(-A_z) - \frac{\partial}{\partial z}(-A_i)\right] + \frac{\partial}{\partial t}\left(\frac{\partial}{\partial x_i}(\phi) - \frac{\partial}{\partial t}(-A_i)\right] = +\vec{j}_i$$

$$(\nu = 2) \quad -\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x_i}(-A_y) - \frac{\partial}{\partial y}(-A_i)\right]$$

$$(\nu = 1) \quad -\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x_i}(-A_x) - \frac{\partial}{\partial x}(-A_i)\right]$$

4th Maxwell eq

Say ① $\mu = 3$, $i = z$ $\Rightarrow$

$$+\frac{\partial}{\partial y}\Big(\overbrace{\frac{\partial}{\partial z}A_y - \frac{\partial}{\partial y}A_z}^{= -B_x}\Big) + \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z} + \frac{\partial}{\partial t}A_z\right)$$

$$+\frac{\partial}{\partial x}\Big(\underbrace{\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z}_{= B_y}\Big)$$

$$B = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix} \qquad \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (\nabla \times B)_z - \frac{\partial}{\partial t}E_z = j_z$$

$$\Rightarrow \boxed{\nabla \times \vec{B} = \frac{\partial}{\partial t}\vec{E} + \vec{j}}$$

Now, let's try $\mu \neq 0$. $\mu \neq \nu \neq \lambda$.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix}$$

say, $\mu = 1$, $\lambda = 2$, $\nu = 3$

$$\partial_1(\partial_3 A_2 - \partial_2 A_3) + \partial_3(\partial_2 A_1 - \partial_1 A_2) + \partial_2(\partial_1 A_3 - \partial_3 A_1) = 0$$

$$\Rightarrow \partial_x\{\partial_z(-A_y) - \partial_y(-A_z)\} + \partial_z[\partial_y(-A_x) - \partial_x(-A_y)] + \partial_y[\partial_x(-A_z) - \partial_z(-A_x)]$$

$$\Rightarrow \partial_x(+B_x) + \partial_z(+B_z) + \partial_y(B_y) = \boxed{\nabla \cdot B = 0}$$

Isn't this too beautiful ?!

Summary: ① from A^μ, j^μ → we conclude that the only Lorentz invariant, gauge invariant. $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j_\mu A^\mu$

② E.O.M ⇒ 4 Maxwell eqs.

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$= \begin{pmatrix} 0, & \partial^0 A^1 - \partial^1 A^0, & \partial^0 A^2 - \partial^2 A^0, & \partial^0 A^3 - \partial^3 A^0 \\ & 0 & \partial^1 A^2 - \partial^2 A^1, & \partial^1 A^3 - \partial^3 A^1 \\ & & 0 & \partial^2 A^3 - \partial^3 A^2 \\ & & & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0, & \partial_t A_x + \partial_x \phi, & \partial_t A_y + \partial_y \phi, & \partial_t A_z + \partial_z \phi \\ & 0, & -\partial_x A_y + \partial_y A_x, & -\partial_x A_z + \partial_z A_x \\ & & 0 & -\partial_y A_z + \partial_z A_y \\ & & & 0 \end{pmatrix} = \begin{pmatrix} 0, & -E_x, & -E_y, & -E_z \\ E_x & 0, & -B_z, & +B_y \\ E_y & +B_z & 0, & -B_x \\ E_z & -B_y & +B_x & 0 \end{pmatrix}$$

⇒ Now, you have a good recipe to get (E', B') in different inertial frame.

or $\frac{dW}{dt} = -\frac{d}{dt}\left(\frac{\varepsilon_0}{2}E^2 + \frac{B^2}{2\mu_0}\right) - \nabla\cdot\left(\frac{E\times B}{\mu_0}\right)$ $\qquad \nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

$F = q\,(E + v\times B)$

$dW = q\vec{F}\cdot d\vec{l} = \vec{F}\cdot\vec{v}\,dt = \vec{E}\cdot \underbrace{q\vec{v}}_{\vec{j}}\,dt$

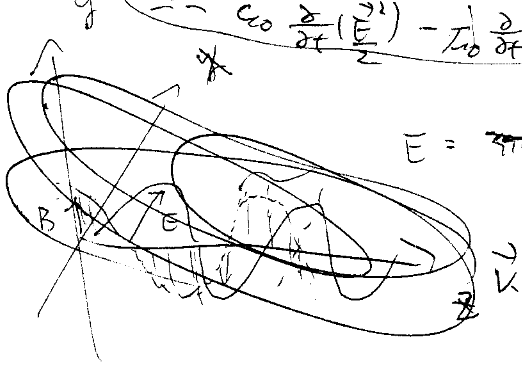
$\frac{dW}{dt} = \vec{E}\cdot\vec{j} = \vec{E}\cdot\left(\cancel{\mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}}\ \frac{1}{\mu_0}\nabla\times\vec{B} - \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$

from field itself

$= -\varepsilon_0\frac{\partial}{\partial t}(\vec{E}^2) + \frac{1}{\mu_0}(\vec{E}\cdot\nabla\times\vec{B})$ $\qquad E_k\,\epsilon^{ijk}\,\partial_i B_j = \partial_i(\epsilon^{ijk}E_k B_j) - \epsilon^{ijk}(\partial_i E_k)B_j$

$= -\varepsilon_0\frac{\partial}{\partial t}\left(\frac{\vec{E}^2}{2}\right) - \frac{1}{\mu_0}\frac{\partial}{\partial t}\left(\frac{\vec{B}^2}{2}\right) + \nabla\cdot\left(\frac{B\times E}{\mu_0}\right)$ $\qquad = \partial_i(B\times E)_i + (\nabla\times E)_j B_j$

$= \nabla\cdot(B\times E)$ $\qquad (\nabla\times E)_j = -\left(\frac{\partial B}{\partial t}\right)_j$



$E = \cancel{\sin(\omega t - kx)}\ \sin(kz - \omega t)\,\hat{x}$

$F = q(E + v\times B)$

$\vec{P} \propto \vec{E}\times\vec{B}$

$[\vec{E}] = \frac{[F]}{q} = \frac{ML}{T^2}$

$[\vec{B}] = \frac{[F]}{q[v]} = \frac{ML}{T^2}\frac{T}{L} = \frac{M}{T}$

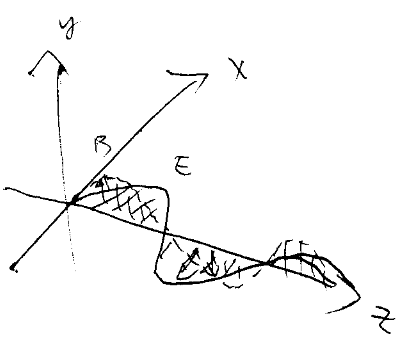
$[P] = \left[\frac{dW}{dt}\right] = \frac{ML^2}{T^3} \propto [\vec{E}][\vec{B}]\frac{L}{q^2 M} \propto \frac{\vec{E}\times\vec{B}}{\mu_0}$

$\left[\frac{q^2}{4\pi\varepsilon_0 r}\right] = \frac{ML}{T^2}$ $\qquad \left[\frac{1}{\varepsilon_0}\right] = \frac{ML}{q^2T^2}$

$\left[\frac{1}{\mu_0\varepsilon_0}\right] = [c^2] = \frac{L^2}{T^2} \Rightarrow \left[\frac{1}{\mu_0}\right] = \frac{q^2L}{M}$

$\nabla\times E = -\frac{\partial B}{\partial t}$ $\qquad \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ B_x & B_y & B_z \end{vmatrix}$ $\qquad -\frac{\partial B_y}{\partial t} = \partial_z E_x$

$\nabla\times B = \frac{\partial E_x}{\partial t}$ in $\hat{x}$



$\hookrightarrow (\partial_y B_z - \partial_z B_y)$

$\Rightarrow -\partial_z B_y = \frac{\partial E_x}{\partial t} = -\omega\cos(kz - \omega t)$

$\Rightarrow \cancel{B_y = \frac{\omega}{k}\sin(kz - \omega t)} \Rightarrow B_y = \frac{\omega}{k}\sin(kz - \omega t)$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \qquad F_{\mu\nu} = g_{\mu\alpha} g_{\nu\beta} F^{\alpha\beta}$$

$$= -\frac{1}{4} \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$= -\frac{1}{4}\left[-2(E_x^2+E_y^2+E_z^2) + 2(B_x^2+B_y^2+B_z^2)\right] = \frac{1}{2}(-B^2+E^2)$$

HW: ① find out $\hat{H} = p\dot{q} - L$

② By using the Lorentz transform properties of $F^{\mu\nu}$ under to find out the E, B in a moving frame relative to a rest charge.

Synchrotron radiation by dimension analysis. (in SI unit)

$$[P] = \left[\frac{dE}{dt}\right] = \frac{ML^2}{T^3} \qquad \vec{P} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

Because Poynting vector $\propto \vec{E}\times\vec{B}$, both E & B are proportional to charge.

$$\Rightarrow P \propto e^2 \underset{\text{acceleration}}{a^\alpha} \left(\frac{1}{\varepsilon_0}\right)^\beta c^\gamma \qquad [V] = \left[\frac{q_1 q_2}{4\pi\varepsilon_0 r}\right] = \frac{1}{[\varepsilon_0][L]} = \frac{[M]L^2}{T^2}$$

$$\Rightarrow \left[\frac{1}{\varepsilon_0}\right] = \frac{ML^3}{T^2}$$

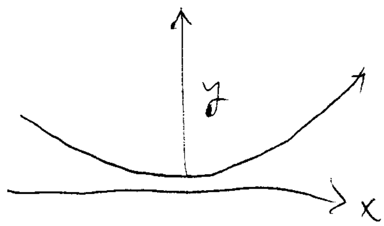
$$\propto e^2 \left(\frac{L}{T^2}\right)^\alpha \left(\frac{ML^3}{T^2}\right)^\beta \left(\frac{L}{T}\right)^\gamma$$

$$= e^2 \frac{L^{\alpha+3\beta+\gamma} M^\beta}{T^{2\alpha+2\beta+\gamma}} \quad \Rightarrow \beta = 1 \qquad \begin{aligned} \alpha+\gamma+3 &= 2 \\ 2\alpha+\gamma+2 &= 3 \end{aligned} \qquad \begin{aligned} \alpha &= 2 \\ \gamma &= -3 \end{aligned}$$

$$\Rightarrow P \propto e^2 \frac{a^2}{\varepsilon_0 c^3}$$

comparing to the classical result ~~which~~ is $\frac{1}{6\pi}\frac{e^2a^2}{\varepsilon_0 c^3}$

In the relativistic limit.



$$a = \ddot{y}$$

In the comoving frame:

$\frac{d^2 y}{d\tau^2}$, In the lab frame. $a = \frac{d^2 y}{dt^2}$

$d\tau = \gamma dt$

$$\begin{pmatrix} d\tau \\ dx' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} dt \\ 0 \end{pmatrix}$$

$$a = \frac{d^2}{dt^2} \simeq \gamma^2 \frac{d^2}{d\tau^2}$$

$\Rightarrow$ In the lab frame. $\boxed{P = \frac{1}{6\pi} \frac{e^2 a^2}{\varepsilon_0 c^3} \gamma^4}$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ an $E = m_0 \gamma$

$$\Rightarrow \quad P = \frac{1}{6\pi} \frac{e^2 a^2}{\varepsilon_0 c^3} \left(\frac{E}{m_0}\right)^4$$

$\Rightarrow$ the lighter charged particle, the more synchrotron radiation.

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