Oct. 23, 2007

L & P for

- charged particle
- 卢瑟福散射公式（下次） $H = p\dot{q} - L$,

  < cross section . some terminology –
- 加速器历史、工作原理，影片，
- 吸收公式
- GZK， 截断公式（GZK → 宇宙射线有一上限）.（ ppt. picture ）
- new ideas for particle acceleration. (picture, clips)
- Demonstration of 带 DZY μ-detector

$$S = -\int d^4x \left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + j^\mu A_\mu\right) - m\int d\tau \qquad j^\mu = (\rho, \vec{j}\,), \quad \vec{j} = \rho\vec{v}$$

$$= -\int d^4x\, \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\int dx_\mu A^\mu[x(\tau)] - m\int d\tau \qquad A^\mu = (\phi, \vec{A})$$

$$= \int d^4x\, \mathcal{L}_{em}(x) + \int dt \left[-m\sqrt{1-\vec{v}^2} - e\phi + e\vec{A}\cdot\vec{v}\right]$$

$\Rightarrow$ the Lagrangian for a point charged particle is

$$L = -m\sqrt{1-\vec{v}^2} - e\phi + e\vec{A}\cdot\vec{v}$$

canonical momentum

$$\vec{p} = \frac{\partial L}{\partial \dot{\vec{q}}} = \frac{m\vec{v}}{\sqrt{1-v^2}} + e\vec{A} \quad \Rightarrow \quad v^2 = \frac{(\vec{p}-e\vec{A})^2}{(m^2+(\vec{p}-e\vec{A})^2)}, \Rightarrow \frac{1}{\sqrt{1-v^2}} = \frac{\sqrt{m^2+(\vec{p}-e\vec{A})^2}}{m}$$

$$H = p\dot{q} - L = \frac{mv^2}{\sqrt{1-v^2}} + e\vec{A}\cdot v + m\sqrt{1-v^2} + e\phi - e\vec{A}\cdot\vec{v}$$

$$= \frac{m}{\sqrt{1-v^2}} + e\phi = \boxed{\sqrt{m^2 + \underbrace{(\vec{p}-eA)^2}_{\text{kinetic momentum}}} + e\phi}$$

$\Rightarrow$ From the Lagrangian, we have the EOM:

$$\underline{\epsilon^{ijk}\epsilon^{abk} = \delta^{ia}\delta^{jb}}$$

$$0 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \frac{d}{dt}\left[\frac{m\vec{v}}{\sqrt{1-v^2}}\right] + e\left(\frac{d}{dt}\vec{A} + \vec{\nabla}\phi\right) - e\vec{\nabla}(\vec{A}\cdot\vec{v}) \qquad \partial_i(A_jV_j) = (\partial_i A_j)V_j$$

$$= V\times(\nabla\times A) - (\vec{V}\cdot\vec{\nabla})\vec{A}$$

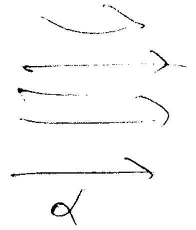
$$\left(\frac{dA}{dt} = \frac{\partial A}{\partial t} + (\vec{V}\cdot\vec{\nabla})A\right) \to \text{comoving}$$

$$\Rightarrow \frac{d}{dt}\frac{m\vec{v}}{\sqrt{1-v^2}} = -e\frac{\partial\vec{A}}{\partial t} - e\vec{\nabla}A^0 + e\,V\times(\nabla\times A) \qquad \epsilon^{ijk}V_j(\nabla\times A)_k$$

$$= e(\vec{E} + \vec{v}\times\vec{B}) \qquad = \epsilon^{ijk}V_j\,\epsilon^{kab}\partial_a A_b$$

$$= V_j(\partial_i A_j) - V_j(\partial_j A_i)$$

Since we can't directly measure the impact parameter, and what we do is throwing a bunch of particle at the target.

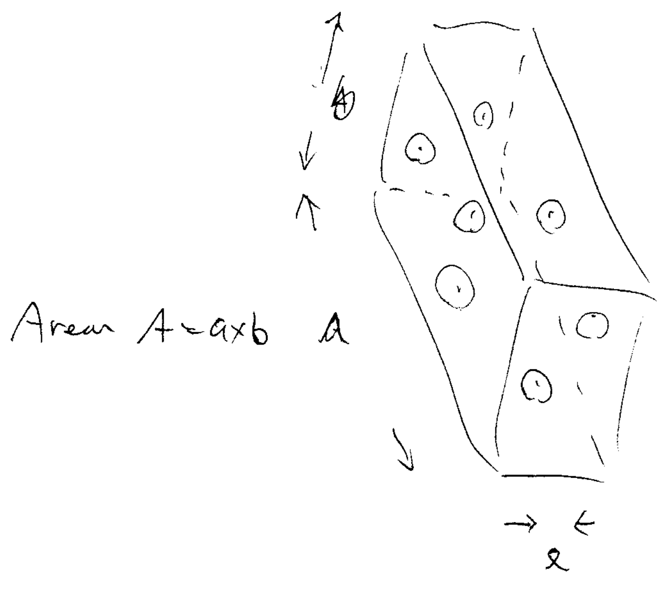


similarly, we can't manipulate each target. So we prepare a distribution of them

Then what we can measure is the deflecting angle

⇒ Then only the impact parameter less than $b(\phi)$ can produce such event.

$\sigma = \pi b^2(\phi)$ is known as the cross section.



$\#$ of nuclei : $A \times \ell \times n$ — density

$\frac{\# \text{ of nuclei} \times \sigma(\phi)}{A}$ → possibility that deflection angle $\geq \phi$

$$P(\phi) = \frac{A\cdot \ell\cdot n \times \pi b^2(\phi)}{A} = \ell\cdot n\,\pi\,\frac{4k_e^2 z^2 e^4}{(m_\alpha v^2)^2 \tan^2\frac{\phi}{2}}$$

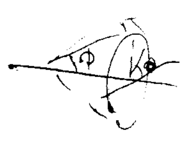
scattering angle $\geq \phi$

In other word $d\sigma(\phi) = 2\pi b\, db = 2\pi\left(\frac{2k_e z e^2}{m_\alpha v^2 \tan\frac{\phi}{2}}\right)\left(\frac{-k_e z e^2}{m_\alpha v^2 \sin^2\frac{\phi}{2}}\right) d\phi$

$$= -4\pi\left(\frac{k_e z e^2}{m_\alpha v^2}\right)^2 \frac{\cos\frac{\phi}{2}}{\sin^3\frac{\phi}{2}}\, d\phi$$

$$= -2\pi\left(\frac{k_e z e^2}{m_\alpha v^2}\right)^2 \frac{\sin\phi\, d\phi}{\sin^4\frac{\phi}{2}}$$

⇒ solid angle $d\Omega = \sin\phi\, d\theta\, d\phi = 2 s\frac{\phi}{2} c\frac{\phi}{2}\, d\theta\, d\phi$

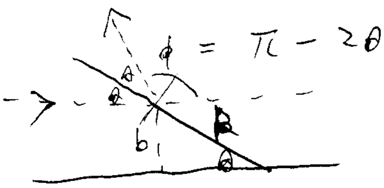


$\theta = (0, 2\pi) \Rightarrow d\Omega = 4\pi\, s\frac{\phi}{2} c\frac{\phi}{2}\, d\phi$

$$\Rightarrow \; d\sigma(\Omega) = \left(\frac{k e Z e^2}{m_\alpha V^2}\right)^2 \frac{d\Omega}{\sin^4\frac{\phi}{2}}$$

* should talk at this first

✩ a simple example: ~~infinit~~ hard sphere scatting (with $m=\infty$)



$$\sin\theta = \frac{b}{R}, \qquad \phi = \pi - 2\sin^{-1}\left(\frac{b}{R}\right)$$

or $\sin\left(\frac{\pi-\phi}{2}\right) = \frac{b}{R}$, $\quad b = R\sin\left(\frac{\pi-\phi}{2}\right)$

$$\Rightarrow \; db = -\frac{R}{2}\cos\left(\frac{\pi-\phi}{2}\right) d\phi$$

$$\begin{aligned} d\sigma &= 2\pi b\, db \\ &= 2\pi R\sin\left(\frac{\pi-\phi}{2}\right)\left(-\frac{R}{2}\right)\cos\left(\frac{\pi-\phi}{2}\right) d\phi \\ &= -\frac{\pi R^2}{2}\sin(\pi-\phi)\, d\phi = -\frac{\pi R^2}{2}\sin\phi\, d\phi \end{aligned}$$

$$\sigma = -\frac{\pi R^2}{2}\int_{\pi}^{0} d\phi \,\sin\phi = -\frac{\pi R^2}{2}(-\cos\phi)\Big|_{\pi}^{0} = \underline{\underline{\pi R^2}}$$

✩ Terminology:

Luminosity $\quad L \equiv f\,\frac{n_1 n_2}{A} \quad \frac{1}{cm^2}\,\frac{1}{sec}$ (f: frequency)

Event rate $\quad R(s) = \underset{\text{in } cm^2}{\sigma(s)} \times L$

$1\,cm^2 = 10^{24}$ barn
$= 10^{27}$ mbarn
$= 10^{33}$ nb
$= 10^{36}$ pb
$= 10^{39}$ fb

1 year $\sim \pi \times 10^7$ sec
$(365 \times 86400 = 3.1536 \times 10^7)$

對撞機, $\sim 10^7$ sec. accumarated Luminosity $\propto \frac{1}{A}$

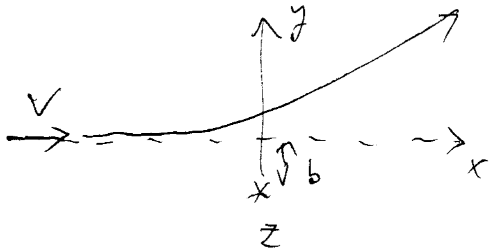
$(10^{33}\, cm^{-2} sec^{-1}) \times 1$ year $= 10\, (fb)^{-1}$

History of accelerator.

Cross section.

Rutherford Scattering.

α - particle was fired at the nuclear (charged Z) with <u>impact parameter</u> b



$$F = \frac{k\, Q_1 Q_2}{r^2}$$

$$\Delta P_y = F_y \Delta t$$

$$m_\alpha V_y \simeq k_e \underbrace{2}_{\text{2 for } \alpha} Z e^2 \int_{-\infty}^{\infty} dt \frac{b}{(x^2+b^2)^{\frac{3}{2}}} \qquad , \quad x \simeq vt$$

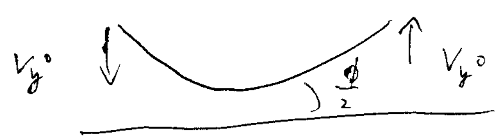
$$= 4k_e Z e^2 b \int_0^\infty \frac{dt}{(v^2t^2+b^2)^{\frac{3}{2}}} \qquad z = vt \ , \quad dt = \frac{dz}{v}$$

$$= \frac{4k_e Z e^2 b}{v} \int_0^\infty \frac{dz}{(z^2+b^2)^{\frac{3}{2}}} \qquad z = b \tan\theta = b\frac{s_\theta}{c_\theta} \qquad dz = \frac{b}{c_\theta^2} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{b\, d\theta}{\cancel{c_\theta^2}} \frac{c_\theta^{\cancel{3}\,1}}{b^3}$$

$$= \frac{4k_e Z e^2}{vb} \int_0^{\frac{\pi}{2}} d\theta\, c_\theta = \frac{4k_e Z e^2}{vb}$$

rotate the coordinate such that the configuration is symmetric.



$$m_\alpha \Delta V_y = \frac{4k_e Z e^2}{vb}$$

$$\text{or} \quad \tan\frac{\phi}{2} \sim \frac{V_y^0}{V_x = v} = \frac{2k_e Z e^2}{m_\alpha v^2 b}$$

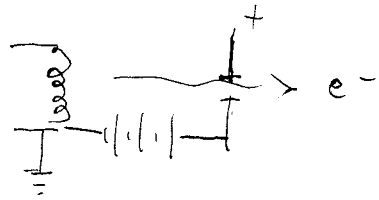
$$\text{or} \quad b \simeq \frac{2k_e Z e^2}{v^2 m_\alpha \tan\frac{\phi}{2}}$$

$$\Rightarrow \quad db \simeq \frac{2k_e Z e^2}{m_\alpha v^2}\left(-\frac{\frac{d\phi}{2}}{\sin^2\frac{\phi}{2}}\right) = \frac{-k_e Z e^2\, d\phi}{m_\alpha v^2 \sin^2\frac{\phi}{2}}$$

Source of probe

① 原子核分裂

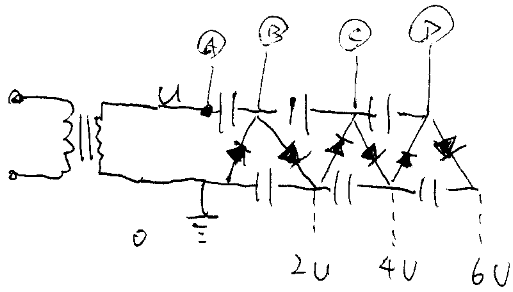
② 粒子加速器

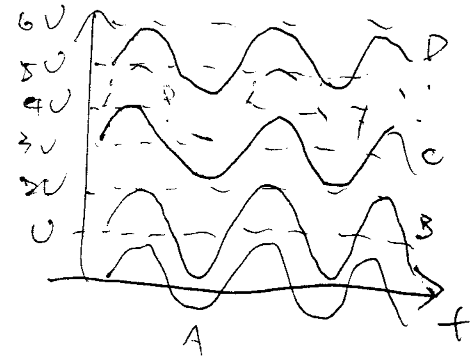


1932 1951 Nobel Prize

Cockcroft-Walton generator

~4MV
~100 mA
~μs

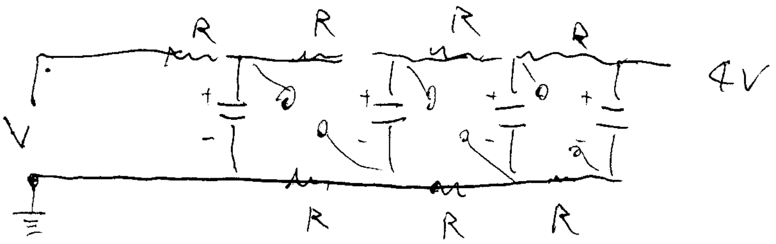




1930s

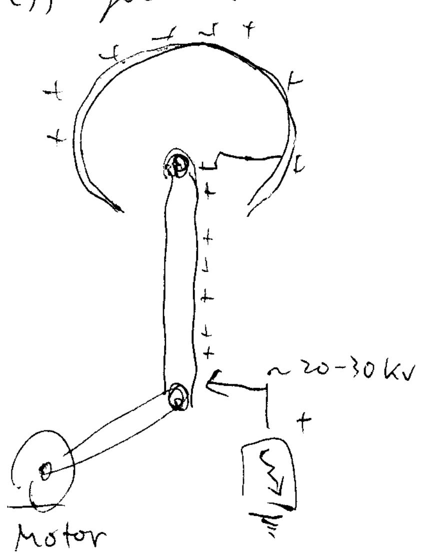
Marx generator

~6MV

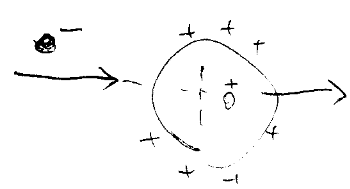


The Van de Graaff generator

~1930
~10MV

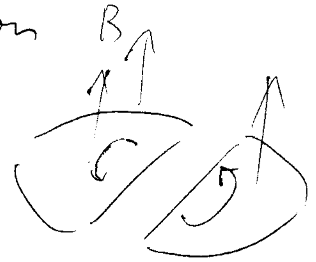


tandem 串级



~1930, Laurence

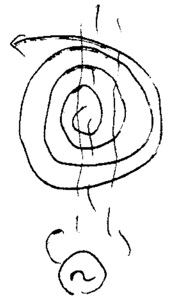
* cyclotron B



$$m r \omega^2 = q r \omega B \quad \text{classical.}$$

$$\Rightarrow \omega = \frac{qB}{m} \quad \Leftarrow \text{cyclotron frequency}$$

$$E = \frac{1}{2} m v^2 = \frac{1}{2} m r^2 \omega^2 \Rightarrow E \propto r^2$$

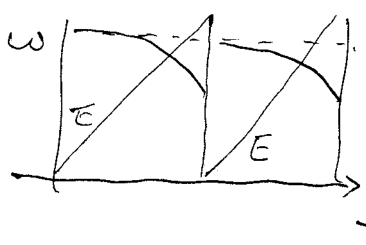


* problem: relativistic effect.

$$\gamma = \frac{E}{m_0}$$

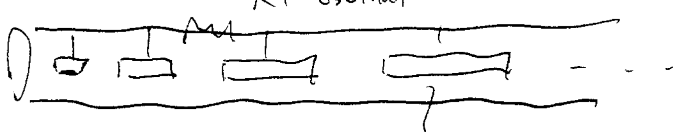
$$\omega = \frac{qB}{m}\sqrt{1-v^2}$$

$$\sim \frac{qB}{E}$$



* Linear accelerator.

UC ~1940s

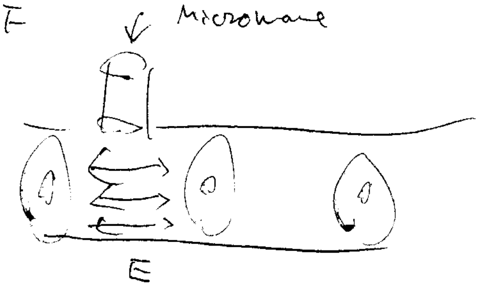


$$l \propto V \sim \sqrt{E}$$

* again, the relativistic effect come in.

$$P = \frac{m_0 V}{\sqrt{1-V^2}}, \quad V = \frac{P}{m_0} = \frac{\sqrt{E^2 - m_0^2}}{m_0} = \sqrt{\left(\frac{E}{m}\right)^2 - 1}, \quad V \sim c, \ V \sim \text{const.}$$

* RF



* Synchrotron $v \sim c$

$$\cancel{eR\omega B = mc^2}$$

$$R = \frac{E}{ecB} \qquad ecB = m\frac{c^2}{R} \sim E\frac{c^2}{R}$$

corentz force

$\Rightarrow$ const. R, vary B-field.

$\Rightarrow$ the $|B|$ must be increase synchronously with the energy E.

$\rightarrow$ synchrotron radiation $\Rightarrow$ energy lost.

for electron beam, now $E \sim 10$ GeV.

$\Rightarrow$ The present technology $B \lesssim 10$ Tesla

RF gradients $\leq 100 \frac{MeV}{m}$

GZK & cosmic ray

$[E][M] \sim \frac{1}{[L]}$

$\rightarrow \sigma \sim \frac{1}{Em}$

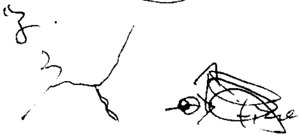
$\sim 1\#/cm^2$ ...

$\Phi_{\mu} \sim 10^4 \,\mu/m^2\, min$

$\omega \qquad \omega'_{\theta}$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2 \left(\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta\right)$$

compton scattering.



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* New idea ~~of~~ for acceleration.

* laser
* plasma wakefield.

Gradient $\sim 0.96\, \underline{n}^2$ (V/cm)

electron density

gas $\Rightarrow 1$ GV/cm

solid $\Rightarrow 100$ GV/cm

LBL $\rightarrow$ 1 GeV/3.3 cm $\Leftrightarrow$ SLAC 64 m

recently, SLAC, 42 GeV/85 cm

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930343	蕭子綱
931704	魏瑪娟
930302	李致宣
930362	黃正峰
930353	李映潔
930313	姚欣佳
930360	潘维平