Oct. 30, 2007

- Noether's theorem ~ 1 hr.
- mid-term ~ 2 hrs
- 收, 發 HW. ? cross section

Assume that a physical system which described by a Lagrangian and enjoys a symetry under following infinitesimal transformation:

$$q \to q' = q + \delta q$$
$$t \to t' = t + \delta t$$

where δq & δt are some kind of functions of q and t

By symetry we mean that the physics are the same before and after this transformation, therefore, the actions must satisfy:

$$S' = S + \text{const.}$$

or $$dt'\, L(q', \dot{q}', t') = dt\, L(q, \dot{q}, t) + \text{const.}$$

$$\dot{q}' \equiv \frac{dq'}{dt'} = \frac{d(q+\delta q)}{d(t+\delta t)} \simeq \frac{d(q+\delta q)}{dt\,(1+\delta\dot{t})} \sim \dot{q} + \delta\dot{q} - \dot{q}\,\delta\dot{t}$$

And the new action can be ~~Taylor~~ Taylor expanded to the first order of the infinitesimal transformation

$$\Rightarrow dt'\, L(q', \dot{q}', t') \simeq dt\,(1+\delta\dot{t})\left[L + \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}(\delta\dot{q} - \dot{q}\,\delta\dot{t}) + \frac{\partial L}{\partial t}\delta t\right]$$

$$= dt\left[L + L\,\delta\dot{t} + \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial t}\delta t + \frac{\partial L}{\partial \dot{q}}\delta\dot{q} - \frac{\partial L}{\partial \dot{q}}\dot{q}\,\delta\dot{t}\right]$$

$$= dt\Big[L + \frac{d}{dt}(L\,\delta t) - \frac{dL}{dt}\delta t + \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial t}\delta t$$
$$+ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\delta q\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\dot{q}\,\delta t\right) + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\dot{q}\right)\delta t\Big]$$

= 0, by E.O.M.

$$= dt\left\{ L + \frac{d}{dt}\left[\left(L - \frac{\partial L}{\partial \dot q}\dot q\right)\delta t + \frac{\partial L}{\partial \dot q}\delta q\right] + \delta t\left[\underbrace{\frac{\partial L}{\partial t} - \frac{dL}{dt}}_{\parallel} + \underbrace{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)\dot q}_{\parallel\,(E.O.M)} + \frac{\partial L}{\partial \dot q}\ddot q\right]\right\}$$

$$-\frac{\partial q}{\partial t}\frac{\partial L}{\partial q} - \frac{\partial \dot q}{\partial t}\frac{\partial L}{\partial \dot q} \quad + \frac{\partial L}{\partial q}\dot q \quad + \frac{\partial L}{\partial \dot q}\ddot q$$

cancel all

$$= dt\left\{ L + \frac{d}{dt}\left[\left(L - \frac{\partial L}{\partial \dot q}\dot q\right)\delta t + \frac{\partial L}{\partial \dot q}\delta q\right]\right\} = dt\,L + \underbrace{d\Omega}_{\text{total derivative}}$$

$$\Rightarrow \quad d\left[\underbrace{\left(L - \frac{\partial L}{\partial \dot q}\dot q\right)\delta t + \frac{\partial L}{\partial \dot q}\delta q}_{\Downarrow} - \Omega\right] = 0$$

or $\Downarrow$ is a ~~const.~~ consered quantity

or $L' = L + \underbrace{\frac{d\Omega}{dt}}$ itself satisfies ~~[illegible]~~ EOM

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translation: $\quad x \to x + \epsilon \qquad \delta q = \epsilon \;,\; \delta t = 0 .$

$$\Rightarrow \quad \frac{\partial L}{\partial \dot q} = \text{const} \quad \Rightarrow \text{momentum conservation}$$

time translation $\quad t \to t + \epsilon \qquad \delta q = 0 , \quad \delta t = \epsilon$

$$\Rightarrow \quad L - \frac{\partial L}{\partial \dot q}\dot q = H = \text{const} \quad \Rightarrow \text{energy conservation}$$

rotation alone the $\hat z$: $\quad x \to x - y\,\delta\theta \;, \quad \delta t = 0$

$\qquad y \to y + x\,\delta\theta$

$\qquad z \to z$

$$\Rightarrow \quad \frac{\partial L}{\partial \dot q}\delta q = \frac{\partial L}{\partial \dot x}(-y\,\delta\theta) + \left(\frac{\partial L}{\partial \dot y}\right)(x\,\delta\theta) + \left(\frac{\partial L}{\partial \dot z}\right)(0)$$

$L = \frac{1}{2}m\dot x^2 - V(x)$

$$= (-P_x y + P_y x)\,\delta\theta = \text{const} \quad \Rightarrow \quad L_z \text{ is conserved.}$$

Similarly, in special relativistic the Lagrangian for a point particle is
$$L = -m\sqrt{1-\dot{\vec{x}}^2}$$

translation

$$\vec{x} \to \vec{x} + \vec{\epsilon}, \qquad \delta\vec{x} = \vec{\epsilon}$$

$$\left(\frac{\partial L}{\partial \dot{x}}\right)\delta x = \frac{m\dot{\vec{x}}}{\sqrt{1-\dot{\vec{x}}^2}}\cdot\vec{\epsilon} \quad \text{is a const.} \Rightarrow \frac{m\dot{\vec{x}}}{\sqrt{1-\dot{\vec{x}}^2}} \quad \text{conserved}$$

$$t \to t + \epsilon \qquad \delta t = \epsilon$$

$$\left(L - \left(\frac{\partial L}{\partial \dot{x}}\right)\dot{x}\right)\delta t = \left(-m\sqrt{1-\dot{\vec{x}}^2} - \frac{m\dot{\vec{x}}^2}{\sqrt{1-\dot{\vec{x}}^2}}\right)\epsilon = \frac{m\,\epsilon}{\sqrt{1-\dot{\vec{x}}^2}} \Rightarrow \frac{m}{\sqrt{1-\dot{\vec{x}}^2}} \quad \text{conserved}$$

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