Int. to HEP - I

Nov. 20, 2007

* Isospin analysis   HW! 4: 26, 27, 29, 37, 39

* Gell-Mann - Nishijima formula $Q = I_3 + \frac{1}{2}(B + S)$ (B: baryon, S: strangeness)

* Discrete symmetry : P, C, CP, CPT

	Q	mass
P	+1	938.28 MeV
n	0	939.57 MeV

Proposed by Heisenberg in 1932
⇒ 2 states of a single nucleon
(if we ignore their difference in charge & mass, especially in strong interaction)

$N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, $p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ just like the 2-component spinor discussed

⇒ isospin, conventionally denoted as I, and I_3 for the 3rd component
(note!! it doesn't have dimension, so there is no ħ associated with)

$p = |\frac{1}{2}\ \frac{1}{2}\rangle$, $n = |\frac{1}{2}\ -\frac{1}{2}\rangle$

~~Heisenberg proposed that~~

⇒ **Strong interaction preserved isosymmetry!!**

* not so in,
E&M $\pi^0 \to 2\gamma$
weak $\Lambda \to p + \pi^-$

by Noether's theorem, the isospin is conserved in all strong int.
It's a very good symmetry for light hadrons

SU(2). nucleons are the 2-dim representation.

πs are the 3-dim representation $I = 0$, $\Lambda = |0\,0\rangle$

$I = 1$ $\pi^+ = |1\,1\rangle$ $\pi^0 = |1\,0\rangle$, $\pi^- = |1, -1\rangle$

	π^+	π^0	π^-
Q	+1	0	−1
mass	139.57	134.98	139.57

$I = \frac{3}{2}$ $\Delta^{++} = |\frac{3}{2}\ \frac{3}{2}\rangle$ $\Delta^{+} = |\frac{3}{2}\ \frac{1}{2}\rangle$ $\Delta^{0} = |\frac{3}{2}\ -\frac{1}{2}\rangle$ $\Delta^{-} = |\frac{3}{2}\ -\frac{3}{2}\rangle$

	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}
Q	+2	+1	0	−1

mass ~1232 − 1233

and so on

* multiplicity $= 2I+1$
* How to assign I_3? by charge Q.

$$Q = I_3 + \frac{1}{2}(B + S)$$ Gell-Mann - Nishijima formula

purely empirical ~

but from quark model, it's trivial

$u = |\frac{1}{2}\ \frac{1}{2}\rangle$, $d = |\frac{1}{2}\ -\frac{1}{2}\rangle$, $s = |0, 0\rangle$

	u	d	s	
I_3	$+\frac{1}{2}$	$-\frac{1}{2}$	0	
S	0	0	−1	⇐ by charge.
Q	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	

$Q = \frac{1}{3}(2u - d + s)$

$B = \frac{1}{3}(u + d + s)$

$I_3 = \frac{1}{2}(u - d)$

$3B + S = u + d$

$2I_3 = u - d$

$u = \frac{1}{2}(2I_3 + 3B + S)$

$d = \frac{1}{2}(3B + S - 2I_3)$

$$\Rightarrow Q = \frac{1}{3}\left(2I_3 + 3B + S - \frac{3}{2}B - \frac{S}{2} + I_3 + S\right)$$

$$= I_3 + \frac{1}{2}(B + S)$$

	c	b	t
I_3	0	0	0
Q	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
c	+1	0	0
b	0	−1	0
t	0	0	+1

⇒ generalized GMN formula

HW 4.26

* isospin does more than classification, it's also dynamical symmetry formed by 2 nucleons:

$|1\,1\rangle = pp$

$|1\,0\rangle = \frac{1}{\sqrt{2}}(pn + np)$

$|1\,-1\rangle = nn$

isotriplet

$|0\,0\rangle = \frac{1}{\sqrt{2}}(pn - np)$

isosinglet, experimentally this is deuteron

(∵ there is no bound state of pp & nn)

* considering the nucleon - nucleon scattering.

(a) $p + p \to d + \pi^+$

$|\tfrac12 \tfrac12\rangle\ |\tfrac12 \tfrac12\rangle \quad |00\rangle \quad |11\rangle$

$= |11\rangle \longrightarrow$

(b) $p + n \to d + \pi^0$

$|\tfrac12 \tfrac12\rangle |\tfrac12 -\tfrac12\rangle \quad |00\rangle \quad |10\rangle$

$= \frac{1}{\sqrt2}(|10\rangle + |00\rangle)$

(c) $n + n \to d + \pi^-$

$|\tfrac12 -\tfrac12\rangle |\tfrac12 -\tfrac12\rangle \quad |00\rangle \quad |1-1\rangle$

$= |1-1\rangle \longrightarrow$

$M_a : M_b : M_c = 1 : \frac{1}{\sqrt2} : 1$

$\Rightarrow \sigma_a : \sigma_b : \sigma_c = 2 : 1 : 2$

$M \propto \langle f|i\rangle, \quad \sigma \propto |M|^2$

Has been checked by exp.

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* $\pi N \to \pi N$ pion - nucleon scattering

elastic processes:

$\pi: I = 1, \quad N: I = \tfrac12, \quad \Rightarrow I = \tfrac32, \ = \tfrac12$ } 2 channels

a $\pi^+ + p \to \pi^+ + p \qquad |\tfrac12 \tfrac12\rangle |11\rangle = |\tfrac32 \tfrac32\rangle \quad \sim M_{\frac32}$

b $\pi^0 + p \to \pi^0 + p \qquad |\tfrac12 \tfrac12\rangle |10\rangle = \sqrt{\tfrac23}\,|\tfrac32 \tfrac12\rangle - \sqrt{\tfrac13}\,|\tfrac12 \tfrac12\rangle$

c $\pi^- + p \to \pi^- + p \qquad |\tfrac12 \tfrac12\rangle |1-1\rangle = \sqrt{\tfrac13}\,|\tfrac32 -\tfrac12\rangle - \sqrt{\tfrac23}\,|\tfrac12 -\tfrac12\rangle$

d $\pi^+ + n \to \pi^+ + n \qquad |\tfrac12 -\tfrac12\rangle |11\rangle = \sqrt{\tfrac13}\,|\tfrac32 \tfrac12\rangle + \sqrt{\tfrac23}\,|\tfrac12 \tfrac12\rangle$

e $\pi^0 + n \to \pi^0 + n \qquad |\tfrac12 -\tfrac12\rangle |10\rangle = \sqrt{\tfrac23}\,|\tfrac32 -\tfrac12\rangle + \sqrt{\tfrac13}\,|\tfrac12 -\tfrac12\rangle$

f $\pi^- + n \to \pi^- + n \qquad |\tfrac12 -\tfrac12\rangle |1-1\rangle = |\tfrac32 -\tfrac32\rangle \quad \sim M_{\frac32}$

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charge - exchange

			$\frac32$	$\frac12$			$\frac32$	$\frac12$
g	$\pi^+ + n \to \pi^0 + p$	$|\tfrac12 -\tfrac12\rangle |11\rangle =$	$\sqrt{\tfrac13}$	$\sqrt{\tfrac23}$	$\longrightarrow$	$|\tfrac12 \tfrac12\rangle |10\rangle =$	$\sqrt{\tfrac23}$	$-\sqrt{\tfrac13}$
h	$\pi^0 + p \to \pi^+ + n$	$|\tfrac12 \tfrac12\rangle |10\rangle =$	$\sqrt{\tfrac23}$	$-\sqrt{\tfrac13}$	$\longrightarrow$	$|\tfrac12 -\tfrac12\rangle |11\rangle =$	$\sqrt{\tfrac13}$	$\sqrt{\tfrac23}$
i	$\pi^0 + n \to \pi^- + p$	$|\tfrac12 -\tfrac12\rangle |10\rangle =$	$\sqrt{\tfrac23}$	$\sqrt{\tfrac13}$	$\longrightarrow$	$|\tfrac12 \tfrac12\rangle |1-1\rangle =$	$\sqrt{\tfrac13}$	$-\sqrt{\tfrac23}$
j	$\pi^- + p \to \pi^0 + n$	$|\tfrac12 \tfrac12\rangle |1-1\rangle =$	$\sqrt{\tfrac13}$	$-\sqrt{\tfrac23}$	$\longrightarrow$	$|\tfrac12 -\tfrac12\rangle |10\rangle =$	$\sqrt{\tfrac23}$	$\sqrt{\tfrac13}$

$\Rightarrow \sigma_a : \sigma_c : \sigma_j = 9|M_{\frac32}|^2 :$ ~~$|M_{\frac32} + 2M_{\frac12}|$: $4M_{\frac32} - M_{\frac12}$~~

$: |M_{\frac32} + 2M_{\frac12}|^2 : 2|M_{\frac32} - M_{\frac12}|^2$

At a CM energy = 1232 MeV, (Fermi 1951), Δ resonance $I = \frac{3}{2}$,

$$M_{\frac{3}{2}} >> M_{\frac{1}{2}} \Rightarrow$$

$$\sigma_a : \sigma_c : \sigma_j = 9 : 1 : 2$$

$\pi^+ p \to \pi^+ p \quad \pi^- p \to \pi^- p \quad \pi^- p \to \pi^0 n$

Experimentally, it's easier to prepare the same initial state & measure the total crossection..

$$\frac{\sigma_{tot}(\pi^+ p) = \sigma_a}{\sigma_{total}(\pi^- p) = \sigma_c + \sigma_j} = \frac{9}{1+2} = 3 \quad \Rightarrow \text{see fig 4.6}$$

In late 1950s → eight-Baryon problem

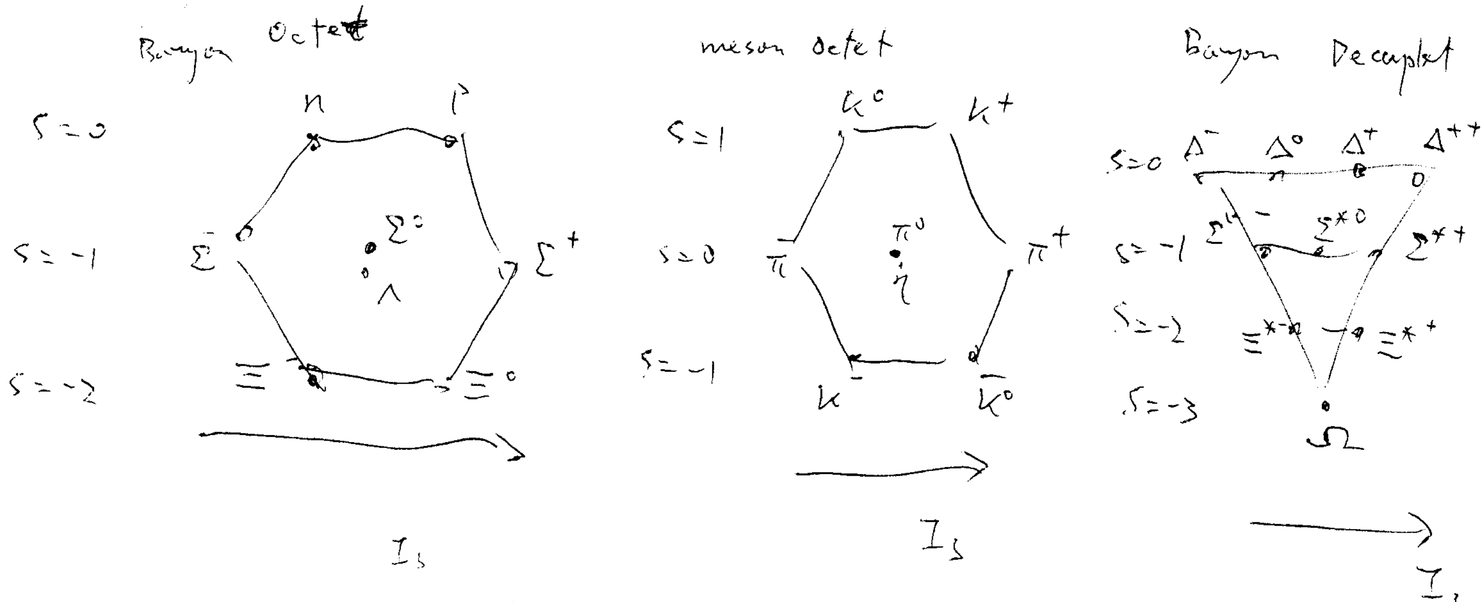
n, p, Λ, Σ's, Ξ's

~940, 1115, 1190 ⟶ 1320 } supermultiplet ⊃ SU(2) iso

Gell-Man → Eightfold way SU(3) (globe) flavor symmetry

not only SU(3) octets, he also predicted decuplet 10-dim rep.

$$SU(3) \begin{matrix} u \\ d \\ s \end{matrix} \to SU(2) \begin{pmatrix} u \\ d \end{pmatrix} + s$$



SU(4), SU(5), SU(6) ?? quark mass hierarchy.

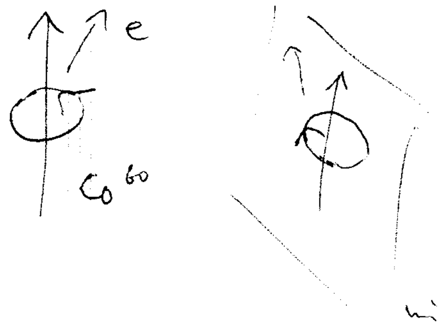
$\theta^+ \to \pi^+ + \pi^0 \quad (\mathbb{P} = +1)$

$\tau^+ \to 3\pi \quad (\mathbb{P} = -1)$

~~$\mathbb{P}$~~

1956 Lee & Yang ⇒ no exp check the $\mathbb{P}$ in weak int.

C. S. Wu.



$\mathbb{P}^2 = I \;\to \lambda = \pm 1$

$\mathbb{P}$

Scalar	$s \leftrightarrow s$
vector	$v \leftrightarrow -v$
pseudo scalar	$p \leftrightarrow -p$
axial vector	$v \leftrightarrow v$

for ~~[illegible]~~ fermions
particle $\mathbb{P}$ +1
anti-particle ⇒ $\mathbb{P} = -1$

maximal $\mathbb{P}$ in SM

⇒ left-handed ν

right-handed $\bar{\nu}$

* helicity $h \equiv \sigma \cdot \hat{p} = \pm 1$

  is not Lorentz invariant

* only for massless particle handness = helicity

*

$\pi^- \to \mu^- + \bar{\nu}_\mu$

can be checked by exp

* for $\pi^0 \to \gamma\gamma$, either LH or RH. (none)

* parity of a composite system in its ground state is the "product" of the parities of its constituents. ⇒ Baryons have intrinsic parity $(+1)^3 = +1$

  Mesons $(+1)(-1) = -1$ $\Big\} (-1)^{\ell+1}$

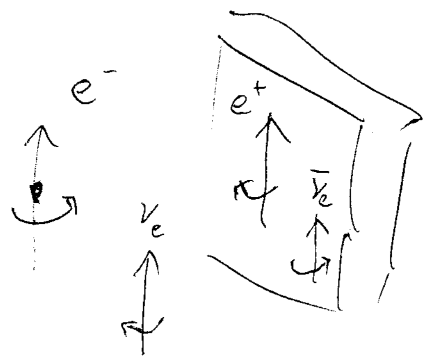
* $\not{CP}$

weak interaction are not invariant under $\mathbb{P}$ & $\mathbb{C}$

$\pi^+ \to (\mu^+)_L + (\nu_\mu)_L \xrightarrow{\mathbb{P}} \pi^+ \to (\mu^+)_R + (\nu_\mu)_R$ ← no such thing $\xrightarrow{\mathbb{C}} \pi^- \to (\mu)_R + (\bar{\nu}_\mu)_R$ O.K.

$\pi^+ \to (\mu^+)_L + (\nu_\mu)_L \xrightarrow{\mathbb{C}} \pi^- \to (\mu)_L + (\bar{\nu}_\mu)_L$ ← no such thing $\xrightarrow{\mathbb{P}} \pi^- \to (\mu)_R + (\bar{\nu}_\mu)_R$

Maybe we are talking about a very bizzare mirror?



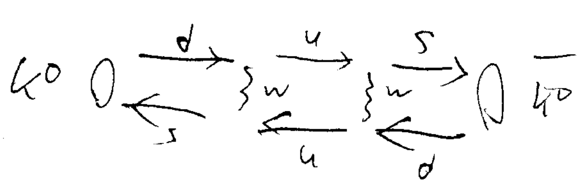
first pointed out by Landau.

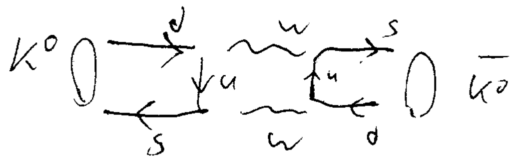
* It looks like ~~See~~ again ν plays a very important role ~~for~~ $\not{\mathbb{P}}$ & $\not{\mathbb{C}}$

However, now we know there are 2 kinds of electrons!!

* $K^0 - \bar{K}^0$ mixing

First pointed out by Gell-Mann & Pais.

$\Rightarrow$ 

or $\Rightarrow$ 

$$\mathbb{P} = (-)^{\ell+1}$$

Because K^0 & $\bar{K}^0$ are pseudo-scalar

$$\mathbb{P}|K^0\rangle = -|K^0\rangle, \quad \mathbb{P}|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

and $\mathbb{C}|K^0\rangle = |\bar{K}^0\rangle, \quad \mathbb{C}|\bar{K}^0\rangle = |K^0\rangle$ particle $\leftrightarrow$ antiparticle

$$\Rightarrow \mathbb{CP}|K^0\rangle = -|\bar{K}^0\rangle, \quad \mathbb{CP}|\bar{K}^0\rangle = -|K^0\rangle$$

therefore we can construct 2 eigenstates of $\mathbb{CP}$

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \quad \& \quad |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

such that
$$\mathbb{CP}|K_1\rangle = +|K_1\rangle$$
$$\mathbb{CP}|K_2\rangle = -|K_2\rangle$$

$$\mathbb{P}|\pi\rangle = -|\pi\rangle, \qquad \mathbb{C} = (-)^{\ell+s} \qquad \ell_\pi = 0, \; s_\pi = 0$$
$$\mathbb{C}|\pi^\pm\rangle = |\pi^\mp\rangle, \quad \mathbb{C}|\pi^0\rangle = |\pi^0\rangle$$

$$\mathbb{CP}|\pi^0\rangle = -|\pi^0\rangle, \quad \mathbb{CP}|\pi^\pm\rangle = -|\pi^\mp\rangle$$

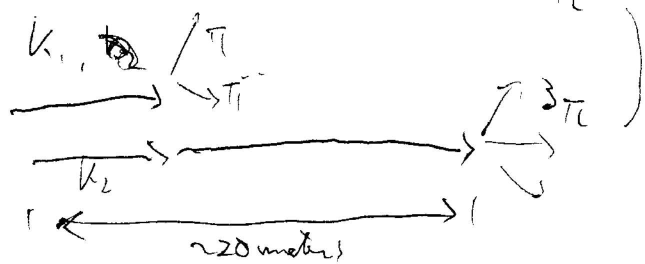
$$\mathbb{CP}\left(\pi^{0\,n}(\pi^+\pi^-)^{m}\right) = (-1)^{n+m+p} = (-)^{\#\pi}$$

if CP is good: $|K_1\rangle \rightarrow$ even # πs mainly 2π $\quad \tau_1 = 0.89\times10^{-10}$ sec ~ few cm

$|K_2\rangle \rightarrow$ odd # πs 3π $\quad \tau_2 = 5.2\times10^{-8}$ sec ~ few meter

K_1, K_2 are their own antiparticle

$$\mathbb{C}|K_1\rangle = -|K_1\rangle, \quad \mathbb{C}|K_2\rangle = +|K_2\rangle \qquad \Delta m = m_2 - m_1 = 3.5\times10^{-6}\ \text{eV} \checkmark$$



2π: $\dfrac{\#\ 45}{\#\ 22700}$

1964
Cronin & Fitch

$\nearrow$ indirect $\not{CP}$, direct $\not{CP}$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_2\rangle + \epsilon|K_1\rangle\right)$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(-\epsilon|K_2\rangle + |K_1\rangle\right)$$

$|K_2\rangle \to 2\pi$ ϵ'

$|K_1\rangle \to 3\pi$

$\epsilon \sim 10^{-3}$

EDM, B meson system
triple correlation

Similarly $\not{CP}$ assymmetry can be observed in K semi-leptonic decay

$$K_L \to \pi^+ + e^- + (\bar{\nu}_e)_R \quad B_1$$

$$\uparrow\downarrow \; CP$$

$$\to \pi^- + e^+ + (\nu_e)_L \quad B_2$$

if CP is good $B_1 = B_2$

However $\frac{B_2 - B_1}{B} \sim 10^{-3}$

☆ with $\not{CP}$, we got a unique way to define what is positive charge.

So if one day we have in contact with E.T. we can check with each other to see whether we both are made of particle

☆ $\not{CP}$ also play an important role in explaining our existence.

Big Bang → hot soup of particle - anti particle.
→ nothing left.

Time reversal & TCP theorem

$T: \; t \leftrightarrow -t$

Rewind the video tape usually look weired.
but we believe in the most fundamental level interaction shall be symmetric in both time directions.

Experimentally, it's very hard to do the test.

Schwinger & Lüders, & Pauli.

TCP theorem: Bose-Einstein

local, QFT, Fermi-Dirac statistics

Lorentz invariant.

$\not{T} \Leftrightarrow \not{CP}$

Every particle must have the same mass, and life time as its antiparticle.

(g_2) ... charge

$\Rightarrow$ $K^0 - \bar{K}^0$ mass difference $< 6\times10^{-15}$ $\mu^+ - e^-$ $< 10^{-13}$

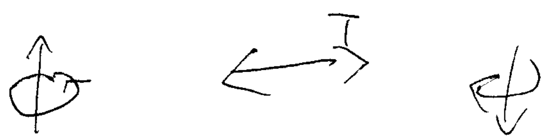
Mass $e^- - e^+$ $< 4\times10^{-8}$, $\Lambda - \bar{\Lambda}$ $< 10^{-6}$, $Q_{p-\bar{p}} < 10^{-5}$

ν, anti-hydrogen.

$d_n < 10^{-25}$ e-cm

$d_e < 10^{-27}$ e-cm

EDM



flip sign.

		T	P
position	r	r	$-r$
momentum	p	$-p$	$-p$
spin	σ, $r\times p$	$-\sigma$	σ
$\vec{E}$	$(=-\nabla\phi)$	$\vec{E}$	$-\vec{E}$
$\vec{B}$	$(=\nabla\times\vec{A})$	$-B$	B
$\vec{\sigma}\cdot\vec{B}$		$\vec{\sigma}\cdot\vec{B}$	$\vec{\sigma}\cdot\vec{B}$
$\vec{\sigma}\cdot\vec{E}$		$-\vec{\sigma}\cdot\vec{E}$	$-\vec{\sigma}\cdot\vec{E}$
$\vec{\sigma}\cdot\vec{p}$		$\sigma\cdot p$	$-\sigma\cdot p$
$\sigma\cdot(p_1\times p_2)$		$-\sigma\cdot(p_1\times p_2)$	$+\sigma\cdot(p_1\times p_2)$

930314 王繼文

930308 林慧婷

930313 姚欣佑

930344 王楨斌

930336 陳柏榮

930349 李品柔

930360 潘紀年

912118 陳[illegible]

930353 李映潔

930343 蕭子綱

930302 李政憲

931704 魏瑪娟

921027 張凱維

930362 黃志峰