Dec. 4, 2007
Int. to HEP-I

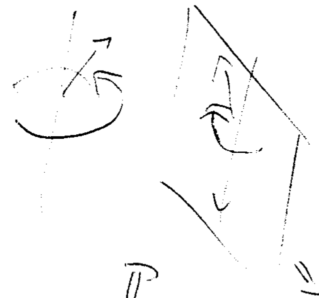
* Parity & mirror
* Hydrogen atom : fine structure.
* positronium

$\mathbb{P}: (x, y, z) \Leftrightarrow (-x, -y, -z)$

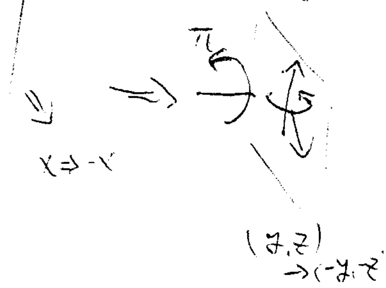
in front of mirror : $(x, y, z) \longleftrightarrow (-x, +y, +z)$

↪ need an extra rotation along x-axis $\hat{\pi}$ to bring it to $(-y, -z)$

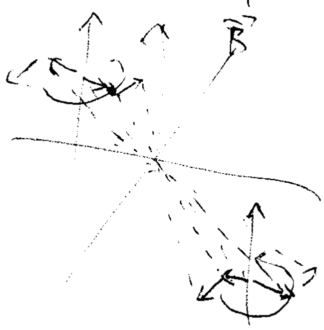
C.S. Wu's Co^{60} expt:



← this diagram is somehow misleading

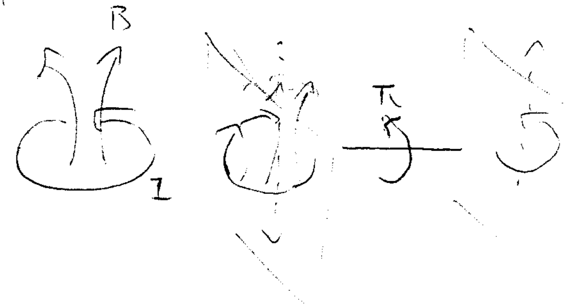


$\sigma \propto r \times p \xrightarrow{\mathbb{P}}$	$+\sigma$
$\vec{p}$	$-\vec{p}$
$\sigma \cdot \vec{p}$	$-\sigma \cdot \vec{p}$
$\vec{B}$	$\vec{B}$



→ still get the same σ

$\vec{B}$



$B = \nabla \times A$

$\vec{E}$

$\Rightarrow \vec{E} \Rightarrow -\vec{E}$

1935 under these

Hellmann - Feynman thm. 1939

$$E_n|\psi_n\rangle = H|\psi_n\rangle$$

$$\underbrace{\left(\frac{\partial}{\partial\lambda}E_n\right)}_{c\text{-}\#}|\psi_n\rangle + E_n\frac{\partial}{\partial\lambda}|\psi_n\rangle = \frac{\partial H}{\partial\lambda}|\psi_n\rangle + H\frac{\partial}{\partial\lambda}|\psi_n\rangle$$

$$\frac{\partial}{\partial\lambda}E_n + \cancel{E_n\langle\psi_n|\frac{\partial}{\partial\lambda}|\psi_n\rangle} = \langle\psi_n|\frac{\partial H}{\partial\lambda}|\psi_n\rangle + \cancel{E_n\langle\psi_n|\frac{\partial}{\partial\lambda}|\psi_n\rangle}$$

$$\frac{dE_n}{d\lambda} = \langle\psi_n|\frac{\partial\hat{H}}{\partial\lambda}|\psi_n\rangle$$

$$E_n = -\frac{me^4}{2\hbar^2(n_r+l+1)^2} \qquad n = n_r + l + 1 \qquad a_0 = \frac{\hbar^2}{me^2} \simeq 0.529\,\text{Å}$$

or $E_n = -\alpha^2 mc^2\left(\frac{1}{2n^2}\right)$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2} - \frac{e^2}{r}$$

$$= -\frac{\hbar^2}{2m}\left(\frac{2}{r}\frac{d}{dr} + \frac{d^2}{dr^2}\right) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2} - \frac{e^2}{r}$$

$$\frac{\partial E}{\partial e} = -\frac{4me^3}{2\hbar^2 n^2} \qquad \frac{\partial\hat{H}}{\partial e} = -\frac{2e}{r} \quad\Rightarrow\quad \left\langle\frac{1}{r}\right\rangle = \frac{me^2}{\hbar^2 n^2} = \frac{1}{a_0 n^2}$$

$$\frac{\partial E}{\partial l} = +\frac{me^4}{\hbar^2 n^3}, \quad \frac{\partial\hat{H}}{\partial l} = \frac{\hbar^2}{2m}\frac{(2l+1)}{r^2} \quad\Rightarrow\quad \left\langle\frac{1}{r^2}\right\rangle = \frac{2m}{(2l+1)\hbar^2}\frac{me^4}{\hbar^2 n^3} = \frac{2}{a_0^2 n^3(2l+1)}$$

for any eigenfunction

$$\langle\psi_n|\left[\frac{d}{dr},\hat{H}\right]|\psi_n\rangle = \langle\psi_n|\left(\frac{d}{dr}\hat{H} - \hat{H}\frac{d}{dr}\right)|\psi_n\rangle = E_n\langle\psi_n|\left(\frac{d}{dr}-\frac{d}{dr}\right)|\psi_n\rangle = 0$$

$$\left[\frac{d}{dr},\hat{H}\right] = -\frac{\hbar^2}{2m}\left(-\frac{2}{r^2}\frac{d}{dr}\right) - \frac{2\hbar^2}{2m}\frac{l(l+1)}{r^3} + \frac{e^2}{r^2}$$

So,
$$0 = \frac{\hbar^2}{m}\langle\psi_n|\frac{1}{r^2}\frac{d}{dr}|\psi_n\rangle - \frac{\hbar^2}{m}l(l+1)\left\langle\frac{1}{r^3}\right\rangle + e^2\left\langle\frac{1}{r^2}\right\rangle$$

or
$$0 = \left\langle\frac{1}{r^2}\frac{d}{dr}\right\rangle - l(l+1)\left\langle\frac{1}{r^3}\right\rangle + \frac{1}{a_0}\left\langle\frac{1}{r^2}\right\rangle$$

$$\left\langle\frac{1}{r^2}\frac{d}{dr}\right\rangle = \int_0^\infty\int d\Omega\, r^2 dr\, \psi^*\frac{1}{r^2}\frac{d}{dr}\psi = \int_0^\infty dr\int d\Omega\, \psi^*\frac{d}{dr}\psi$$

$\psi = N\, Y_{lm}(\cdot\cdot)\, R$

$$= \int_0^\infty dr\, \underbrace{R^*_{nl}\frac{d}{dr}}_{\shortparallel} R_{nl}$$

$$= \int_0^\infty dr\,\frac{1}{2}\frac{d}{dr}(R^*_{nl}R_{nl}) = -\frac{1}{2}|R(0)|^2$$

$$\Rightarrow \left\langle\frac{1}{r^3}\right\rangle = \frac{1}{l(l+1)}\left(-\frac{1}{2}|\psi_{nlm}(0)|^2 + \frac{1}{a_0^3 n^3(l+\frac{1}{2})}\right)$$

for $l \neq 0$, $|\psi_{nlm}(0)|^2 = 0$, $\left\langle\frac{1}{r^3}\right\rangle = \frac{1}{a_0^3 n^3 l(l+\frac{1}{2})(l+1)}$

$l = 0$, div.

γ Relativistic correction

$$T = \sqrt{p^2c^2 + m^2c^4} - mc^2 = \frac{p^2}{2m} - \frac{p^4}{8m^3c^2} + \cdots$$

$$\Delta H_{rel} = -\left(\frac{1}{8m^3c^2}\right)p^4$$

$$\frac{p^2}{2m} = E_n - V, \quad \Rightarrow \quad \Delta H_{rel} = -\frac{1}{2mc^2}(E_n - V)^2$$

$$a_0 = \frac{\hbar^2}{me^2}$$

$$\Rightarrow \langle \Delta H_{rel} \rangle = -\frac{1}{2mc^2}\left(E_n^2 - 2E_n\langle V\rangle + \langle V^2\rangle\right)$$

$$E_n = -\frac{me^4}{2\hbar^2n^2} = -\frac{e^2}{2a_0n^2}$$

$$= -\frac{1}{2mc^2}\left(\left(\frac{me^4}{2\hbar^2n^2}\right)^2 + 2\frac{me^4e^2}{\hbar^2n^2}\left\langle\frac{1}{r}\right\rangle + e^4\left\langle\frac{1}{r^2}\right\rangle\right)$$

$$= -\frac{1}{2mc^2}\left\{\frac{e^4}{4a_0^2n^4} + \frac{e^4}{a_0n^2}\frac{1}{a_0n^2} + \frac{2e^4}{a_0^2n^3(2l+1)}\right\}$$

$$= -\frac{e^4}{2mc^2a_0n^2}\left\{\frac{1}{4n^2} + \frac{1}{n} + \frac{2}{n(2l+1)}\right\}$$

$$E_n = -\alpha^2\frac{1}{2n^2}$$

$$= -\frac{1}{2mc^2}\left(\left(-\frac{e^2}{2a_0n^2}\right)^2 - 2\left(\frac{-e^2}{2a_0n^2}\right)\frac{-e^2}{n^2a_0} + \frac{e^4}{n^3a^2(l+\frac{1}{2})}\right)$$

$$\boxed{\alpha^2mc^2 = \frac{e^2}{a}}$$

$$= -\frac{e^4}{2mc^2n^2a_0^2}\left[\frac{1}{4n^2} - \frac{1}{n^2} + \frac{1}{n(l+\frac{1}{2})}\right]$$

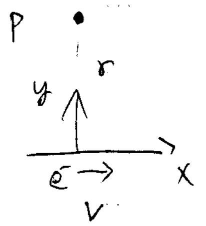
$$\frac{1}{a} = \frac{\alpha^2mc^2}{e^2}$$

$$\alpha \equiv \frac{e^2}{\hbar c} \qquad = -\frac{e^4}{4mc^2a_0^2n^4}\left(\frac{2n}{l+\frac{1}{2}} - \frac{3}{2}\right)$$

$$= -\frac{\alpha^4m^2c^4}{4mc^2n^4}\left(\frac{2n}{l+\frac{1}{2}} - \frac{3}{2}\right)$$

$$= -\frac{\alpha^4mc^2}{4n^4}\left(\frac{2n}{l+\frac{1}{2}} - \frac{3}{2}\right)$$

Spin - Orbit Coupling



$E_y = -\frac{e}{r^2}\hat{y}$ @ proton's rest frame.

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ & 0 & -B_z & B_y \\ & & 0 & -B_x \\ & & & 0 \end{pmatrix} = \begin{pmatrix} 0, & 0, & +\frac{e}{r^2}, & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{e}{r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

To electron's frame, proton move backward

$$\Lambda = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{pmatrix} \gamma & -\gamma\beta & 0 & \\ -\gamma\beta & \gamma & 0 & \\ 0 & 0 & 1 & \\ & & & 1 \end{pmatrix}$$

we are interested in B_z'

$$B_z' = F^{21}_{(e)} = \Lambda^2{}_\alpha \Lambda^1{}_\beta F^{\alpha\beta}_{(p)} = \Lambda^1{}_\beta F^{2\beta}_{(p)} = \Lambda^1{}_0 F^{20} = +\gamma\beta\frac{e}{r^2} \;\hat{=}\; \frac{v}{c}E$$

A magnetic field $\boxed{B \sim \frac{ve}{cr^2}\hat{z}}$ is seen @ electron's frame.
$\sim \frac{v}{c}E$

$\because$ $L = rmv$ or $v = \frac{L}{rm}$

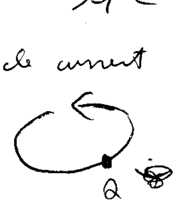
$$\Rightarrow \boxed{\vec{B} = \frac{e}{cmr^3}\vec{L}}$$

for a spinning ~~object~~ particle.

$$\vec{\mu} = -\frac{ge}{2mc}\vec{S}$$

$\vec{\mu} = \int \vec{r}' \times \vec{j}(\vec{r}')\, dr'^3$ $= I \cdot \pi r^2 = \frac{Q}{2m}\hat{L}$

a circle current



@ $L = mrv$

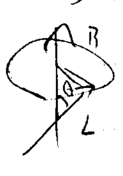
$I = Qf = Q\frac{v}{2\pi r}$

$= \frac{QL}{2\pi m r^2}$

$g = 2$. from Dirac equation.

$$\Delta H = -\vec{\mu}\cdot\vec{B} = \frac{e^2}{m^2c^2r^3}(\vec{L}\cdot\vec{S})$$

Larmor frequency

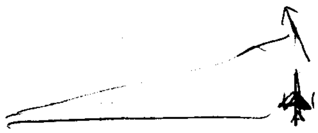


$$\vec{\tau} = \mu \times B = \frac{d\vec{L}}{dt} = \frac{\omega\, dt\, L\sin\theta}{dt} = \omega_L L\sin\theta = \mu B\sin\theta$$

$$\Rightarrow \omega_L = \frac{\mu B}{L} = +\frac{e\,L\,B}{2mc\,L} = \frac{eB}{mc} = \frac{ev}{mc^2}E$$

Thomas precession: (Wigner rotation)

↳ This is a purely relativistical kinematic effect.
Also apply to the sattelite moving around the earth



$$\epsilon = \frac{2\pi}{N} = \tan^{-1}\frac{L_\perp}{L_\parallel}$$

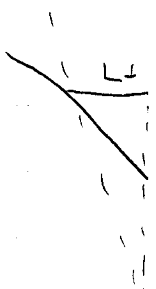
$$\tan\epsilon = \frac{L_\perp}{L_\parallel}, \quad \epsilon \sim \frac{L_\perp}{L_\parallel}$$

After N steps, the plane complltes a circle.

$$N\cdot\epsilon = 2\pi.$$

However, as seen in plane's prospect.

$L_\parallel \to \frac{1}{\gamma}L_\parallel$ or the angle it really needs to turn is. : $\epsilon_a \simeq \frac{L_\perp}{L_\parallel^{(a)}} \sim \gamma\epsilon$



So every infinitisimal steps.
to the rest frame, the plane. miss

$$\delta\epsilon \sim -(\gamma-1)\epsilon$$

After N steps. it has missed

$$N\,\delta\epsilon = -(\gamma-1)N\epsilon = -(\gamma-1)2\pi$$

$$= -\left(\frac{1}{\sqrt{1-\beta^2}}-1\right)2\pi \sim -\frac{\beta^2}{2}(2\pi)$$

$$\Rightarrow \omega_T = -\frac{\beta^2}{2}\omega_0$$

↳ 由轉一圈的時間決定

$$\omega_0 = \frac{2\pi v}{2\pi r} = \frac{v}{r}, \quad eE = \frac{e^2}{r^2} = \frac{mv^2}{r} \text{ or, } \frac{1}{r} = \frac{mv^2}{e^2}$$

$$= \frac{mv^3}{e^2} = \frac{v}{}\frac{eE}{mv^2} = \frac{eE}{mv} \quad \text{or } \frac{1}{r} = \frac{eE}{mv^2}$$

$$\Rightarrow \boxed{\omega_T = -\frac{1}{2}\frac{v^2}{c^2}\frac{eE}{mv} = -\frac{veE}{c^2 m}}$$

It happens that $\omega_T = -\frac{1}{2}\omega_L$

or effectively,

for more detailed discussion see. A.J.P. 40, 1772 (1972)
A.J.P. 72, 943 (2004)

$$\Delta H' = \frac{1}{2}\Delta H = \frac{e^2}{2m^2c^2r^3}(\vec{L}\cdot\vec{S})$$

$$J = L + S$$

$$J^2 = j(j+1)\hbar^2 = \hbar^2\left(\ell(\ell+1) + \frac{3}{4}\right) + 2\vec{L}\cdot\vec{S}$$

$$\Rightarrow \langle \Delta H^{s.o.} \rangle = \frac{e^2\hbar^2}{2m^2c^2}\,\frac{j(j+1)-\ell(\ell+1)-\frac{3}{4}}{2}\left\langle \frac{1}{r^3}\right\rangle$$

$$= \frac{e^2\hbar^2}{4m^2c^2}\left\{j(j+1)-\ell(\ell+1)-\frac{3}{4}\right\}\frac{1}{a_0^3\, n^3\,\ell(\ell+\frac{1}{2})(\ell+1)}$$

$$= \left(\cancel{\frac{\hbar^2}{me^2}}\right)^{a_0}\left(\frac{e^4}{4mc^2}\right)\left(\frac{mc}{e^2 a_0}\right)^3 \frac{1}{n^3}\,\frac{j(j+1)-\ell(\ell+1)-\frac{3}{4}}{\ell(\ell+\frac{1}{2})(\ell+1)}$$

$$= \frac{e^4}{4mc^2}\left(\frac{\alpha^3 mc^2}{e^2}\right)^2\frac{1}{n^3}\cdots$$

$$= \frac{\alpha^4 mc^2}{4n^3}\,\frac{j(j+1)-\ell(\ell+1)-\frac{3}{4}}{\ell(\ell+\frac{1}{2})(\ell+1)}$$

$$\langle e\phi(r+\delta r)\rangle = e\phi + e\frac{1}{2}\frac{\partial^2\phi}{\partial x_i\partial x_j}\langle\delta x_i\delta x_j\rangle$$

$$= e\phi + \frac{e}{6}\frac{\partial_i^2\phi}{m^2}$$

$$= e\phi - \frac{e}{6m^2}\nabla\cdot E$$

$\langle\delta x_i \delta x_j\rangle \sim \frac{\delta_{ij}}{3}\langle\delta x^2\rangle$

$$H > \frac{-e}{8m^2}\nabla\cdot\vec{E}$$

The problem with $\ell=0$ can be removed by considering Darwin term

---

$$\langle \Delta H^{so}\rangle + \langle \Delta H^{rel}\rangle = \frac{\alpha^4 mc^2}{4n^4}\left[-\frac{2n}{\ell+\frac{1}{2}} + \frac{3}{2} + \frac{n\left(j(j+1)-\ell(\ell+1)-\frac{3}{4}\right)}{\ell(\ell+\frac{1}{2})(\ell+1)}\right]$$

case-(1). $\ell = j - \frac{1}{2}$

$$\left(-\frac{2n}{j} + \frac{3}{2} + \frac{n\left(j(j+1) - (j-\frac{1}{2})(j+\frac{1}{2}) - \frac{3}{4}\right)}{(j-\frac{1}{2})\, j\,(j+\frac{1}{2})}\right)$$

$$= \left(\frac{-2n}{j} + \frac{3}{2} + \frac{n}{j(j+\frac{1}{2})}\right) = \left(\frac{-2n}{j+\frac{1}{2}} + \frac{3}{2}\right)$$

case-(2). $\ell = j + \frac{1}{2}$,

$$\left(\frac{-2n}{j+1} + \frac{3}{2} + \frac{n\left(j(j+1) - (j+\frac{1}{2})(j+\frac{3}{2}) - \frac{3}{4}\right)}{(j+\frac{1}{2})(j+1)(j+\frac{3}{2})}\right)$$

$$= \left\{\frac{3}{2} - \frac{2n}{j+1} - \frac{n}{(j+\frac{1}{2})(j+1)}\right] = \left(\frac{3}{2} - \frac{2n}{j+\frac{1}{2}}\right)$$

Hyper fine structure.

$$\vec{\mu}_p = g_p \frac{e}{m_p c} \vec{S}_p \qquad g_p \simeq 2.8. \quad \overset{\simeq 2 \text{ Lande factor}}{}$$

(we will try to calculate the # in quark model)

Therefore, the proton spin - electron orbital interaction giving

$$\Delta H_{pso} = -\left(\frac{g_p e \vec{S}_p}{m_p c}\right) \cdot \left(\frac{-e}{c m r^3} \vec{L}\right) = (+) \frac{g_p e^2}{m_p m c^2 r^3} (\vec{S}_p \cdot \vec{L})$$

<u>Note</u>! there is no Thomas precession $\frac{1}{2}$

In the same time, the proton spin ~~att~~ generates a magnetic field:

$$B = \nabla \times \left[\Big(\underbrace{\frac{g_p e}{m_p c} \vec{S}_p}_{\sim \vec{\mu}_p} \Big) \times \vec{\nabla} \frac{1}{4\pi r} \right]$$

$$= \frac{1}{r^3} \left[3 \frac{(\vec{\mu}_p \cdot \vec{r}) \vec{r}}{r^2} - \vec{\mu}_p \right] + \frac{8\pi}{3} \vec{\mu}_p \delta^3(r)$$

you shall go back to derive this formula, ~~from~~ your E&M textbook and

$$\Delta H_{s.s} = -\left(\frac{g_p e}{m_p c}\right)\left(\frac{-2e}{2mc}\right) \left[\underbrace{\frac{1}{r^3} \left[3(\vec{S}_p \cdot \hat{r})(\vec{S}_e \cdot \hat{r}) - \vec{S}_p \cdot \vec{S}_e \right]} + \frac{8\pi}{3} \vec{S}_p \cdot \vec{S}_e \delta^3(r) \right]$$

for spherically symmetric WF, $\Rightarrow 0$

$l = 0$,

$$\Delta H_{ss} = + \frac{g_p e^2}{m_p m c^2} \frac{8\pi}{3} (\vec{S}_p \cdot \vec{S}_e) \delta^3(r)$$

$$a_0 = \frac{\hbar^2}{m e^2} \qquad \frac{e^2}{a_0} = \alpha^2 m c^2$$

$$\langle \delta^3(r) \rangle = |\psi_{n00}(r=0)|^2 = \frac{1}{\pi n^3 a_0^3}$$

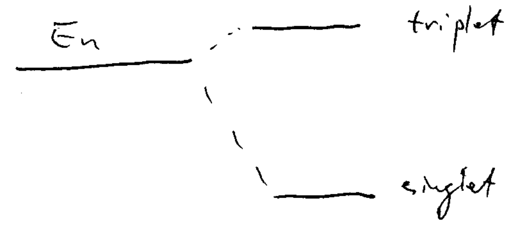
$$F = L + S_e + S_p = J + S_p, \qquad l = 0, \qquad f(f+1)\hbar^2 = \frac{3}{2}\hbar^2 + 2\vec{S}_e \cdot \vec{S}_p$$

$$\Rightarrow \langle \Delta H_{Hyperfine} \rangle = \frac{g_p e^2}{m_p m c^2} \frac{8\pi}{3} \frac{1}{\pi n^3 a_0^3} \frac{\hbar^2}{2} \left(f(f+1) - \frac{3}{2} \right)$$

$$= \frac{g_p e^4}{m_p c^2 a_0^2} \frac{4}{3 n^3} \left(f(f+1) - \frac{3}{2} \right) = \frac{g_p}{m_p} \frac{\alpha^4 m^2 c^2}{n^3} \left(\frac{4}{3} f(f+1) - 2 \right)$$

$$= \left(g_p \alpha \frac{m}{m_p}\right) \frac{\alpha m c^2}{n^3} \left(\underbrace{\frac{4}{3} f(f+1) - 2}_{} \right)$$

$$f = \begin{cases} 1 & \text{triplet} \quad \frac{2}{3} \\ 0 & \text{singlet} \quad -2 \end{cases}$$



$l = 0$

$$\Delta\left(E_{trip} - E_{singlet}\right) = \frac{16}{3} g_p \left(\frac{m}{m_p}\right) \alpha^2 \boxed{\frac{\alpha^2 m c^2}{2}}$$

$n = 1$

$$\frac{1}{137} \frac{16}{3} \times 2.8 \times \frac{1}{137} \times \frac{1}{2000} \times 13.6 \text{ eV}$$

$$\simeq \frac{0.102}{137} \text{ eV} \simeq 1.2 \times 10^{-22} \text{ J}.$$

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.58 \times 10^{-16} \text{ eV-sec}) \times (3 \times 10^{10} \text{ cm/sec})}{\lambda} = 0.102 \text{ eV}$$

$$= \frac{6.6 \times 10^{-34} \text{ Js} \times 3 \times 10^{10} \text{ cm/sec}}{\lambda} = 1.2 \times 10^{-22} \text{ J}$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^{10}}{1.2 \times 10^{-22}} \text{ cm}$$

$$= \frac{32}{3} g_p \frac{1}{m_p c^2} \left(\frac{\alpha^2 m c^2}{2}\right)^2 = \frac{32}{3} \times 2.8 \frac{(13.6 \text{ eV})^2}{(938 \text{ GeV})} \sim 5.9 \times 10^{-6} \text{ eV}$$

$$E = h\nu = \frac{6.6 \times 10^{-34} \text{ J sec}}{\lambda} \left(3 \times 10^{10} \text{ cm/sec}\right) = 5.9 \times 10^{-6} \times 1.6 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{3.3 \times 10^{-24}}{1.9 \times 10^{-25}} \qquad \lambda = 21 \text{ cm}$$

This is the famous 21 cm frequency for radio astronomy.

For $l \neq 0$,

$$\Delta H_{hyper\text{-}fine} = \frac{g_p e^2}{m_p m c^2 r^3} (\vec{S}_p \cdot \vec{L}) + \frac{g_p e^2}{m_p m c^2} \left[\frac{8\pi}{3} \cancel{(\vec{S}_p \cdot \vec{S}_e)\, \delta^3(r)} + \frac{1}{r^3} \left[3(\vec{S}_p \cdot \hat{r})(\vec{S}_e \cdot \hat{r}) - \vec{S}_p \cdot \vec{S}_e \right] \right]$$

need a trick: $r^2 \delta_{ij} - 3 x_i x_j = -\frac{\langle r^2 \rangle}{(2l+3)(2l-1)} \left[2 \vec{L}^2 \delta_{ij} - 3 (L_i L_j + L_j L_i) \right]$

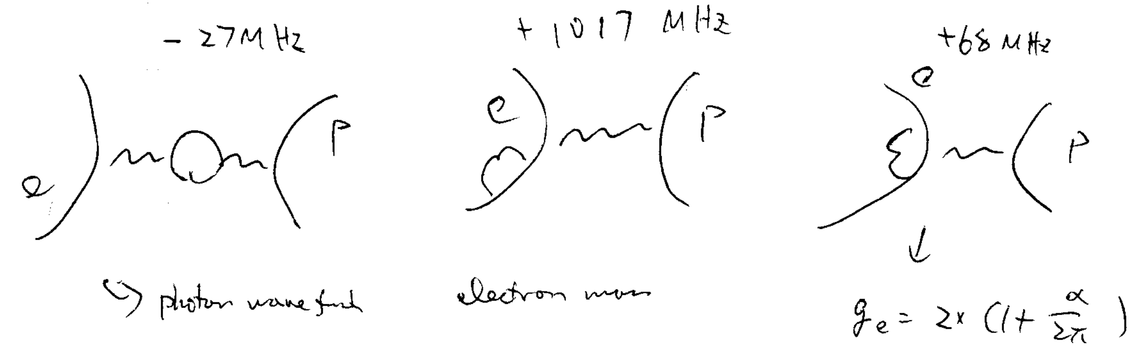
For those are interested in how to get this, consul Landau or Bethe-Salpeter

Anyway, $\Delta E_{hf} = \left(\frac{m}{m_p}\right) \alpha^4 m c^2 \frac{g_p}{2n^3} \frac{[f(f+1) - j(j+1) - \frac{3}{4}]}{j(j+1)(l+\frac{1}{2})}$

$$\frac{g_p e^2}{4 m_p m c^2}\left(\frac{\vec{S}_p\cdot\vec{L}}{r^3} + \frac{1}{r}\frac{2}{(2l+3)(2l-1)}\left\{l(l+1)\,\vec{S}_p\cdot\vec{S}_e - 3(\vec{L}\cdot\vec{S}_p)(\vec{L}\cdot\vec{S}_e)\right\}\right)$$

$$= \frac{g_p e^2}{4 m_p m c^2}\left(\frac{(\vec{S}_p\cdot\vec{L})}{a^3 n^3 l(l+\frac{1}{2})(l+1)} + \frac{2}{a n^2 (2l+3)(2l-1)}\left\{l(l+1)(\vec{S}_p\cdot\vec{S}_e) - 3(L\cdot S_p)(L\cdot S_e)\right\}\right)$$

Lamb shift. $\nu_{lamb}^{exp} \simeq 1058$ MHz



$2S_{\frac{1}{2}}$ $n=2$, $l=0$, $j=\frac{1}{2}$

$2P_{\frac{1}{2}}$ $n=2$, $l=1$, $j=\frac{1}{2}$

For $l=0$, $\Delta E_{Lamb} = \alpha^5 mc^2 \frac{1}{4n^3} k(n,0)$

~ 13.0 for any n.

$l\neq 0$ $\Delta E_{lamb} = \alpha^5 mc^2 \frac{1}{4n^3}\left\{0.05 \pm \frac{1}{\pi(j+\frac{1}{2})(l+\frac{1}{2})}\right\}$ for $j = l\pm\frac{1}{2}$

In all

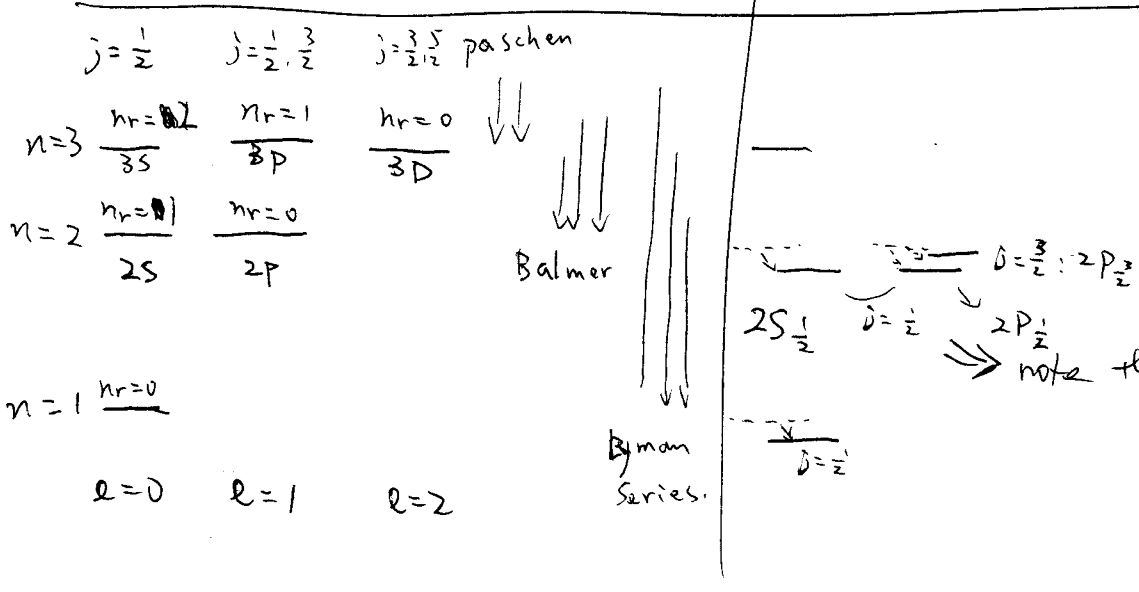
Tree.

$$E_n = \frac{-\alpha^2 mc^2}{2n^2}$$

$n = n_r + l + 1$

with fine structure

$$\Delta E_{f.s.} = \frac{\alpha^4 mc^2}{4n^4}\left(\frac{3}{2} - \frac{2n}{j+\frac{1}{2}}\right)$$



$2S_{\frac{1}{2}}$ → $\frac{\alpha^5 mc^2}{4\times 8}\times 13.$

$2P_{\frac{1}{2}}$ → $\frac{\alpha^5 mc^2}{4\times 8}\times(\sim -0.$

$\Delta E \sim \frac{\alpha^5 mc^2}{\cancel{46}\times 2}\times 13.2$

$\sim \frac{\alpha^3}{16} 13.2\times E_1$

$= 4.4\times 10^{-6}$ eV

$= 7\times 10^{-25}$ J

⇓

$\nu_{lamb} \sim 10^9$ Hz

Positronium : bound state of e^+e^-

eCM: $m_1 v_1 + m_2 v_2 = 0$, $m_1 v_1 = -m_2 v_2 = p$

$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{p^2}{2m_1} + \frac{p^2}{2m_2} = \frac{p^2}{2m_{red}}$ $m_{red} = \left(\frac{1}{\frac{1}{m_1}+\frac{1}{m_2}}\right) = \frac{m_1 m_2}{m_1+m_2}$

$E = \frac{p^2}{2m_{red}} + V(r)$ $m_{red} = \frac{1}{2}m_e$, $V(r_1 - r_2) = -\frac{e^2}{r}$

$\hat{H} = \frac{P_1^2}{2m_{red}} - \frac{e^2}{r_{12}}$ $E = -\frac{\alpha^2 m_{red} c^2}{2n^2} = -\frac{\alpha^2 m_e c^2}{4n^2}$

or $a^{pos} = 2a \sim 1\,\AA$ $m \to \frac{1}{2}m$

* relativistic correction

$T_1 + T_2 = \sqrt{p_1^2c^2+m_1^2c^4} + \sqrt{p_2^2c^2+m_2^2c^4} - 2m_e = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - \frac{p^4}{8m_1^3c^2} - \frac{p^4}{8m_2^3c^2}$

$\Rightarrow \Delta H^{rel} = -\frac{p^4}{4m_e^3c^2} = -\frac{p^4}{4\times 8 m_{red}^3 c^2} = -\frac{1}{4}\frac{p^4}{8m_{red}^3c^2}$

also, from previous we know $\langle \Delta H^{rel} \rangle \propto m_{red}$

$\Rightarrow \Delta H^{rel}_{pos} = \frac{1}{8}\Delta H^{rel}_{Hydrogen}$

hyper fine stronger ~ 2000 times than Hydrogen

TCP test, $m_{e^+} = m_{e^-}$, & $|\mu_{e^+}| = |\mu_{e^-}|$. any deviation can be seen in spectrum

$e^+ \underset{C}{\leftrightarrows} e^-$. Charge conjugation invariance & intrinsic electron-positron parity.

$m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$

$m_1 \dot{\vec{r}}_1 + m_2 \dot{\vec{r}}_2 = 0$

$m_1 \ddot{\vec{r}}_1 + m_2 \ddot{\vec{r}}_2 = 0$

$\frac{1}{2}m_1 \dot{\vec{r}}_1^{\,2} + \frac{1}{2}m_2 \dot{\vec{r}}_2^{\,2}$ $\frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)(\dot{\vec{r}}_1 - \dot{\vec{r}}_2)^2$

$= \frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)(\dot{\vec{r}}_1)^2\left(1+\frac{m_1}{m_2}\right)^2$

$= \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\frac{(m_1+m_2)^2}{m_2^2}(\dot{\vec{r}}_1^{\,2})$

$= \frac{1}{2}\frac{m_1}{m_2}(m_1+m_2)(\dot{\vec{r}}_1^{\,2})$

$= \frac{1}{2}m_1(\dot{\vec{r}}_1)^2 + \frac{1}{2}m_2\left(\frac{m_1}{m_2}\dot{\vec{r}}_1\right)^2$

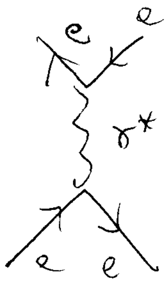
Can be seen as e^+ at rest and e^- with a reduced mass circling around e^+

Hyperfine now is indistinguishable from spin-orbital –

$$\Delta H_{ret} = -\frac{e^2}{2m^2c^2}\frac{1}{r}\left[p^2+(\vec{p}\cdot\hat{r})^2\right] \quad \leftarrow \text{retarded potential}$$

$$E^{pos}_{fs} = \alpha^4 mc^2 \frac{1}{2n^3}\left(\frac{11}{32n} - \frac{1+\frac{\epsilon}{2}}{2l+1}\right) \qquad \epsilon = \begin{cases} \dfrac{-(3l+4)}{(l+1)(2l+3)} & j = l+1 \\ \dfrac{1}{l(l+1)} & j = l \\ \dfrac{(3l-1)}{l(2l-1)} & j = l-1 \end{cases}$$

$$S = S_1 + S_2, \quad J = L + S$$



$$\Delta E_{annih} = \frac{\alpha^4 mc^2}{4n^3}$$

$l = 0$, $s = 1$ ↗ because γ are spin-1

↓

to have $|\psi(0)|^2 \neq 0$.

Lamb shift is small compared to the "hyper-fine"

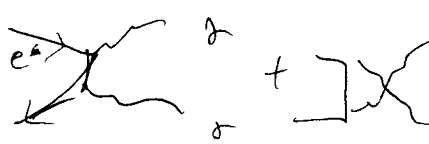
$(-1)^{l+s} = (-1)^n$ ← # of photon.

$l = 0$ to annihilate

$s = 1$ triplet → 3γ, 5γ.

$s = 0$ singlet → 2γ, 4γ.

singlet → 2γ. $\sigma = 4\pi\alpha^2\left(\frac{\hbar^2}{m^2cv}\right)$



$$\Gamma = \sigma v|\psi(0)|^2 = \frac{\alpha^5 mc^2}{2\hbar n^3}, \quad \tau = \Gamma^{-1} = 1.25\times 10^{-10}\ \text{sec}.$$

930302 李玟慧
921704 魏瑀嫺
921027 張凱維
931416 林柏年
930357 林家正
930360 潘維君
912118 陳致
930336 陳柏榮
930354 林鴻臏
930343 蕭子綱
930349 李昌泰
930345 陳在榮