* 下週 Dec. 18. 考試, open book, bring calculator, C-G coefficient table.
* HW: chapter 5: 28, 30, 31, structure const. of SU(3)

* Plan: 5.7: Quarkonium — slightly touch.
  - 5.8: light quark meson. ✓
    - Intr. to SU(3).
  - 5.9 Baryon WF & color degree of freedom ✓
  - 5.10, Baryon mass & magnetic moment. ✓

  ⇐ focusing on these 3. add $\boxed{\text{Young tableau}}$

* 5.7. You can read most of it by yourself.

$$V(r) = \underbrace{-\frac{4}{3}\frac{\alpha_s}{r}}_{\text{1-gluon exchange}} + \underbrace{F_0 r}_{\text{confinement, Regge slope}}$$

will be explained in QCD, chapt. 8.

* light quark (u, d, s) bound states hard to get.

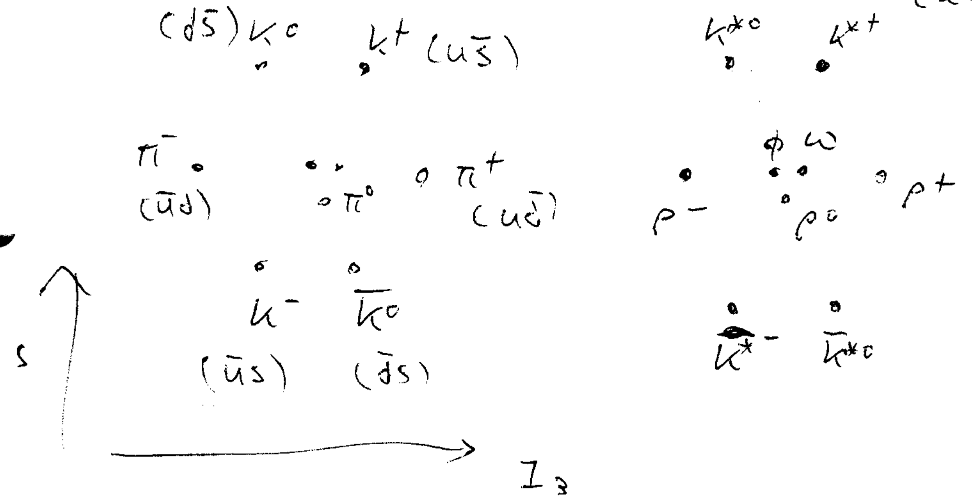
  $M = m_1 + m_2 + E$ ← for heavy quarks (c, b, t) (1.5 GeV, 4.6 GeV; no time)

  charmonium, bottomonium

§ 5.8.

So, how do we deal with the light quark (u d s) mesons?
Symmetry again comes to rescue us —

Pseudo scalar nonet ($l=0, s=0$)   vector nonet ($l=0, s=1$)



$l = 0$

$\mathbb{P} = (-1)^{l+1}$ ← intrinsic parity for quark antiquark

Before intro. SO(3). something about SU(2) isosymmetry

$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\tfrac{1}{2}, \tfrac{1}{2}\rangle$, $\quad d = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$

antiparticle carries the opposite quantum # ~~of~~ particle

$I_3 : +\tfrac{1}{2} \Leftrightarrow I_3 = -\tfrac{1}{2}$

It's natural to think $\quad \bar{d} \sim |\tfrac{1}{2}\ \tfrac{1}{2}\rangle$, & $\bar{u} \sim |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$

However, $\begin{pmatrix} u \\ d \end{pmatrix}$ is an SU(2) doublet. $\quad \begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}$ is not.

It has to be $\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$.

$u, d \overset{C}{\Longleftrightarrow} \bar{u}, \bar{d}$

$$\begin{pmatrix} u \\ d \end{pmatrix} \to \underbrace{e^{i\frac{\pi}{2}\sigma_2}}_{\substack{\| \\ i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}} \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} d \\ -u \end{pmatrix}$$

same SU(2) rotation

C

if $\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}$ is an SU(2) doublet. $\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix} \to \begin{pmatrix} \bar{u} \\ -\bar{d} \end{pmatrix}$

不同

$C \quad \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \longrightarrow \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$ or $\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}$

However if it is $\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \xrightarrow{\text{SU(2) rotation}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$

$$\begin{array}{ccc} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} & & \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \\ \Updownarrow C & & \Updownarrow C \\ \begin{pmatrix} -d \\ u \end{pmatrix} & \longrightarrow & \begin{pmatrix} u \\ d \end{pmatrix} \end{array}$$

$\begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$

$\boxed{i\sigma_2}$

$\Rightarrow \quad \bar{d} = -|\tfrac{1}{2}\ \tfrac{1}{2}\rangle, \quad \bar{u} = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$

This sign is important.

$$\Rightarrow \begin{aligned} |1,1\rangle &= -u\bar{d} \\ |1,0\rangle &= \tfrac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) = \tfrac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\ |1,-1\rangle &= d\bar{u} \end{aligned} \quad \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \text{ or } \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix}$$

$|0,0\rangle = (u\bar{u} + d\bar{d})/\sqrt{2}$ isosinglet.

$Q = I_3 + Y = \left(\frac{b+s}{2}\right)$ (3)

I_3, Y (or S)

SU(3): we see that we need two "axises", or 2 quantum #, to label the nonet. If there is a symmetry, its "rank" must be 2.

# of simultaneously diagonal elements.

L^2, $\overset{m = \sum m_i}{L_z}$ / SU(2) $|\lambda_3, \lambda_8\rangle$ $\lambda_3 = \sum_i \lambda_i$, $\lambda_8 = \sum_i \lambda_8$.

SU(3) generators. (Gell-Mann matrices)

SU(n)

$= \underset{\text{complex}}{2n^2} \underset{n \times n}{} \underset{\det = 1}{-1} \underset{\text{unitary}}{-n^2}$

$= n^2 - 1$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

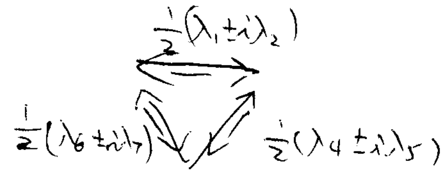
$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$\left[\frac{\lambda_i}{2}, \frac{\lambda_j}{2}\right] = i \underbrace{f_{ijk}}_{\text{structure coefficient}} \frac{\lambda_k}{2}$$

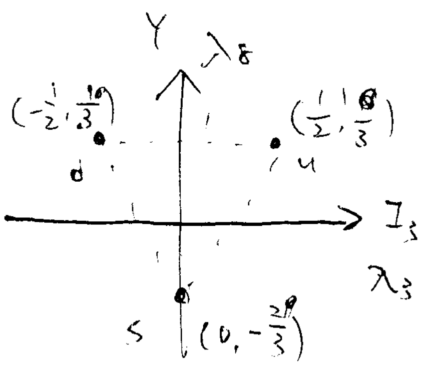
HW
(a) f_{ijk} is totally antisymmetric
(b) find out all non-zero f_{ijk}

	I_3	Y
u	$\frac{1}{2}$	$\frac{1}{3}$
d	$-\frac{1}{2}$	$\frac{1}{3}$
s	0	$-\frac{2}{3}$

	I_3	Y
$\bar{u}$	$-\frac{1}{2}$	$-\frac{1}{3}$
$\bar{d}$	$\frac{1}{2}$	$-\frac{1}{3}$
$\bar{s}$	0	$+\frac{2}{3}$



$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

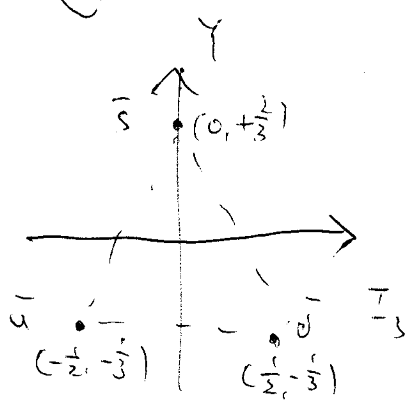


$\left(\frac{1}{2}\right)^2 + N^2 = 1$

$N = \frac{\sqrt{3}}{2}$

$Y = \frac{\lambda_8}{\sqrt{3}}$

$I_3 = \lambda_3$

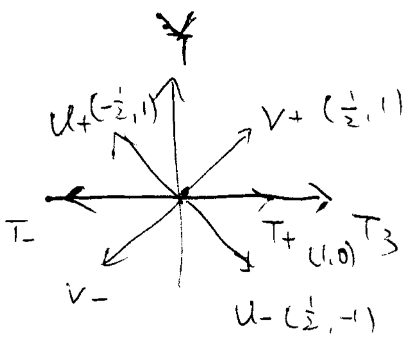


$T^{\pm}$

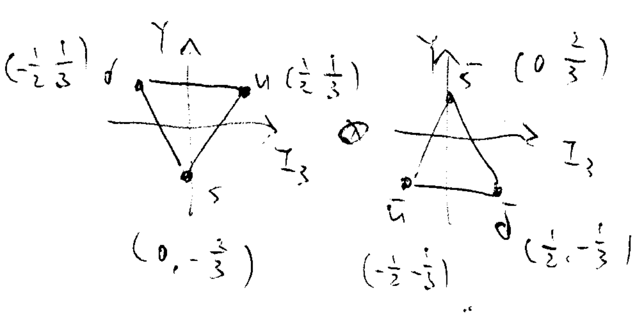
$$\frac{\lambda_1 + i\lambda_2}{2} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \frac{\lambda_4 \pm i\lambda_5}{2} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ (1) & 0 & 0 \end{pmatrix}, \quad \frac{\lambda_6 \pm i\lambda_7}{2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & (1) & 0 \end{pmatrix}$$

$$\frac{\lambda_1 - i\lambda_2}{2} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

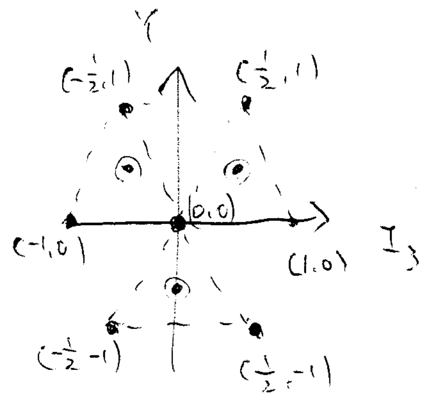
$V^{\pm}$ $\qquad$ $U^{\pm}$



$3 \times \bar{3}$



$\Longrightarrow$

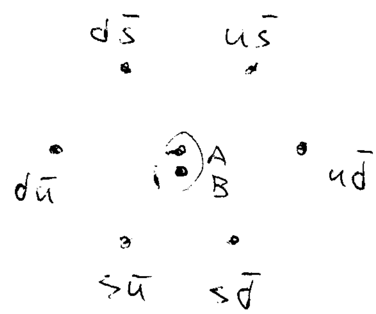


$3 \times 3 = 9$

there is a siglet. $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$
which will not transfom under all SU(3) transfomet

$3 \times \bar{3} = 1 + \underline{\underline{8}}$
octect, which will transfom among themself –

$\Longrightarrow$



$+$ C $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$ (η')

one of A & B is π^0: $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \equiv A$

$B = (\alpha\, u\bar{u} + \beta\, d\bar{d} + \lambda\, s\bar{s})$

$\langle B | C \rangle \propto \quad \alpha + \beta + \lambda = 0$

$\langle B | A \rangle \propto \quad \alpha - \beta = 0$

$\Rightarrow \alpha : \beta : \lambda = 1 : 1 : -2$

$\Rightarrow \quad B = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \quad (\eta)$

However. A. B. C mix for (0^-) $\eta' \sim B + \epsilon C$

for (1^-) $\phi \simeq s\bar{s}$ $\omega \simeq \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

If SU(3) is an exact symmetry, all octect should share the same mass

but, $m_s > m_u, m_d$. $\overset{484}{m_K} > m_\pi$

But that's not the whole story, otherwise $\overset{140}{m_\pi} = \overset{780}{m_\rho}$

This must be attributed to spin-spin interaction.

$\underline{0^-}, \underline{1^-}$

recall the hyperfine splitting in Hydrogen. ($l = 0$)

$$\Delta E_{hf} = \frac{4\pi g_p e^2}{3 m m_p c^2} (\vec{S}_e \cdot \vec{S}_p) |\psi_{100}(0)|^2 \quad , \quad \vec{\mu} = \frac{g e}{2mc} \vec{S}$$

QED

$\Rightarrow$ phenomenological

unknown QCD

$$M_{ij} = m_1 + m_2 + A \frac{(\vec{S}_1 \cdot \vec{S}_2)}{m_1 m_2}$$

$$S(S+1)\hbar^2 = S^2 = (S_1 + S_2)^2 = \frac{3}{2}\hbar^2 + 2 S_1 \cdot S_2$$

$$S_1 \cdot S_2 = \hbar^2\left(\frac{S(S+1)}{2} - \frac{3}{4}\right) = \hbar^2 \begin{cases} \frac{1}{4} & \text{triplet} \\ -\frac{3}{4} & \text{singlet.} \end{cases}$$

$\Rightarrow$ $m_u = m_d = 310$ MeV. $m_s = 483$ MeV. $A = \left(\frac{2m_u}{\hbar}\right)^2 160$ MeV

		fit	exp
3	π	140	138
4	K	484	496
1	η	559	549
3	ρ	780	776
4	K^*	780	783
1	ω	846	892
1	ϕ	1032	1020

$\Rightarrow$ 1% accuracy.

Baryon : 3-body problem.
pauli exclusion principle , antisymmetrized for identical fermion
symmetrized for identical boson.

2 angular momentum. l_1, l_2.
for ground state $l_1 = l_2 = 0$.
$\Rightarrow$ totally from spin.

$\underline{\frac{1}{2} \times \frac{1}{2}} \times \frac{1}{2}$

$|1,1\rangle = \uparrow\uparrow$
$|1,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$
$|1,-1\rangle = \downarrow\downarrow$
$|0\,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

from C-G table. ($1 \times \frac{1}{2}$)

$|\frac{3}{2}\,\frac{3}{2}\rangle = |1,1\rangle\uparrow$
$|\frac{3}{2},\frac{1}{2}\rangle = \frac{1}{\sqrt{3}}|1,1\rangle\downarrow + \sqrt{\frac{2}{3}}|1\,0\rangle\uparrow$
$|\frac{3}{2}\,{-\frac{1}{2}}\rangle = \sqrt{\frac{2}{3}}|1\,0\rangle\downarrow + \sqrt{\frac{1}{3}}|1\,{-1}\rangle\uparrow$
$|\frac{3}{2}\,{-\frac{3}{2}}\rangle = |1\,{-1}\rangle\downarrow$

$|\frac{3}{2}\,\frac{3}{2}\rangle = \uparrow\uparrow\uparrow$
$|\frac{3}{2}\,\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}\uparrow\uparrow\downarrow + \sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)\right)\uparrow = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$
$|\frac{3}{2}\,{-\frac{1}{2}}\rangle = \sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)\downarrow\right) + \sqrt{\frac{1}{3}}\downarrow\downarrow\uparrow = \frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow)$
$|\frac{3}{2}\,{-\frac{3}{2}}\rangle = \downarrow\downarrow\downarrow$

all symmetric

P_{12}
$|\frac{1}{2}\,\frac{1}{2}\rangle$: $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow$
$|\frac{1}{2}\,{-\frac{1}{2}}\rangle$: $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\downarrow$

1-2 antisymmetric
constructed from the singlet state $|0\,0\rangle$

P_{23} : $|\frac{1}{2}\,\frac{1}{2}\rangle$: $\frac{1}{\sqrt{2}}\uparrow(\uparrow\downarrow - \downarrow\uparrow)$
$|\frac{1}{2}\,{-\frac{1}{2}}\rangle$: $\frac{1}{\sqrt{2}}\downarrow(\uparrow\downarrow - \downarrow\uparrow)$

2-3 antisymmetric

$\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)$
$= \frac{1}{\sqrt{3}}(\uparrow_1 P_{23} + \uparrow_2 P_{13})$

1-2 symmetric : constructed from $|1\rangle$

$|\frac{1}{2}\,\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|1\,1\rangle\downarrow - \sqrt{\frac{1}{3}}|1\,0\rangle\uparrow = \sqrt{\frac{2}{3}}\uparrow\uparrow\downarrow - \sqrt{\frac{1}{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) = \sqrt{\frac{1}{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$
$|\frac{1}{2}\,{-\frac{1}{2}}\rangle = \sqrt{\frac{1}{3}}|1,0\rangle\downarrow - \sqrt{\frac{2}{3}}|1\,{-1}\rangle\uparrow = \sqrt{\frac{1}{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow) - \sqrt{\frac{2}{3}}\downarrow\downarrow\uparrow = \sqrt{\frac{1}{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$

P_{13} : $|\frac{1}{2}\ \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) = P_{12}^{\uparrow} + P_{23}^{\uparrow} = \frac{1}{\sqrt{2}}(\uparrow\cancel{\downarrow\uparrow} - \downarrow\uparrow\uparrow) + \frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \cancel{\uparrow\downarrow\uparrow})$

$|\frac{1}{2}\ -\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\downarrow\uparrow) = P_{12}^{\downarrow} + P_{23}^{\downarrow} = \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \cancel{\downarrow\uparrow\downarrow}) + \frac{1}{\sqrt{2}}(\cancel{\downarrow\uparrow\downarrow} - \downarrow\downarrow\uparrow)$

$\Rightarrow$ really independent combination.

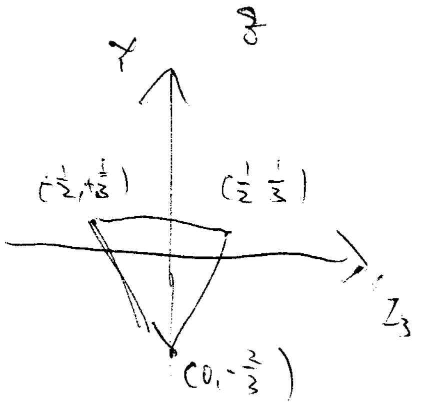
$\underbrace{\text{spin-}\frac{3}{2}}_{\text{totally symmetric}}$ $\qquad$ $\underbrace{\text{spin }\frac{1}{2}}$ $\quad$ $\underbrace{\text{spin }\frac{1}{2}}$ partial symmetric

$2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$

Another example is the SU(3) flavor

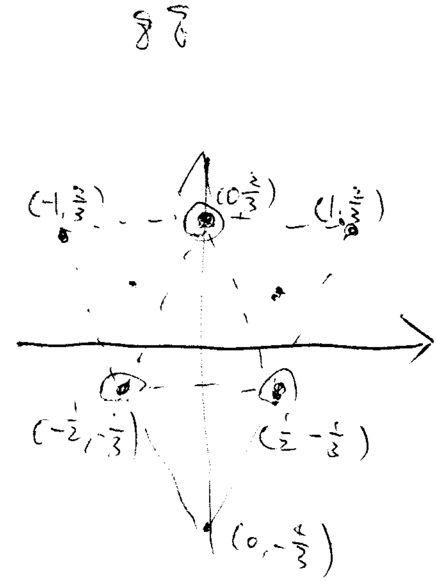
$q_1 q_2 q_3 \qquad q_i \in u, d, s$

$q \qquad\qquad qq \qquad\qquad qqq$



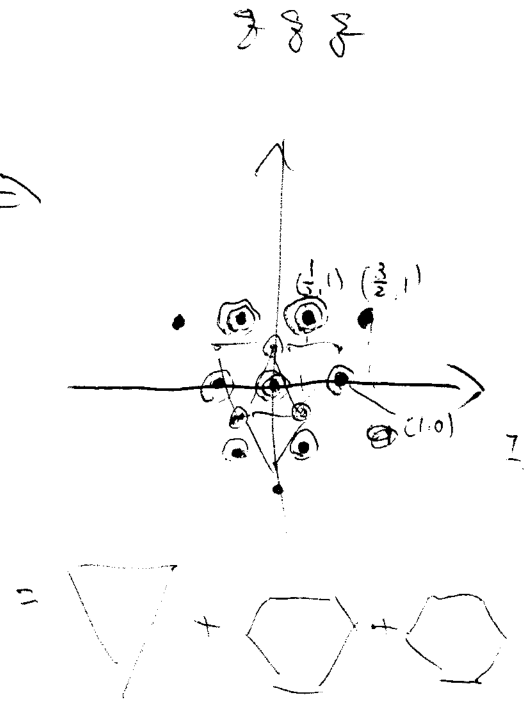
3

$\Rightarrow$



$3 + 6 = 9$

$\Rightarrow$



$10 + 8 + 8 + 1$

930302 李政憲
931704 魏瑪嫺
921027 張凱維
930313 姚欣佑
930349 李昭希
930343 蕭子綱
930353 李映潔
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930314 王繼文
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