Int. to HEP (2)

Dec. 25, 2007

① 發 final 考卷, 講解問題 ~1 hr.

check 你們成績 (so far), 上網檢查自己的分數.

Next HW: 5, 28, 30, 31 Due on Jan 2, Wed. 5pm. 助教信箱

②

p, n wave function and magnetic moment.

$$\psi = \underbrace{\psi_{space}(l_1 = l_2 = 0)}_{1} \; \underbrace{\psi_{color}}_{\text{totally anti-sy}} \; \underbrace{\psi_{spin}\,\psi_{flavor}}_{\text{the combination must be totally sym.}}$$

mass spectrum.

③ Young tableau.

Dirac - g factor for point particle

$$\mu_p = \sum_{i=1}^{3} \mu_i = \sum_i \frac{2\hbar}{2m_{q}c} Q_i \hat{S}_{z_i}$$

$$\mu_p = \frac{\hbar}{c}\left\langle \psi(\text{proton}) \middle| \frac{Q_1}{m_{q_1}}\hat{S}_1 + \frac{Q_2}{m_{q_2}}\hat{S}_2 + \frac{Q_3}{m_{q_3}}\hat{S}_3 \middle| \psi(\text{proton}) \right\rangle$$

$$= \frac{\hbar}{c}\frac{N^2}{4}\left\langle \middle| \left(\frac{Q_1}{m_{q_1}}\hat{S}_1 + \frac{Q_2}{m_{q_2}}\hat{S}_2 + \frac{Q_3}{m_{q_3}}\hat{S}_3\right) \middle| \begin{array}{l} udu(2\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) \\ + uud(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \\ + duu(2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) \end{array} \right\rangle$$

$$= \frac{\hbar}{c}\frac{1}{4}\times\frac{2}{9}\left\{ 2\left(\frac{+\frac{2}{3}}{m_u}\times\frac{1}{2} + \frac{(-\frac{1}{3})}{m_d}\times(-\frac{1}{2}) + \frac{+\frac{2}{3}}{m_u}(\right.\right.$$

or
$$= \frac{1}{4}\times\frac{2}{9}\left\{ \begin{array}{l} 4(2\mu_u - \mu_d) + (-\mu_u + \mu_d + \mu_u) + (\mu_u + \mu_d - \mu_u) \\ + 4(2\mu_u - \mu_d) + (\mu_u - \mu_u + \mu_d) + (-\mu_u + \mu_u + \mu_d) \\ + 4(-\mu_d + 2\mu_u) + (\mu_d - \mu_u + \mu_d) + (\mu_d + \mu_u - \mu_u) \end{array} \right)$$

$$= \frac{1}{18}\{8\mu_u - 2\mu_d) \times 3 = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d \,!$$

In baryon

	MeV	
u	363	$\simeq \frac{m_p}{2.6}$
d	363	
s	538	

$$\mu_p = \frac{4}{3}\left(+\frac{2}{3}\frac{\hbar}{m_u c}\frac{1}{2}\right) - \frac{1}{3}\left(-\frac{1}{3}\frac{\hbar}{m_d c}\frac{1}{2}\right)$$

$$= \frac{8}{9}\left(\frac{\hbar}{2m_u c}\right) + \frac{1}{9}\left(\frac{\hbar}{2m_d c}\right) \simeq 2.6\ \frac{\hbar}{2m_p c}$$

$$|\psi(\text{proton})|^2 = \frac{N^2}{4}\left(\begin{array}{l}(4+1+1)\\(4+1+1)\\(4+1+1)\end{array}\right) = \frac{18}{4} \Rightarrow N = \frac{\sqrt{2}}{3}$$

work out the magnetic moment of proton & neutron.

constructing a baryon WF.

$$\psi(\text{octet}) = \frac{\sqrt{2}}{3}\Big(\psi_{12}(\text{spin})\,\psi_{12}(\text{flavor}) + \psi_{23}(\text{spin})\,\psi_{23}(\text{flavor}) + \psi_{13}(\text{spin})\,\psi_{13}(\text{flavor}) \Big)$$

~~Proton~~ The normalization factor. check the spin $\uparrow$ WF

spin

$$\psi_{12} = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$

$$\psi_{13} = \tfrac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) = \tfrac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) = \psi_{12} + \psi_{23}$$

$$\psi_{23} = \tfrac{1}{\sqrt{2}}\uparrow(\uparrow\downarrow - \downarrow\uparrow) = \tfrac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)$$

flavor

$$\psi_{12} = (q_1 q_2 - q_2 q_1) q_3 = q_1 q_2 q_3 - q_2 q_1 q_3$$

$$\psi_{23} = q_1 (q_2 q_3 - q_3 q_2) = q_1 q_2 q_3 - q_1 q_3 q_2$$

$$\psi_{13} = q_1 q_2 q_3 - q_3 q_2 q_1$$

$$\psi_{12} + \psi_{23} = 2 q_1 q_2 q_3 - q_2 q_1 q_3 - q_1 q_3 q_2$$

under P_{13}: $2 q_3 q_2 q_1 - q_2 q_3 q_1 - q_3 q_1 q_2$

$$p = (uud)\uparrow$$

$$|\psi(\text{proton})|^2 = \frac{N^2}{4}\begin{pmatrix} 4+1+1 \\ 4+1+1 \\ 4+1+1 \end{pmatrix} = \frac{9}{2}N^2 \Rightarrow N = \frac{\sqrt{2}}{3}$$

$$\psi(\text{proton}) = N\Big(\tfrac{1}{\sqrt{2}}(ud - du)u \;\tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow + \tfrac{1}{\sqrt{2}}u(ud - du)\;\tfrac{1}{\sqrt{2}}\uparrow(\uparrow\downarrow - \downarrow\uparrow) + \tfrac{1}{\sqrt{2}}(uud - duu)\;\tfrac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) \Big)$$

$$= \frac{N}{2}\Big((\underline{udu} - duu)(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + (uud - \underline{udu})(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow) + (uud - duu)(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) \Big)$$

$$= \frac{N}{2}\Big(udu(2\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow) + uud(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + duu(2\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow) \Big)$$

$\therefore \quad (udd)\uparrow$

$$\psi(\text{neutron}) = N \begin{Bmatrix} \frac{1}{\sqrt2}(ud-du)d\ \frac{1}{\sqrt2}(\uparrow\downarrow-\downarrow\uparrow)\uparrow \\ +\frac{1}{\sqrt2}\ d(ud-du)\ \frac{1}{\sqrt2}\uparrow(\uparrow\downarrow-\downarrow\uparrow) \\ +\frac{1}{\sqrt2}(udd-ddu)\ \frac{1}{\sqrt2}(\uparrow\uparrow\downarrow-\downarrow\uparrow\uparrow) \end{Bmatrix}$$

$$= \frac{N}{2}\begin{bmatrix} (\underline{udd}-dud)(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow) \\ +(dud-ddu)(\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow) \\ +(\underline{udd}-ddu)(\uparrow\uparrow\downarrow-\downarrow\uparrow\uparrow) \end{bmatrix}$$

$$= \frac{N}{2}\begin{bmatrix} udd(-2\downarrow\uparrow\uparrow+\uparrow\downarrow\uparrow+\uparrow\uparrow\downarrow) \\ +dud(-2\uparrow\downarrow\uparrow-\uparrow\uparrow\downarrow-\downarrow\uparrow\uparrow) \\ +ddu(-2\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow) \end{bmatrix}$$

$$|\psi_N|^2 = \frac{N^2}{4}\begin{pmatrix} 1\ (4+1+1) \\ +1\ (4+1+1) \\ +1\ (4+1+1) \end{pmatrix} = \frac{9}{2}N^2 \qquad \Rightarrow N = \frac{\sqrt2}{3}$$

$$\langle\psi_N|\vec\mu|\psi_N\rangle = \frac{N^2}{4}\begin{pmatrix} 4(-\mu_u+2\mu_d)+\mu_u+\mu_u \\ +4(-\mu_u+2\mu_d)+2\mu_u \\ +4(2\mu_d-\mu_u)+2\mu_u \end{pmatrix}$$

$$= \frac{1}{4}\times\frac{2}{9}\times 3\,(8\mu_d-2\mu_u)$$

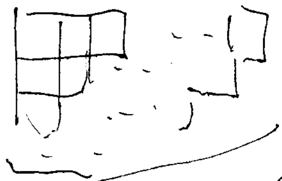
$$= \frac{1}{3}(4\mu_d-\mu_u)$$

$$\mu_N = \frac{4}{3}\left(\left(-\frac{1}{3}\right)\frac{\hbar}{2m_d c} - \left(\frac{2}{3}\right)\frac{\hbar}{2m_u c}\right)$$

$$\simeq -\frac{6}{9}\frac{(2.6)\hbar}{2m_p c} \simeq -1.73\frac{\hbar}{2m_p c}$$

Young Tableau for SU(N)

l_1
l_2
$\vdots$
l_{N-1}



① the length of each row ~~decrease or~~ decrease or no longer than the above one

② dimensionality of an irreducible representation is given by

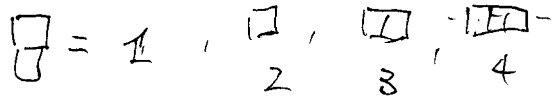
$$d(\lambda_1 \cdots \lambda_{N-1}) = \left(\frac{\lambda_1+1}{1}\right)\left(\frac{\lambda_2+1}{1}\right)\cdots\left(\frac{\lambda_{N-1}+1}{1}\right)\left(\frac{\lambda_1+\lambda_2+2}{2}\right)\left(\frac{\lambda_2+\lambda_3+2}{2}\right)\cdots\times\left(1+\frac{\lambda_1+\lambda_2+\cdots\lambda_{N-1}}{N-1}\right)$$

(1)a every Young tableau can be characterized by a sequence of number

$$(l_1-l_2,\ l_2-l_3, \cdots, l_{N-1}) = (\lambda_1, \lambda_2, \cdots \lambda_{N-1})$$

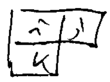
example.

- $SU(2)$, the



$= 1$, 2, 3, 4 ...

$$d(\lambda_1) = \frac{\lambda+1}{1} = 1+\lambda_1$$



means: $i \leftrightarrow j$ symmetrize, $i \leftrightarrow k$ antisymmetrize

$$\psi_{ij;k} \equiv (\psi_{ij} + \psi_{ji})_{;k}$$

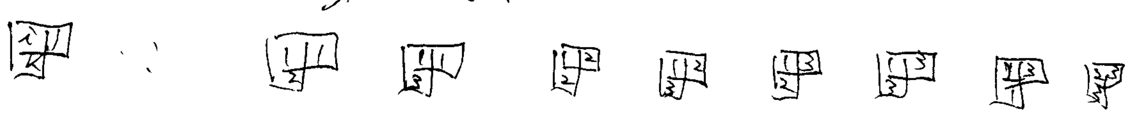
same row: symmetrize
same column: antisymmetrization

$$= \psi_{ij;k} + \psi_{ji;k} - \psi_{kj;i} - \psi_{jk;i}$$

Standard tableau for n objects
the index # do not decrease when going $\longrightarrow$
and always increase from $\downarrow$

e.g. $n=3$

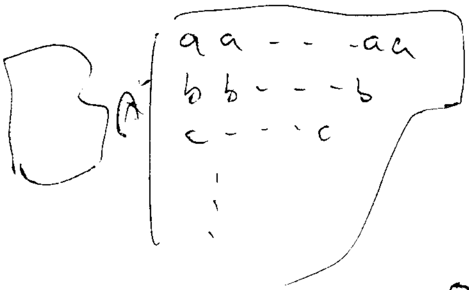
Standard.



#8

non-standard

$= 0$ are not independent from the above 8 multiplet production



(1) allowed sequence:
at any point.

$\# a \geq \# b \geq \# c \ldots$

OK: abac, aabcb, abcd

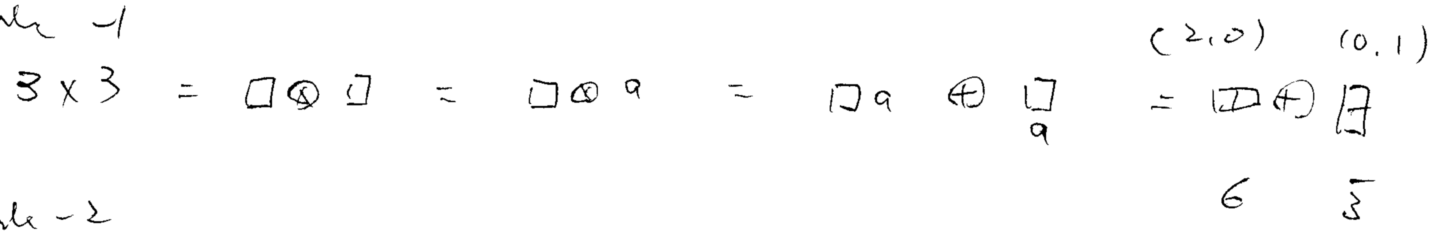
X: acb, abb ..

(2) Attach "a" first, to the "base" each column can ~~only~~ have one "a" at most.

(3). Attach "b" to the new one. ~~& repeat~~

(4) remove the non allowed diagrams from top row down and repeat the whole process. read from right to left!

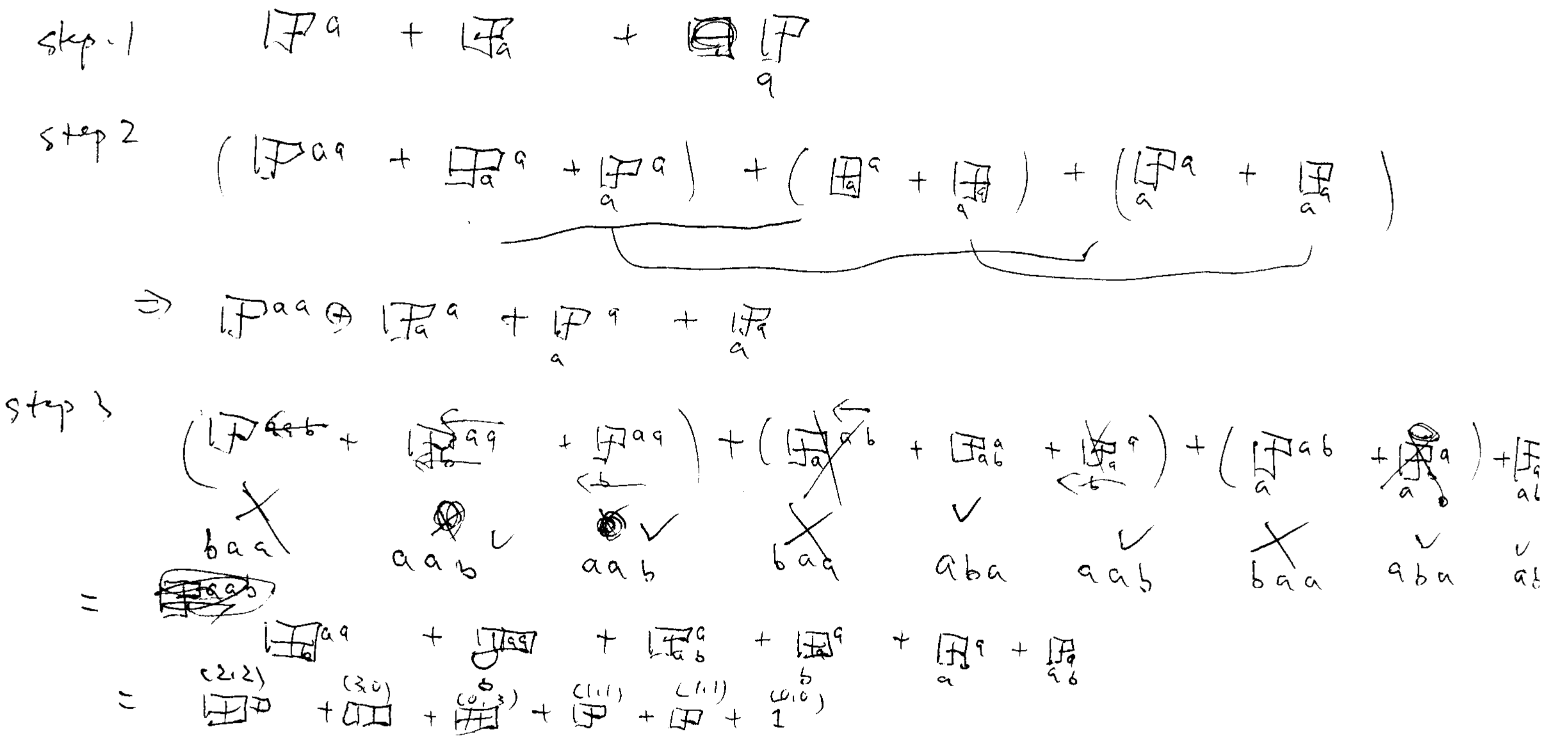
example -1



example - 2

step. 1

step 2



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