$SU(4)_L \times U(1)_X$ electroweak gauge group with little Higgs

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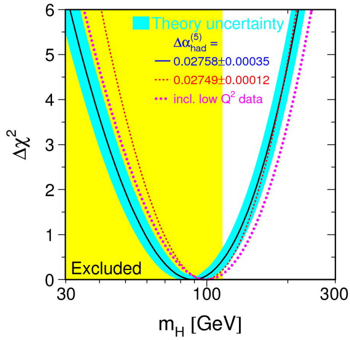
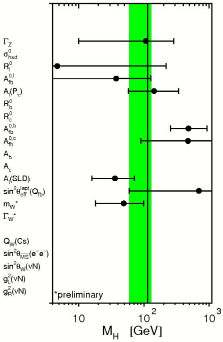
Outline

Introduction - Experimental Background

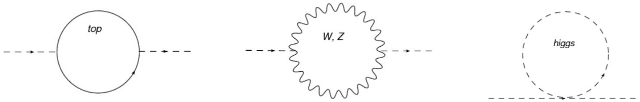




⋆ Precision electroweak data are (known to be) in good agreement with the Standard Model with a Higgs mass less than about 200 GeV.
(lepewwg.web.cern.ch/LEPEWWG)

Introduction- Motivation

- Due to quantum corrections, the Higgs mass is quadratically sensitive to the cutoff scale: $\sim \Lambda^2$ ($\rightarrow$ naturalness problem?)



$$
\begin{aligned}
m_h^2 &= m_{H^0}^2 - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \frac{9}{64\pi^2} g^2 \Lambda^2 + \frac{1}{16\pi^2} \lambda^2 \Lambda^2 \\
(200\ \mathrm{GeV})^2 &= m_{H^0}^2 + \left[-(2\ \mathrm{TeV})^2 + (700\ \mathrm{GeV})^2 + (500\ \mathrm{GeV})^2 \right] \left(\frac{\Lambda}{10\ \mathrm{TeV}} \right)^2 \\
&\Rightarrow \quad \Lambda \sim 1\ \mathrm{TeV} \quad \rightarrow \quad \text{Fine-Tuning Issue}
\end{aligned}
$$

- Note that cutoff Λ for non-renormalizable operators such as $|H^\dagger D_\mu H|^2/\Lambda^2$ should be greater than about 5 TeV.

Introduction - Little Higgs approach

- As an alternative solution to the naturalness problem (and the fine-tuning issue), little Higgs models (LHM) has been recently introduced. (Arkani-Hamed, Cohen, and Georgi 2001)
- LHMs adopts the early idea that Higgs can be considered as a Nambu Goldstone boson from global symmetry breaking at some higher scale $\Lambda \sim 4\pi f$.
  (Dimopoulos, Preskill 1982; Georgi, Kaplan 1984; Banks 1984)
- Higgs acquires a mass radiatively through symmetry breaking at the EW scale v, by collective breaking.
- Consequently, quadratic divergences absent at one-loop level $\Rightarrow$ cancellation among same spin states!

Introduction - Various little Higgs models

Model	Global Group	Gauge Group	EW-scale scalars	TeV-scale scalars
Minimal Moose	$SU(3)^8/SU(3)^4$	$SU(3)\times SU(2)\times U(1)$	h_1, h_2, ϕ, σ	(none)
Minimal Moose with $SU(2)_C$	$SO(5)^8/SO(5)^4$	$SO(5)\times SU(2)\times U(1)$	h_1, h_2	$\phi^r, \sigma^\pm, \sigma^r$
Moose with T-parity	$SO(5)^{10}/SO(5)^5$	$(SU(2)\times U(1))^3$	h_1, h_2	$h_{3,4,5}, \phi^r_{1,2,3}, \sigma_{1,2,3,4,5}, \eta_{1,2,3}$
Littlest Higgs	$SU(5)/SO(5)$	$(SU(2)\times U(1))^2$	h	ϕ
$SU(6)/Sp(6)$ model	$SU(6)/Sp(6)$	$(SU(2)\times U(1))^2$	h_1, h_2	σ
Littlest Higgs with $SU(2)_C$)	$SO(9)/SO(5)\times SO(4)$	$SU(2)^3\times U(1)$	h	ϕ, ϕ^r, η
Littlest Higgs with T-parity	$SU(5)/SO(5)$	$(SU(2)\times U(1))^2$	h	ϕ
$SU(3)$ simple group	$SU(3)^2\times U(1)/SU(2)^2\times U(1)$	$SU(3)\times U(1)$	h, η	(none)
$SU(4)$ simple group	$SU(4)^4\times U(1)/SU(3)^4\times U(1)$	$SU(4)\times U(1)$	h_1, h_2, η_1, η_2	$\sigma_1, \sigma_2, \sigma_3$
SU(9)/SU(8) simple group	$SU(9)/SU(8)$	$SU(3)\times U(1)$	h_1, h_2	σ_1, σ_2

⋆ In this talk, we discuss the simple group model with $SU(4)_L \times U(1)_X$ electroweak gauge group (suggested by Kaplan and Schmaltz, 2003) by embedding the anomaly-free fermion spectra.

Outline

Model Construction - Scalar Sector

- The Coleman-Weinberg mechanism (radiative corrections) produces the Higgs potential:

$$V = \delta m^2 h^\dagger h + \delta\lambda (h^\dagger h)^2$$

- In $SU(3)$ LHM (simplest), generated Higgs mass is somewhat too large, so a tree level "μ" term is included by hand and partially cancels the Higgs mass.

- "μ" term explicitly breaks the spontaneously broken global symmetry and gives a mass to the would-be Nambu-Goldstone boson η:

$$\begin{aligned} -V &= \mu^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \\ &= 2 f_1 f_2 \mu^2 \cos\left(\frac{f\eta}{\sqrt{2} f_1 f_2}\right)\left[1 - \frac{f^2}{2 f_1^2 f_2^2}(h^\dagger h) + \frac{f^4}{24 f_1^3 f_2^3}(h^\dagger h)^2 + \dots\right] \end{aligned}$$

- In $SU(4)$ LHM, however, the model produces a tree-level quartic coupling from the following terms, so doesn't reqire the above μ term:

$$\kappa_{11}|\Phi_1^\dagger \Psi_1|^2 + \kappa_{22}|\Phi_2^\dagger \Psi_2|^2 + \kappa_{12}|\Phi_1^\dagger \Psi_2|^2 + \kappa_{21}|\Phi_2^\dagger \Psi_1|^2$$

Model Construction - Scalar Sector

- Start from a non-linear sigma model $[SU(4)/SU(3)]^4$ with four complex quadruplets scalar fields $\Phi_{1,2}$ and $\Psi_{1,2}$

  $\Longrightarrow$ Diagonal $SU(4)$ is gauged.

- Gauge symmetry breaking: $SU(4)_w \times U(1)_X \to SU(2)_w \times U(1)_Y$

  $\Longrightarrow$ 12 new gauge bosons with masses of order the scale f.

- Global symmetry breaking: $[SU(4)]^4 \to [SU(3)]^4$

  $\Longrightarrow$ 12 of the 28 degrees of freedom in the Φ_i and Ψ_i are eaten by the Higgs mechanism when $SU(4)_w$ is broken.

  $\Longrightarrow$ Remaining 16 consist of two complex doublets h_u and h_d, three complex $SU(2)$ singlets σ_1, σ_2 and σ_3, and two real scalars η_u and η_d.

  (Kaplan and Schmaltz 2003)

Model Construction - Scalar Sector

One possible parametrization of the non-linear sigma model fields Φ_i and Ψ_i with general f_i ($SU(4)$ breaking is not aligned):

$$\Phi_1 = e^{+i\mathcal{H}_u \frac{f_{12}}{f_{11}}} \begin{pmatrix} 0 \\ 0 \\ f_{11} \\ 0 \end{pmatrix} \qquad \Phi_2 = e^{-i\mathcal{H}_u \frac{f_{11}}{f_{12}}} \begin{pmatrix} 0 \\ 0 \\ f_{12} \\ 0 \end{pmatrix}$$

$$\Psi_1 = e^{+i\mathcal{H}_d \frac{f_{22}}{f_{21}}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_{21} \end{pmatrix} \qquad \Psi_2 = e^{-i\mathcal{H}_d \frac{f_{21}}{f_{22}}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_{22} \end{pmatrix}$$

where

$$\mathcal{H}_u = \begin{pmatrix} 0 & 0 & h_u & 0 \\ 0 & 0 & & 0 \\ h_u^\dagger & & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} / f_1 \qquad \mathcal{H}_d = \begin{pmatrix} 0 & 0 & 0 & h_d \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 \\ & h_d^\dagger & 0 & 0 \end{pmatrix} / f_2$$

$$f_i^2 = \sum_{j=1,2} f_{ij}^2$$

Model Construction - Gauge Boson Sector

- Most general expression for the electric charge generator:

$$Q = aT_{3L} + \frac{1}{\sqrt{3}} bT_{8L} + \frac{1}{\sqrt{6}} cT_{15L} + XI_4$$

- Correct embedding of the SM isospin $SU(2)_L$ doublets gives $a = 1$.
- Charge operator acting on the representations 4 and $\bar{4}$ of $SU(4)_L$:

$$Q[4] = diag\left(\frac{1}{2} + \frac{b}{6} + \frac{c}{12} + X, -\frac{1}{2} + \frac{b}{6} + \frac{c}{12} + X, -\frac{2b}{6} + \frac{c}{12} + X, -\frac{3c}{12} + X\right)$$

$$Q[\bar{4}] = diag\left(-\frac{1}{2} - \frac{b}{6} - \frac{c}{12} + X, \frac{1}{2} - \frac{b}{6} - \frac{c}{12} + X, \frac{2b}{6} - \frac{c}{12} + X, \frac{3c}{12} + X\right)$$

- If we demand for gauge bosons with electric charges 0 and ±1 only, there are not more than four different possibilities:

$$b = c = 1; \quad b = c = -1; \quad b = 1 \;\&\; c = -2; \quad \underbrace{b = -1 \;\&\; c = 2}$$

We focus on this scenario

Model Construction - Gauge Boson Sector

- Covariant derivative for 4-plets:

$$iD_\mu = i\partial_\mu + gT^\alpha A_\mu^\alpha + g_X X A_\mu^X$$

- There are 15 gauge bosons associated with $SU(4)_L$:

$$T^\alpha A_\mu^\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} D_{1\mu}^0 & W_\mu^+ & Y_\mu^0 & X_\mu'^+ \\ W_\mu^- & D_{2\mu}^0 & X_{1\mu}^- & Y_\mu'^0 \\ \bar{Y}_\mu^0 & X_{1\mu}^+ & D_{3\mu}^0 & W_\mu'^+ \\ X_\mu'^- & \bar{Y}_\mu'^0 & W_\mu'^- & D_{4\mu}^0 \end{pmatrix}$$

where

$$D_1^{0\mu} = A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{12} \qquad D_2^{0\mu} = -A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{12}$$

$$D_3^{0\mu} = -2A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{12} \qquad D_4^{0\mu} = -3A_{15}^\mu/\sqrt{12}$$

Model Construction - Gauge Boson Sector

- Mass matrix for the neutral Hermitian gauge bosons:

$$\frac{1}{4}g^2 \begin{pmatrix} \frac{v^2}{2} & -\frac{t}{2\sqrt{2}}v^2 & 0 & 0 \\ -\frac{t}{2\sqrt{2}}v^2 & \frac{t^2}{2}f^2 & \frac{t}{\sqrt{2}}\left(f^2 - \frac{v^2}{2}\right) & \frac{t}{\sqrt{2}}\triangle f^2 \\ 0 & \frac{t}{\sqrt{2}}\left(f^2 - \frac{v^2}{2}\right) & f^2 - \frac{v^2}{2} & \triangle f^2 \\ 0 & \frac{t}{\sqrt{2}}\triangle f^2 & \triangle f^2 & f^2 \end{pmatrix} \qquad \begin{matrix} A_3 \\ A_X \\ \frac{1}{\sqrt{3}}A_8 - \frac{2}{\sqrt{6}}A_{15} \\ \frac{1}{\sqrt{3}}A_8 + \frac{1}{\sqrt{6}}A_{15} \end{matrix}$$

where

$$f^2 = f_1^2 + f_2^2, \quad \triangle f^2 = f_1^2 - f_2^2, \quad v^2 = v_1^2 + v_2^2 \ (\gg \triangle v^2), \quad t = g_X/g$$

Model Construction - Gauge Boson Sector

- Physical Gauge Boson Masses:
  - $M_W^2 = \frac{1}{4} g^2 \left(v_1^2 + v_2^2\right), \qquad M_Y^2 = \frac{1}{2} g^2 f_1^2, \qquad M_{Y'}^2 = \frac{1}{2} g^2 f_2^2$
  - $M_X^2 = \frac{1}{4} g^2 \left(2f_1^2 - v_1^2 + v_2^2\right), \qquad M_{X'}^2 = \frac{1}{4} g^2 \left(2f_2^2 + v_1^2 - v_2^2\right)$
  - $M_{W'}^2 = \frac{1}{4} g^2 \left(2f_1^2 + 2f_2^2 - v_1^2 - v_2^2\right)$
  - $M_Z^2 = \frac{1}{2} g^2 \frac{1+t^2}{2+t^2} \left(v_1^2 + v_2^2 - \frac{t^4}{(2+t^2)^2} \frac{v_1^2 v_2^2}{f_1^2 + f_2^2} \right)$
  - $M_{Z'}^2 = \frac{1}{4} g^2 (2+t^2) \left(f_1^2 + f_2^2\right) - M_Z^2, \qquad M_{Z''}^2 = \frac{1}{4} g^2 \left(f_1^2 + f_2^2\right)$

  where

$$v_1^2 \equiv v_u^2 - \frac{v_u^2}{6f_1^4} \left(\frac{f_{12}^4}{f_{11}^2} + \frac{f_{11}^4}{f_{12}^2} \right), \quad v_2^2 \equiv v_d^2 - \frac{v_d^2}{6f_2^4} \left(\frac{f_{22}^4}{f_{21}^2} + \frac{f_{21}^4}{f_{22}^2} \right)$$

Model Construction - Fermion Sector

- Prerequisite for the fermion sector construction:
  - Accommodate the known left-handed quark and lepton isodoublets in the two upper components of 4 and $\bar{4}$ (or $\bar{4}$ and 4)
  - Don't allow for electrically charged antiparticles in the two lower components of the multiplets (antiquarks violate $SU(3)_c$, and e^+, μ^+ and τ^+ violate lepton number at tree level)
  - Forbid the presence of exotic electric charges in the possible models

$\Longrightarrow$ The electric charge of the third and fourth components in 4 and $\bar{4}$ must be equal either to the charge of the first and/or second component.

$\Longrightarrow$ b and c can take only the four sets of values.

Model Construction - Fermion Sector

- For example, each fermion 4-plet takes the following form:

$$b = c = 1 : \begin{pmatrix} u \\ d \\ D \\ D' \end{pmatrix} \quad or \quad b = c = -1 : \begin{pmatrix} d \\ u \\ U \\ U' \end{pmatrix}$$

$$b = 1 \& \ c = -2 : \begin{pmatrix} u \\ d \\ D \\ U \end{pmatrix} \quad or \quad b = -1 \& \ c = 2 : \begin{pmatrix} u \\ d \\ U \\ D \end{pmatrix}$$

- Our choice is to have the duplicated extra heavy fermions which can remove the quadratic divergences due to their SM fermion partners.

Model Construction - Fermion Sector

- There are total five possible ways to construct anomaly-free fermion spectra for $b = -1$ & $c = 2$.
  (Ponce, Gutiérrez, and Sánchez 2004)
- We choose the following set of fermion spectra which would be feasible enough to complete this little Higgs model (Kong 2003):

	$U(1)_Y$-states		
$(3_C, 4_L, \frac{1}{6})$	$\frac{1}{6}[Q]$	$\frac{2}{3}(T)$	$\frac{-1}{3}(B)$
$2\ (3_C, \bar{4}_L, \frac{1}{6})$	$2\ \frac{1}{6}[2\ Q]$	$2\ \frac{-1}{3}(D, S)$	$2\ \frac{2}{3}(U, C)$
$3\ (l_C, 4_L, \frac{-1}{2})$	$3\ \frac{-1}{2}[3\ L]$	$3\ \ 0(3\ N)$	$3\ \ \text{-}1(3\ E^-)$
$6\ (\bar{3}_C, 1_L, \frac{-2}{3})$	$6\ \frac{-2}{3}\ (\bar{u}, \bar{c}, \bar{t}, \bar{U}, \bar{C}, \bar{T})$		
$6\ (\bar{3}_C, 1_L, \frac{1}{3})$	$6\ \frac{1}{3}\ (\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S}, \bar{B})$		
$6\ (1_C, 1_L, 1)$	$3\ 1\ (e^+, \mu^+, \tau^+)$		$3\ \ 1(3\ E^+)$

$\Longrightarrow$ Anomaly cancellation is achieved when $N_f = N_c = 3 \Rightarrow$ one of the best features of this model.

Outline

Phenomenological Constraints

- Custodial $SU(2)$ symmetry violating shift in the Z mass:

$$\delta\rho \equiv \alpha T \simeq \frac{t^4}{4(2+t^2)^2}\frac{v^2}{f^2} \qquad (f^2 = f_1^2 + f_2^2)$$

- For $T \lesssim 0.15$ (95% CL), $f_{SU(4)} \gtrsim 1.07$ TeV
  $\Longrightarrow$ comparable to $f_{SU(3)} \gtrsim 2.3$ TeV
- Also, combined analysis of Z pole observables and the atomic parity violation data gives (for $f_1 \approx f_2$):

$$M_{Z'} \gtrsim 0.67\ \mathrm{TeV} \quad \Rightarrow \quad f_{SU(4)} \gtrsim 1.23\ \mathrm{TeV}$$

- Further phenomenological study (especially of fermion sector) is required to test the model itself - *In progress*

Outline

Summary

- As an alternative to SUSY, a little Higgs model is proposed to describe a TeV scale effective theory.
- It is natural to complete the LHM with all nice features of the SM such as the gauge anomaly cancellation conditions which dictate the SM fermionic spectrum.
- Sensible phenomenological analysis of such models has to start with feasible complete global quantum number assignment to all multiplets.
- Combined analysis of Z pole observables and the atomic parity violation data gives $\Lambda \sim 4\pi f \gtrsim 15$ TeV.
- Further phenomenological study is in progress.