Neutrino Mass and Mixing with Discrete Flavor Symmetries

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Outline

- A Brief introduction to neutrino physics
- Zero 1-3 mixing scenario (A Z_2 x Z_2 model)
- *An* S_3 model
- A_4 models
- Summary

The Standard Model

Higgs mechanism + EWSB

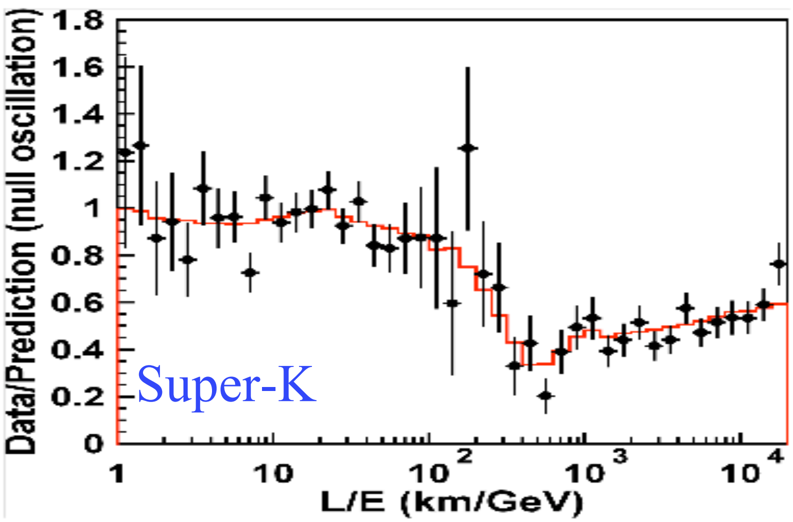
$$\bar{F}_L H F_R + h.c.$$

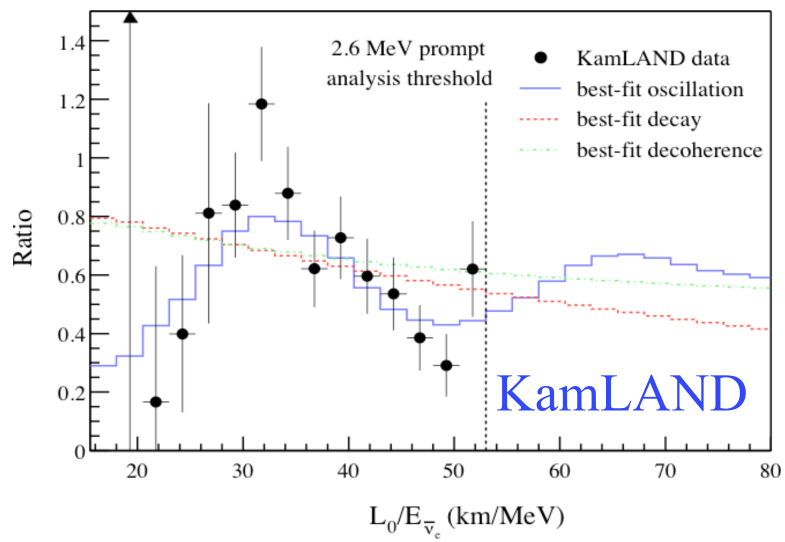
no right-handed neutrinos

$\downarrow$

massless neutrinos

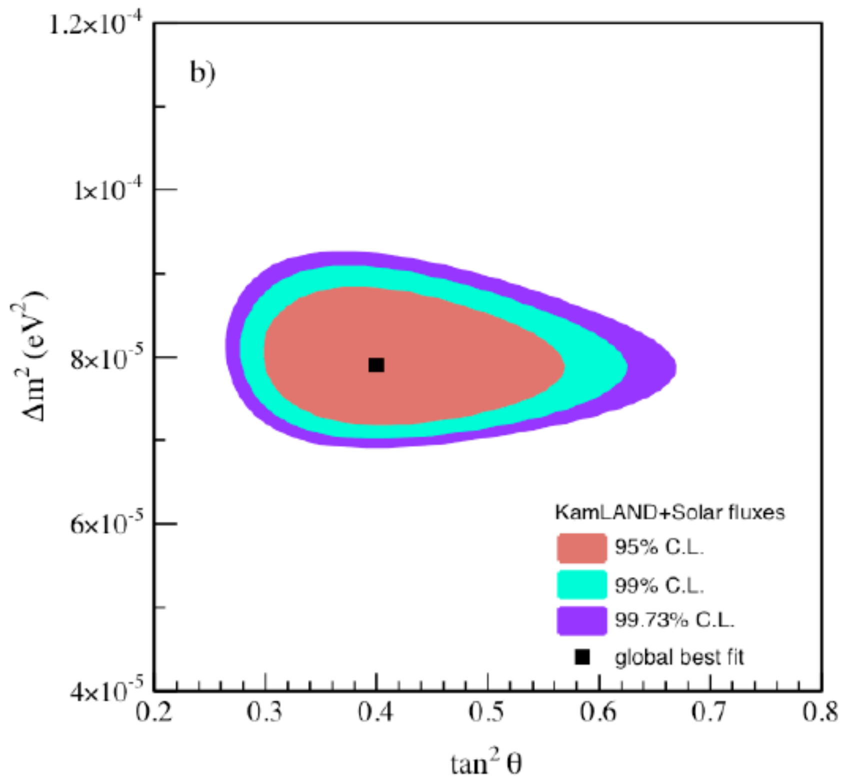
Neutrino masses, mixing and oscillations

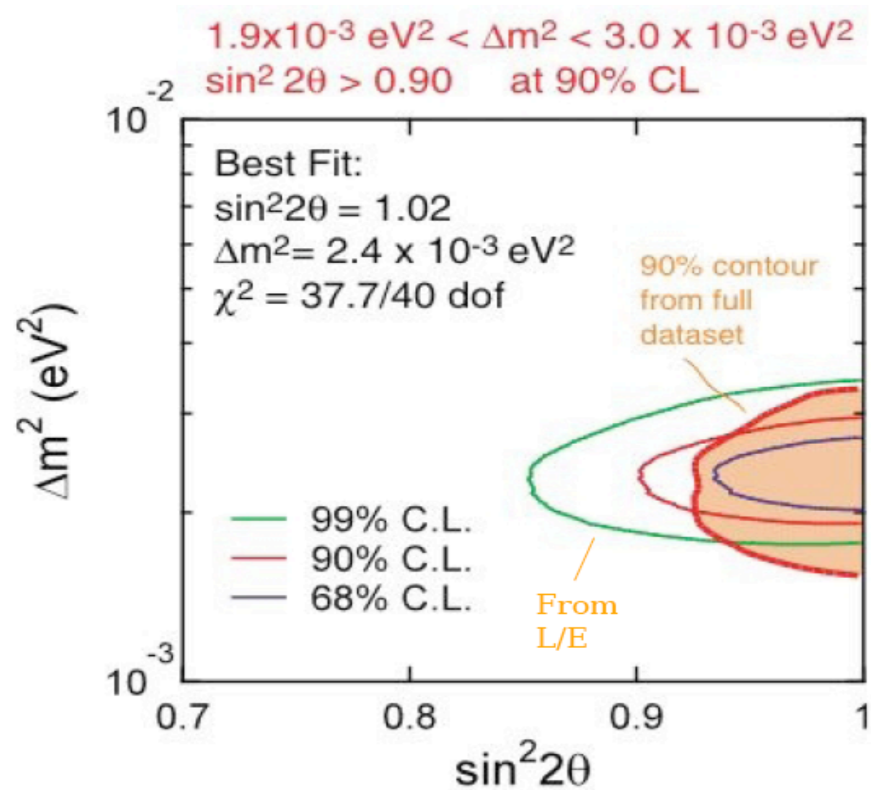




Neutrino Oscillation

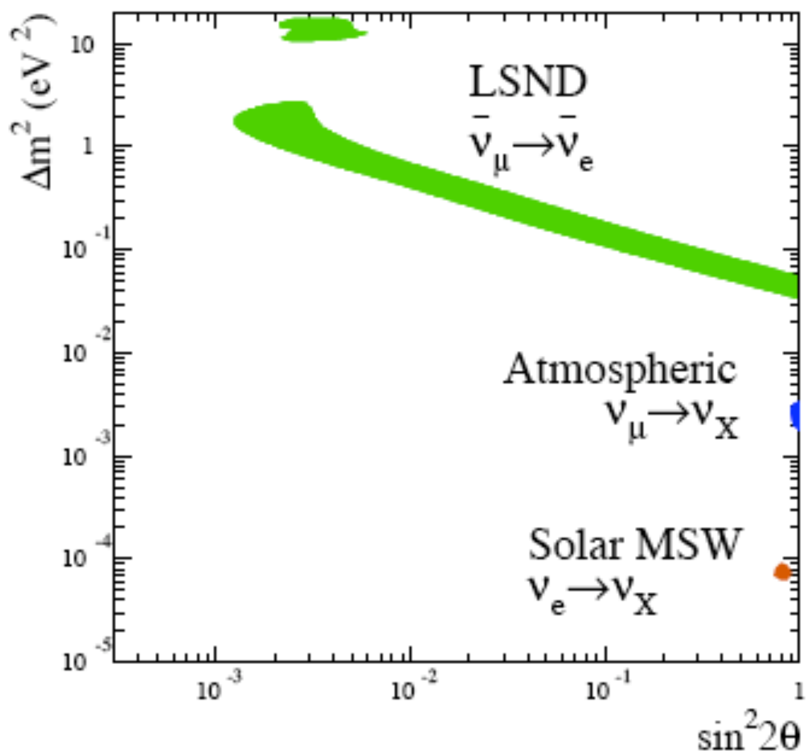
Neutrino masses, mixing and oscillations



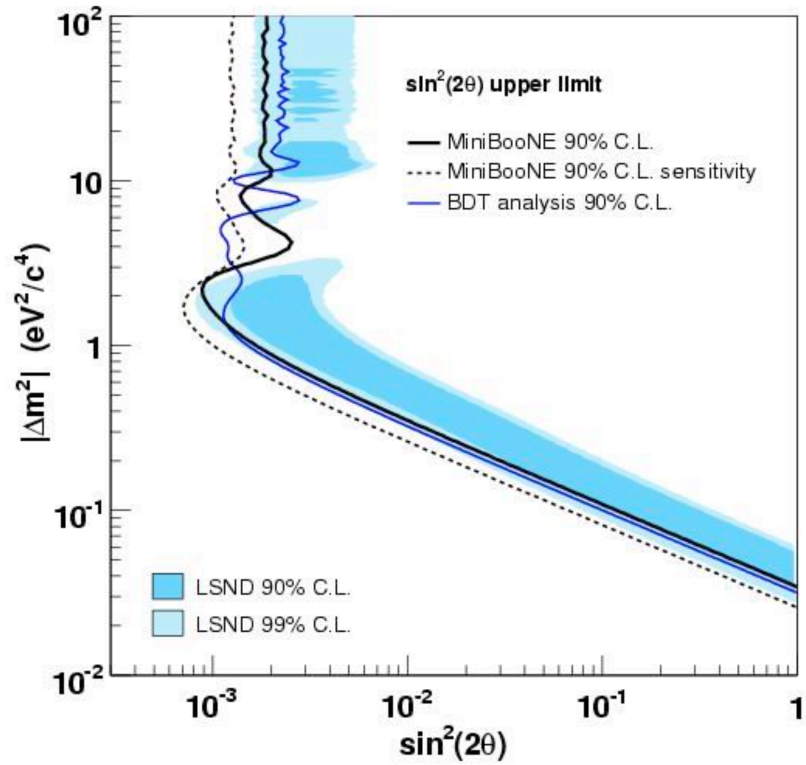


Solar (left) and atmospheric (right) mass squared difference and mixing angle

LSND - MiniBooNE



accelerator exp. (LSND)
$\Delta m^2 \sim 1$ eV^2



MiniBooNE
says no

Current interpretation of data (-LSND)

$$M_\nu = U_{\rm MNS} M_\nu^{\rm diag} U_{\rm MNS}^T, \quad M_e = M_e^{\rm diag}$$

$$U_{\rm MNS} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric } \nu,\ \text{K2K}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{reactor } \nu,\ \text{CP violation}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar } \nu,\ \text{KamLAND}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho} & 0 \\ 0 & 0 & e^{i\sigma} \end{pmatrix}}_{\text{Majorana phases}}$$

Oscillation parameter		central value	99% CL range
solar mass splitting	Δm^2_{12} =	$(8.0 \pm 0.3)\, 10^{-5} {\rm eV}^2$	$(7.2 \div 8.9)\, 10^{-5} {\rm eV}^2$
atmospheric mass splitting	$\|\Delta m^2_{23}\|$ =	$(2.5 \pm 0.3)\, 10^{-3} {\rm eV}^2$	$(1.7 \div 3.3)\, 10^{-3} {\rm eV}^2$
solar mixing angle	$\tan^2 \theta_{12}$ =	0.45 ± 0.05	$30^\circ < \theta_{12} < 38^\circ$
atmospheric mixing angle	$\sin^2 2\theta_{23}$ =	1.02 ± 0.04	$36^\circ < \theta_{23} < 54^\circ$
'CHOOZ' mixing angle	$\sin^2 2\theta_{13}$ =	0 ± 0.05	$\theta_{13} < 10^\circ$

Neutrino masses, mixing and oscillations

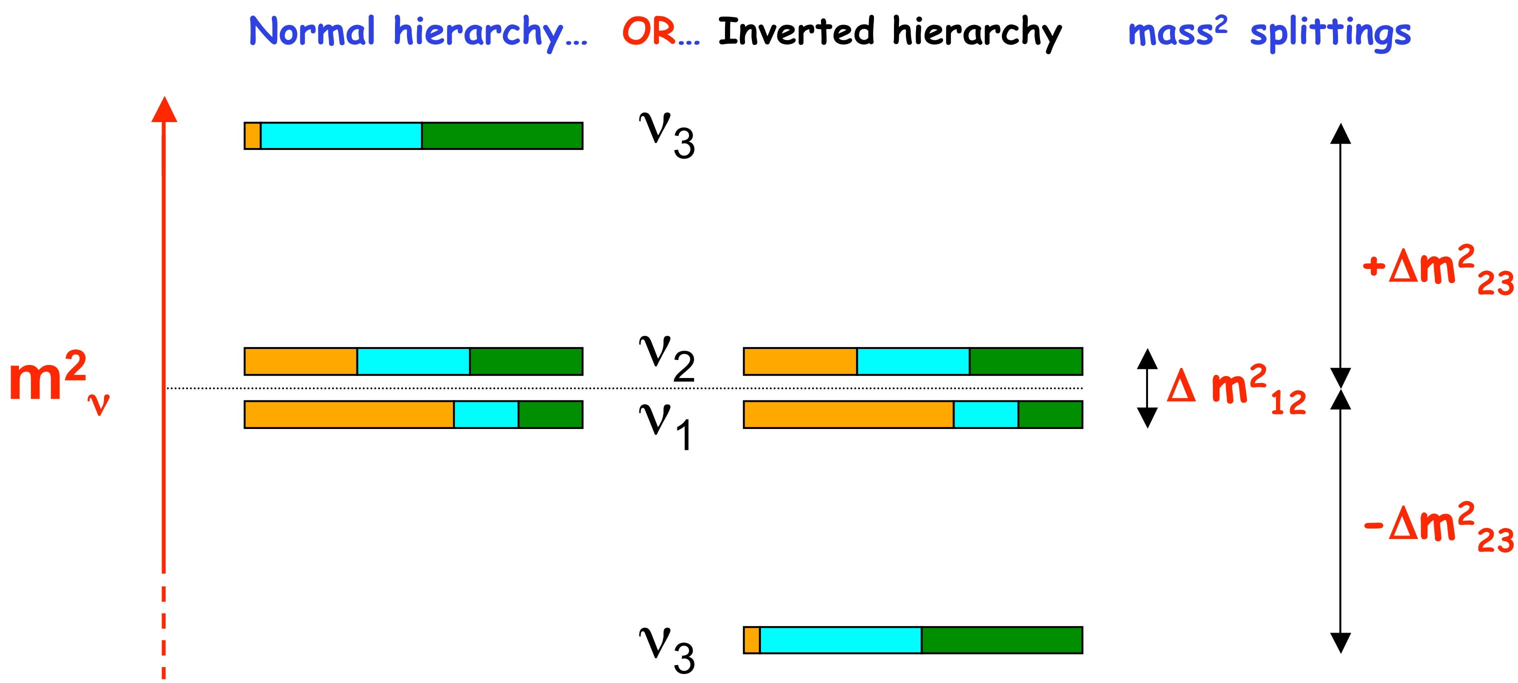
$$-\mathcal{L} = \frac{g}{\sqrt{2}} \bar{u}_L \gamma_\mu U_u^\dagger U_d d_L W^\mu + \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma_\mu U_\ell^\dagger U_\nu \nu_L W^\mu + h.c$$

$$V_{CKM} = U_u^\dagger U_d$$

$$U_{MNSP} = U_\ell^\dagger U_\nu$$

Quark mixing: $\theta_{12} = 13^\circ$; $\theta_{23} = 2.3^\circ$; $\theta_{13} = 0.2^\circ$; 90% C.L.

Lepton mixing: $\theta_{12} = 30^\circ \sim 38^\circ$; $\theta_{23} = 36^\circ \sim 54^\circ$; $\theta_{13} < 10^\circ$; 99% C.L.



- The cosmological bound on neutrino masses

$$\sum m_i \leq 0.69\,\mathrm{eV}\,(95\%\,\mathrm{C.L.})$$

…under some assumptions.

Mass generate mechanism

- Dirac Neutrino Mass

too small Yukawa couplings

- Majorana Neutrino Mass

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{\mathcal{O}_5}{\Lambda} + \frac{\mathcal{O}_6}{\Lambda^2}.$$

$$\frac{f_{ij}}{2\Lambda}\nu_i\nu_j\phi^0\phi^0 \Rightarrow (\mathcal{M}_\nu)_{ij} = f_{ij}\frac{\langle\phi^0\rangle^2}{\Lambda}.$$

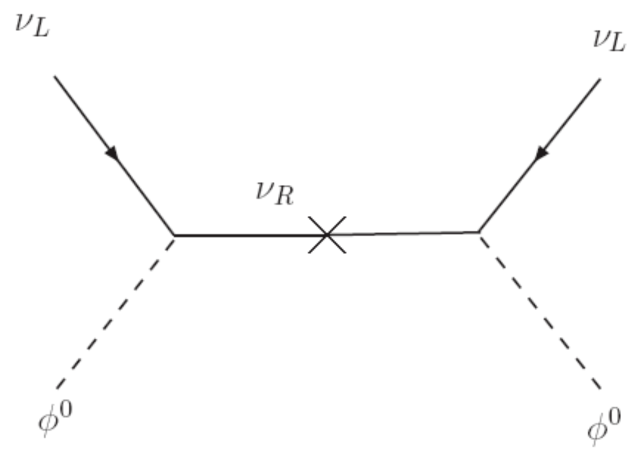
seesaw type mechanism

Type-I and Type-II Seesaw

Type-I seesaw: SM + ν_R

$$\mathcal{M} = \begin{pmatrix} 0 & \mathcal{M}_D \\ \mathcal{M}_D^T & \mathcal{M}_R \end{pmatrix}$$

$$\mathcal{M}_\nu = -\mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T.$$

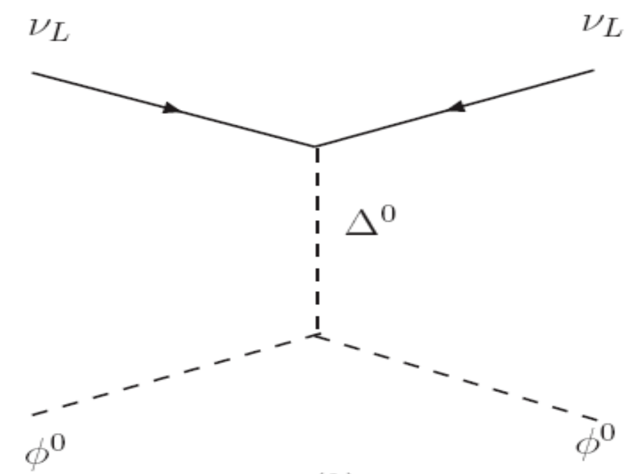


(a)

Type-II seesaw: SM + triplet Higgs

$$f L^T L \Delta + h.c.$$

$$m_L = f \langle \Delta \rangle.$$



(b)

$$\mathcal{M}_\nu = \mathcal{M}_L - \mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T$$

hybrid or mixed seesaw

Model building: Top-down and Bottom-up approach

- Top-down:
  - starts with a general unified theory framework (GUTs, extra-dimension theory, etc.);
- Bottom-up:
  - reconstruct the neutrino mass matrix in the flavor basis using the available information on masses and mixings;
  - identify the symmetry and mechanism of symmetry violation;

Discrete family symmetry

$$G_{SM} \otimes G_F$$

- Different fermion generations related by a symmetry;
- Discrete groups have more low dimensional rep. than continuous Lie groups;
- Non-Abelian groups can relate different generations, because of irreducible representations with dimension larger than one;
- Features of fermion mixing can be related to the structure of the Higgs sector.

S_3 : Phys. Lett. B73, 61 (1978); Phys. Lett. B78, 459 (1978); Phys. Rev. D19, 317 (1979); Phys. Rev. D43, 2761 (1991) ; Prog. Theor. Phys. 109, 795 (2003) ; Phys. Lett. B568, 83 (2003); Phys. Rev. D70, 055004 (2004); Phys. Rev. D61, 033012 (2000); JHEP 0508, 013(2005) ; Phys. Lett. B622, 327 (2005); Phys. Rev. Lett. 75, 3985 (1995); hep-ph/0610061; etc.

A_4 : Phys. Rev. D 19, 3369 (1979); Phys. Rev. D 64,113012 (2001); Phys. Lett. B 552, 207 (2003); Nucl. Phys. B 720, 64 (2005); Phys. Lett. B630(2005) 58; Nucl. Phys. B 741(2006)215, etc.

S_4 : Phys. Lett. B329,463 (1994); Phys. Rev. D69,053007 (2004) ; Phys. Lett. B632, 352 (2006); hep-ph/0602244; etc.

D_4 : Phys. Lett. B572, 189 (2003); JHEP 0407, 078 (2004); Nucl. Phys. B704, 3 (2005); etc.

D_5 : hep-ph/0409288; hep-ph/0604265; etc.

D_6 : Phys. Rev. D60, 096002 (1999); etc.

$\Delta(27)$: hep-ph/0607056;, hep-ph/0607045.

$Q_{8(4)}$: Phys. Rev. D 71, 011901(R) (2005)

$Q_{12(6)}$: Phys. Rev. D71, 056006 (2005).

$\Sigma(81)$: hep-ph/0701016.

…… and more to count.

Non-Abelian discrete groups

order	6	8	10	12	24	...
S_N	S_3				S_4	...
D_N	$D_3(=S_3)$	D_4	D_5	D_6		...
Q_N		$Q_8(Q_4)$		$Q_{12}(Q_6)$		...
T				$T(A_4)$		...

Z_2 x Z_2 Model

- Experimental data:

  Nearly maximal 2-3 mixing

  Small 1-3 mixing

- How small is θ_{13} ?
- Vanishing θ_{13} ⇔ Discrete Symmetry?

$$\theta_{23} = \pi/4 \quad \theta_{13} = 0$$

⇔ μ-τ symmetry

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b+2c & d & d \\ d & b & a+b \\ d & a+b & b \end{pmatrix}$$

arbitrary θ_{12}

Some Discrete Models For $\theta_{13} = 0$

- C. Low, hep-ph/0404017 & 0501251,

  (Abelian Symmetries Classification)
- W. Grimus and L. Lavoura, hep-ph/0310050
- W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, M. Tanimoto, hep-ph/0407112

  (Non-abelian discrete)

$$\boxed{\theta_{13} = 0}$$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\theta_{13} = 0$$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{PMNS} = U_l^\dagger U_\nu$$

$$\mathcal{M}_l = \begin{pmatrix} X & 0 & 0 \\ 0 & X & X \\ 0 & X & X \end{pmatrix} \quad \mathcal{M}_\nu = \begin{pmatrix} X & X & 0 \\ X & X & 0 \\ 0 & 0 & X \end{pmatrix}$$

$$\theta_{13} = 0$$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$Z_2 \times Z_2$ Group

$(+,+)$, $(+,-)$, $(-,+)$, $(-,-)$

3 Family Leptons

2 Higgs Doublets

2 Higgs Triplets

	$Z_2 \times Z_2$
$(\nu_1, l_1), l_1^c$	$(+, -)$
$(\nu_2, l_2), l_2^c$	$(-, +)$
$(\nu_3, l_3), l_3^c$	$(-, -)$
(ϕ_1^0, ϕ_1^-)	$(+, +)$
(ϕ_2^0, ϕ_2^-)	$(+, -)$
$(\xi_1^{++}, \xi_1^+, \xi_1^0)$	$(+, +)$
$(\xi_2^{++}, \xi_2^+, \xi_2^0)$	$(-, -)$

- Charge Lepton Matrix

$$\mathcal{M}_l = \begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ 0 & e & c \end{pmatrix}, \quad \left\{ \begin{array}{lcl} \langle \phi_1^0 \rangle & \rightarrow & a, b, c \\ \langle \phi_2^0 \rangle & \rightarrow & d, e \end{array} \right.$$

- Neutrino Mass Matrix (Type-II seesaw)

$$\mathcal{M}_\nu = \begin{pmatrix} A & D & 0 \\ D & B & 0 \\ 0 & 0 & C \end{pmatrix}, \quad \left\{ \begin{array}{lcl} \langle \xi_1^0 \rangle & \rightarrow & A, B, C \\ \langle \xi_2^0 \rangle & \rightarrow & D \end{array} \right.$$

- Vanishing 1-3 mixing is realized under $Z_2 \times Z_2$ symmetry;
- This model does not constrain any mass or mixing except $\theta_{13} = 0$.

μ-τ Sector Lepton Flavor Violation

$$M_l^{2-3} = \begin{pmatrix} bv_1 & dv_2 \\ ev_2 & cv_1 \end{pmatrix} = \begin{pmatrix} c_L & s_L \\ -s_L & c_L \end{pmatrix} \begin{pmatrix} m_\mu & 0 \\ 0 & m_\tau \end{pmatrix} \begin{pmatrix} c_R & -s_R \\ s_R & c_R \end{pmatrix}$$

- 5 physical Higgs bosons:

$$h^\pm = \frac{v_2\phi_1^\pm - v_1\phi_2^\pm}{\sqrt{v_1^2+v_2^2}} ,$$

$$H^0 = \frac{\sqrt{2}(v_1 \Re e\phi_1^0 + v_2 \Re e\phi_2^0)}{\sqrt{v_1^2+v_2^2}} , \quad \text{(CP even)}$$

$$h^0 = \frac{\sqrt{2}(v_2 \Re e\phi_1^0 - v_1 \Re e\phi_2^0)}{\sqrt{v_1^2+v_2^2}} , \quad \text{(CP even)}$$

$$A^0 = \frac{\sqrt{2}(v_2 \Im m\phi_1^0 - v_1 \Im m\phi_2^0)}{\sqrt{v_1^2+v_2^2}} . \quad \text{(CP odd)}$$

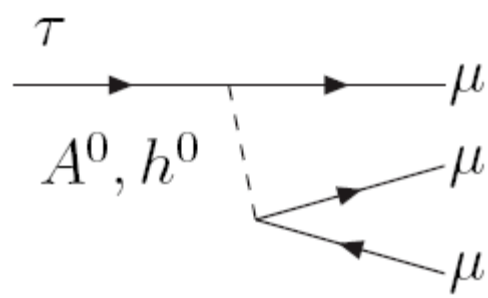
- Higgs decay to leptons

$$\frac{\Gamma(h^- \to \mu^-\nu)}{\Gamma(h^- \to \tau^-\nu)} = \frac{\sin^2 2\theta_R}{\cos^2 2\beta + \cos^2 2\theta_R} , \quad \text{not } \frac{m_\mu^2}{m_\tau^2} \text{ (MSSM)}$$

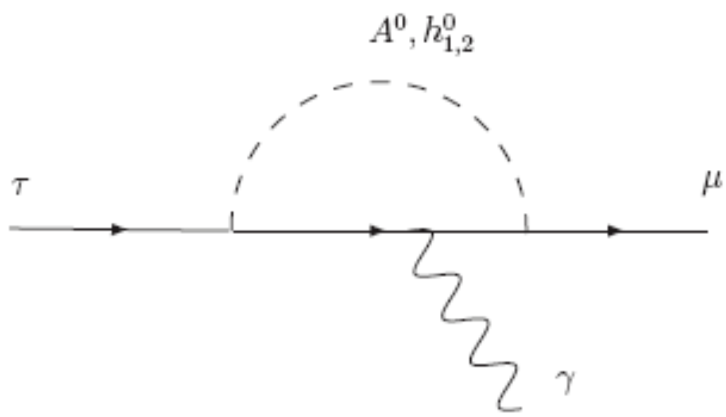
$$\frac{\Gamma(A^0 \to \mu^+\tau^-, \mu^-\tau^+)}{\Gamma(A^0 \to \tau^+\tau^-)} = \frac{\cos^2 2\theta_R}{\cos^2 2\beta} , \quad \frac{\Gamma(A^0 \to \mu^+\mu^-)}{\Gamma(A^0 \to \tau^+\tau^-)} = \frac{\sin^2 2\theta_R}{\cos^2 2\beta} .$$

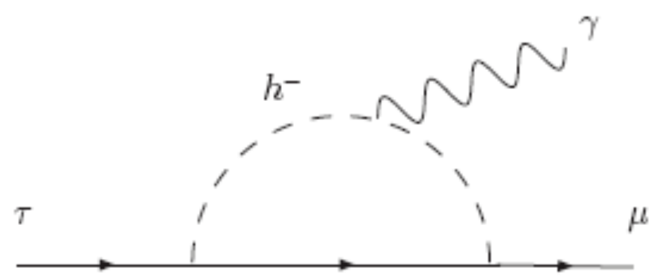
Lepton Flavor Violation mediated by **h^0, A^0, h^-** :

$$\tau \longrightarrow 3\mu$$



$$\tau \longrightarrow \mu\gamma$$





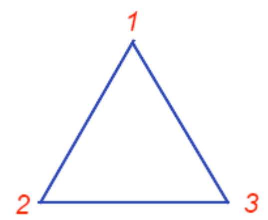
the anomalous magnetic moment of the muon

The permutation group S_3

- The smallest non-abelian group:

  6 elements (symmetry group of equilateral triangle)

E;
(123), (132);
(12), (23), (13).



Irreducible representations: 1, 1', 2
1 x 1' =1'; 1' x 1'=1; 2 x 2 = 1 + 1' + 2.

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \in \mathbf{2} \quad \Rightarrow \quad \begin{array}{l} (\psi_1\phi_2 + \psi_2\phi_1) \in \mathbf{1} \\ (\psi_1\phi_2 - \psi_2\phi_1) \in \mathbf{1'} \end{array} \quad \begin{pmatrix} \psi_2\phi_2 \\ \psi_1\phi_1 \end{pmatrix} \in \mathbf{2}$$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \in \mathbf{2} \quad \Rightarrow \quad \begin{pmatrix} \psi_2^\dagger \\ \psi_1^\dagger \end{pmatrix} \in \mathbf{2}$$

Origin of the (nearly) maximal 2-3 mixing

$$\mathbf{M_l} = \begin{pmatrix} m_\mu & 0 \\ 0 & m_\tau \end{pmatrix} \qquad \mathbf{M_\nu} = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

dominant 2-3 block

$$M_\nu = \begin{pmatrix} & & \\ & \mathcal{O}(1) & \mathcal{O}(1) \\ & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

⇒ **large 2-3 mixing and normal hierarchy**

An S_3 model:

Lepton 2-3 sector

- 2 Higgs doublets
- 2 Higgs triplets

$$\begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix}, \quad \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \quad \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \in \mathbf{2}.$$

$$\mu^c \in \mathbf{1}, \quad \tau^c \in \mathbf{1}'.$$

- Invariant terms:

$$f_1\,(\tau\phi_1^0 + \mu\phi_2^0)\mu^c, \quad f_2\,(\tau\phi_1^0 - \mu\phi_2^0)\tau^c, \quad f_3\,(\nu_\mu\nu_\mu\xi_1^0 + \nu_\tau\nu_\tau\xi_2^0)$$

- S_3 symmetry ⇔ the $V(\varphi_1, \varphi_2)$ is minimized by $v_1=v_2=v$

$$M_l = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \sqrt{2}f_1 v & 0 \\ 0 & \sqrt{2}f_2 v \end{pmatrix},$$

$$M_\nu = \begin{pmatrix} f_3 u_1 & 0 \\ 0 & f_3 u_2 \end{pmatrix}.$$

S_3 model:

electron sector

- Higgs doublet ϕ_3 provides mass to the electron

$$L_e \, , \; e^c \, , \; \Phi_3 \in \mathbf{1} \, .$$

- Invariants

$$(\tau\phi_1^0 + \mu\phi_2^0)e^c \, , \quad ee^c\phi_3^0 \, , \quad e\mu^c\phi_3^0 \, , \quad (\nu_\tau\xi_1^0 + \nu_\mu\xi_2^0)\nu_e$$

- Mass matrix

$$M_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} f_4 v_3 & f_5 v_3 & 0 \\ 0 & \sqrt{2} f_1 v & 0 \\ 0 & 0 & \sqrt{2} f_2 v \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} 0 & f_6 u_2 & f_6 u_1 \\ f_6 u_2 & f_3 u_1 & 0 \\ f_6 u_1 & 0 & f_3 u_2 \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} 0 & f_6 u_2 & f_6 u_1 \\ f_6 u_2 & f_3 u_1 & 0 \\ f_6 u_1 & 0 & f_3 u_2 \end{pmatrix} = f_3 u_2 \begin{pmatrix} 0 & \epsilon_f & \epsilon_f \epsilon_u \\ \epsilon_f & \epsilon_u & 0 \\ \epsilon_f \epsilon_u & 0 & 1 \end{pmatrix}$$

- Normal hierarchy: $m_1 < m_2 < m_3$
- After diagonalization of charged lepton mass matrix, we have

$$m_{ee} \equiv |(M_\nu^{fl})_{11}| \approx \sqrt{2\Delta m^2_{atm}} |\epsilon_f| \theta^l_{12} \lesssim 10^{-2}\text{eV} \cdot \theta^l_{12}$$

tiny neutrinoless double beta decay

- Correlation between 1-3 mixing and solar and atmospheric parameters:

$$\theta_{13} \approx \frac{1}{2} \sin 2\theta_{12} \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \; .$$

$0.008 \lesssim \theta_{13} \lesssim 0.032$ 90% C.L. allowed ranges

S_3 model:

quark sector

- Let $$\begin{pmatrix} Q_2 \\ Q_3 \end{pmatrix} \in \mathbf{2}\,, \quad Q_1,\, u^c,\, c^c,\, d^c,\, s^c \in \mathbf{1}\,, \quad b^c,\, t^c \in \mathbf{1}'$$
- Maximal 2-3 mixings cancel for quarks

$$\theta_{23}^{q} = \theta_{23}^{u} - \theta_{23}^{d} = 0$$

- If v_2-v_1 =δv, a quark-lepton relation is predicted:

$$\theta_{23}^{q} \approx 2\left(\frac{\pi}{4} - \theta_{23}\right) = \frac{\delta v}{v}\,.$$

δv can be induced by S_3 soft-breaking term.

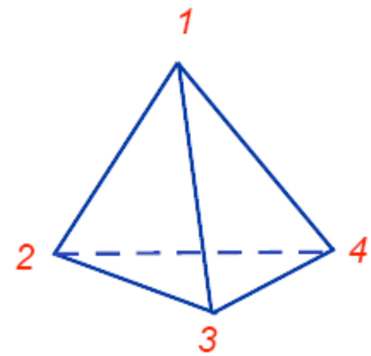
- CKM matrix:

$$\begin{aligned} U_{CKM} &\equiv U_{23}U_{13}U_{12} = \\ &= (U_{23}^{u}U_{12}^{u})^{\dagger}(U_{23}^{d}U_{12}^{d}) \equiv U_{12}^{u\dagger}U_{23}^{q}U_{12}^{d} \end{aligned}$$

Tetrahedron Group A_4

- A_4 is the even permutation of 4 objects. It is also the rotational symmetry group of the tetrahedron.

  - 12 elements:



E;
(243), (134), (142), (123);
(234), (143), (124), (132);
(12)(34), (13)(24), (14)(23).

  - Character table of A_4:

Class	n	h	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{3}$
$\mathcal{C}_1$	1	1	1	1	1	3
$\mathcal{C}_2$	4	3	1	ω	ω^2	0
$\mathcal{C}_3$	4	3	1	ω^2	ω	0
$\mathcal{C}_4$	3	2	1	1	1	-1

$$\omega = e^{i2\pi/3}$$

- Three 1-dimensional and one 3-dimensional irreducible representations

The irreducible representation multiplication rules of A_4

3 x 3 = 1 + 1' + 1'' + 3 + 3 and **1' x 1''=1**

If $\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \sim \mathbf{3}$,

We have:

$$\begin{aligned} (\psi_1\varphi_1 + \psi_2\varphi_2 + \psi_3\varphi_3) &\sim \mathbf{1}, \\ (\psi_1\varphi_1 + \omega^2\psi_2\varphi_2 + \omega\psi_3\varphi_3) &\sim \mathbf{1}', \\ (\psi_1\varphi_1 + \omega\psi_2\varphi_2 + \omega^2\psi_3\varphi_3) &\sim \mathbf{1}'', \end{aligned}$$

$$\begin{pmatrix} \psi_2\varphi_3 \\ \psi_3\varphi_1 \\ \psi_1\varphi_2 \end{pmatrix}, \begin{pmatrix} \psi_3\varphi_2 \\ \psi_1\varphi_3 \\ \psi_2\varphi_1 \end{pmatrix} \sim \mathbf{3}.$$

$$3 \times 3 = 1(11+22+33)+1'(11+\omega^2 22+\omega 33)+1''(11+\omega 22+\omega^2 33)$$
$$+3(23, 31, 12)+3(32, 13, 21)$$

$$(\nu_i, l_i) \sim 3 \quad \Rightarrow \quad \mathcal{M}_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+b\omega+c\omega^2 & d \\ e & d & a+b\omega^2+c\omega \end{pmatrix}$$

Type-II seesaw terms

Two Cases for charged lepton sector:

(I) $l_i^c \sim 1, 1', 1''$ **with** $(\phi_i^0, \phi_i^-) \sim 3$

$$\mathcal{M}_l = \begin{pmatrix} h_1 v_1 & h_2 v_1 & h_3 v_1 \\ h_1 v_2 & h_2 v_2 \omega & h_3 v_2 \omega^2 \\ h_1 v_3 & h_2 v_3 \omega^2 & h_3 v_3 \omega \end{pmatrix} \Rightarrow (\mathbf{I}): \quad U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

(II) $l_i^c \sim 3$ **with** $(\phi_i^0, \phi_i^-) \sim 1, 1', 1''$

$$\begin{pmatrix} m_e \\ m_\mu \\ m_\tau \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} h_1 v_1 \\ h_2 v_2 \\ h_3 v_3 \end{pmatrix} \Rightarrow (\mathbf{II}): \quad U_L = 1.$$

Tri-bamaximal realization under A_4 (E.Ma, PRD70,031901)

$$U_{l\nu} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

P. Harrison, D. Perkins, G. Scott (2002)
X.-G. He, A. Zee (2003)

$$l_i^c \sim 1, 1', 1'' \quad (\phi_i^0, \phi_i^-) \sim 3 \quad \xi_{1,2,3} \sim 1, 1', 1'', \xi_{4,5,6} \sim 3$$

if $\langle \xi_{5,6} \rangle = 0 \qquad b = c$

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = \begin{pmatrix} a + 2d/3 & b - d/3 & b - d/3 \\ b - d/3 & b + 2d/3 & a - d/3 \\ b - d/3 & a - d/3 & b + 2d/3 \end{pmatrix}$$

$$= U_{l\nu} \begin{pmatrix} a - b + d & 0 & 0 \\ 0 & a + 2b & 0 \\ 0 & 0 & -a + b + d \end{pmatrix} (U_{l\nu})^T$$

From $(\mathbf{II}):\quad U_L = 1.$

$l_i^c \sim \mathbf{3} \quad (\phi_i^0, \phi_i^-) \sim \mathbf{1}, \mathbf{1}', \mathbf{1}'' \quad \xi_{1,2,3} \sim \mathbf{1}, \mathbf{1}', \mathbf{1}'', \xi_{4,5,6} \sim \mathbf{3}$

if $\langle \xi_4 \rangle = \langle \xi_5 \rangle = \langle \xi_6 \rangle$

$$\mathcal{M}_\nu = \begin{pmatrix} a+b+c & d & d \\ d & a+b\omega+c\omega^2 & d \\ d & d & a+b\omega^2+c\omega \end{pmatrix}$$

$b = c \quad \Rightarrow$ **under the basis** $[\nu_e, (\nu_\mu+\nu_\tau)/\sqrt{2}, (-\nu_\mu+\nu_\tau)/\sqrt{2}]$

$$\mathcal{M}_\nu = \begin{pmatrix} a+2b & \sqrt{2}d & 0 \\ \sqrt{2}d & a-b+d & 0 \\ 0 & 0 & a-b-d \end{pmatrix}$$

maximal 2-3 mixing
vanishing 1-3 mixing

Approximate tri-bimiximal

An A_4 Model with Hybrid Seesaw :

- Hybrid seesaw

$$\mathcal{M}_\nu = \mathcal{M}_L - \mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T$$

- The texture zeroes of $\mathcal{M}_R$:

$$\mathcal{M}_R = \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$$

- $\mathcal{M}_R^{-1}$ has 3 zero sub-determinants. Assuming $\mathcal{M}_D$ diagonal:

$$\mathcal{M}_\nu^I = \begin{pmatrix} a & b & c \\ b & b^2/a & -bc/a \\ c & -bc/a & c^2/a \end{pmatrix}$$

…cannot reproduce all present data

- Add the contribution from Type-II seesaw:

$$\mathcal{M}_\nu = \begin{pmatrix} d+a & b & c \\ b & d+b^2/a & -bc/a \\ c & -bc/a & d+c^2/a \end{pmatrix}$$

- This pattern can fit all present data.
- It can be stabilized by a family symmetry A_4.

An A_4 Model (hybrid seesaw)

- Three families of Leptons

$$(\nu_i, l_i),\ l_i^c,\ \nu_i^c \sim \mathbf{3}$$

- Three Higgs doublets

$$\Phi_i \sim \mathbf{1}, \mathbf{1'}, \mathbf{1''}$$

- One Higgs triplet

$$\xi \sim \mathbf{1}$$

- Three Higgs singlets

$$\Sigma_i \sim \mathbf{3}$$

- Higgs doublets $\langle \Phi_i \rangle \Rightarrow$ diagonal $\mathcal{M}_D$ & $\mathcal{M}_\ell$

$$\begin{pmatrix} m_e \\ m_\mu \\ m_\tau \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} y_{l1}\langle \Phi_1 \rangle \\ y_{l2}\langle \Phi_2 \rangle \\ y_{l3}\langle \Phi_3 \rangle \end{pmatrix}$$

- Higgs triplet $\langle \xi \rangle \Rightarrow \mathcal{M}_L = d\,\mathbf{I}$

$$(\nu_1\nu_1 + \nu_2\nu_2 + \nu_3\nu_3)\xi^0$$

- Higgs singlets $\langle \Sigma_i \rangle \Rightarrow \mathcal{M}_R$

$$(\mathcal{M}_R)_{ij} \propto \langle \Sigma_k \rangle \text{ with } i \neq j \neq k$$

So under A4 symmetry we get:

$$\mathcal{M}_\nu = \begin{pmatrix} d+a & b & c \\ b & d+b^2/a & -bc/a \\ c & -bc/a & d+c^2/a \end{pmatrix}$$

Special case:

$$\theta_{23} = \pi/4 \text{ and } \theta_{13} = 0 \Leftrightarrow b^2 = c^2$$

Case b=c:

- In the basis $\{\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2}, (\nu_\tau - \nu_\mu)/\sqrt{2}\}$

$$\mathcal{M}_\nu = \begin{pmatrix} d+a & \sqrt{2}b & 0 \\ \sqrt{2}b & d & 0 \\ 0 & 0 & d+2b^2/a \end{pmatrix}$$

- mixing angle θ_{12} :

$$\tan 2\theta_{12} = \frac{2\sqrt{2}|[2Re(bd^*)+ab^*]|}{|d|^2-|d+a|^2}$$

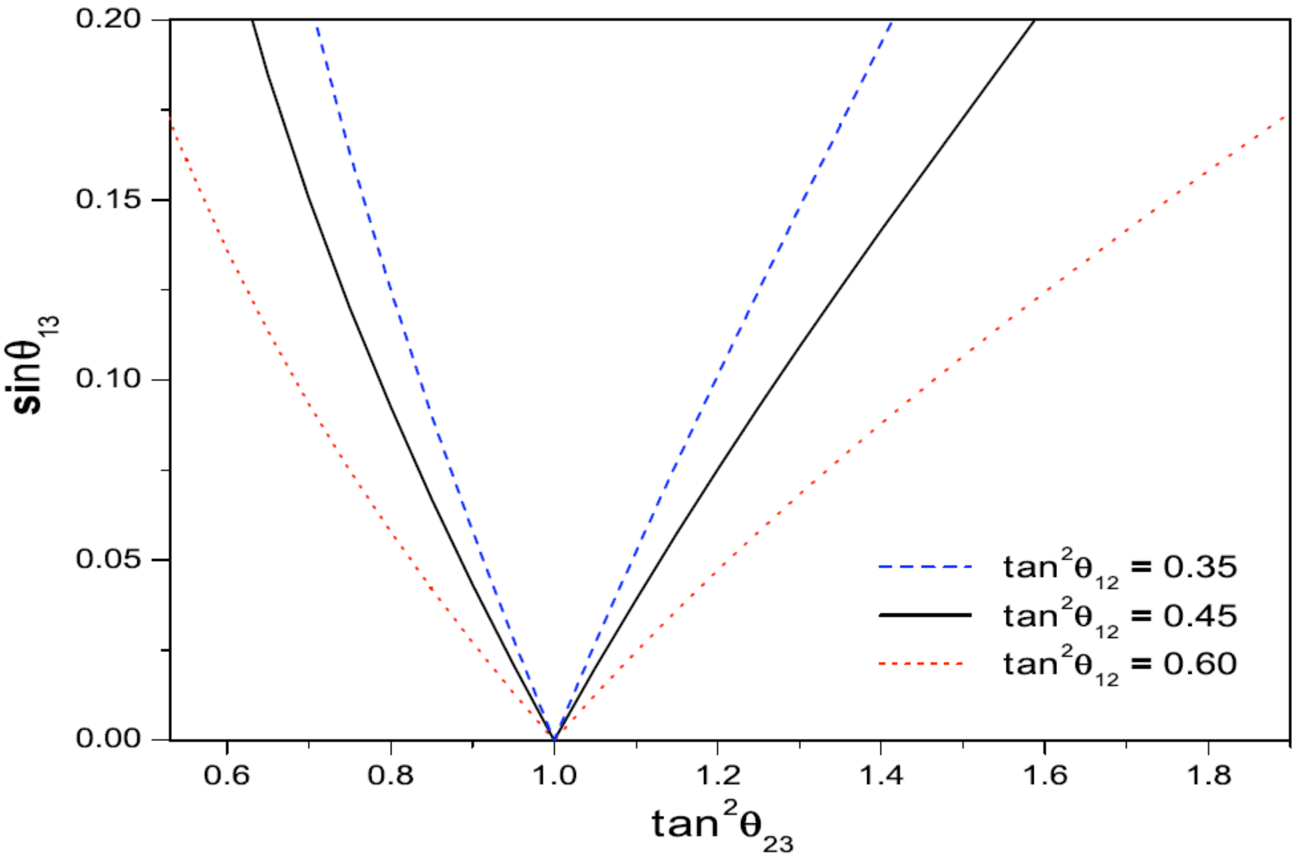
- The mass squared differences:

$$\Delta m^2_{sol} = \frac{|d|^2-|d+a|^2}{\cos 2\theta_{12}} ,$$

$$\pm\Delta m^2_{atm} = 4\left|\frac{b^2}{a}\right|^2 + 4Re\left(\frac{d^* b^2}{a}\right) - 2|b|^2 + \frac{1}{2}(|d|^2 - |d+a|^2)$$

Case b≠c:

$$\sin\theta_{13} \approx \frac{1}{\tan 2\theta_{23}} \frac{2}{\tan 2\theta_{12}}$$



No complex phases case

Summary:

- The discovery of neutrino masses and attempts to understand the flavor puzzle have made it quite natural to expect the existence of a family symmetry;
- **Discrete symmetries** are suitable to be the family symmetry:
    - can explain texture zeros and equalities in the mass matrix,
    - Can reproduce all the current data, (nearly maximal 2-3 mixing and tiny 1-3 mixing)
- The non-abelian discrete symmetries in general prove to be more restrictive and more predictive.
- Precision measurements of the deviation of the 2-3 mixing to maximal and non-zero 1-3 mixing are important to understand the physics.
- Extended Higgs sector (more Higgs doublets and triplets);
- A_4 family symmetry is successfully applied to get tri-bimaximal mixing and a hybrid seesaw scenario is realized under A_4 symmetry.

$$\delta a_\mu = \frac{m_\mu^2 m_\tau^2}{32\pi^2(v_1^2+v_2^2)\sin^2 2\beta}\left\{\cos^2 2\theta_R\left[\frac{1}{3m_{h^0}^2}+\frac{1}{3m_A^2}\right]+\right.$$
$$\left. + \sin^2 2\theta_R\left[\sum_{i=1}^{2}\frac{k_i}{m_i^2}\left(-\frac{7}{3}-2\log\frac{m_\mu^2}{m_i^2}\right)+\frac{1}{m_A^2}\left(\frac{11}{3}+2\log\frac{m_\mu^2}{m_A^2}\right)-\frac{1}{3m_{h^-}^2}\right]\right\}$$

$$\Gamma(\tau\to 3\mu) = \left[\frac{m_\tau^2\sin 2\theta_R\cos 2\theta_R}{2\sin^2 2\beta(v_1^2+v_2^2)}\right]^2\frac{m_\tau^5}{4096\pi^3}\left(\frac{1}{m_A^4}+\frac{1}{m_{h^0}^4}+\frac{2}{3m_A^2 m_{h^0}^2}\right)$$

$$\Gamma(\tau\to\mu\gamma) = \frac{\alpha_{em}m_\tau^5}{(64\pi^2)^2}(|A_L|^2+|A_R|^2),$$

where

$$A_L = \frac{1}{3}\left[\frac{m_\tau^2\sin 2\theta_R\cos 2\theta_R}{2\sin^2 2\beta(v_1^2+v_2^2)}\right]\left[\frac{1}{m_A^2}+\frac{1}{m_{h^0}^2}-\frac{1}{m_{h^-}^2}\right],$$

and

$$A_R = \left[\frac{m_\tau^2\cos 2\theta_R\cos 2\beta}{2\sin^2 2\beta(v_1^2+v_2^2)}\right]\left[\sum_{i=1}^{2}\frac{k_i}{m_i^2}\left(\frac{8}{3}+2\ln\frac{m_\tau^2}{m_i^2}\right)-\frac{1}{m_A^2}\left(\frac{10}{3}+2\ln\frac{m_\tau^2}{m_A^2}\right)\right]$$

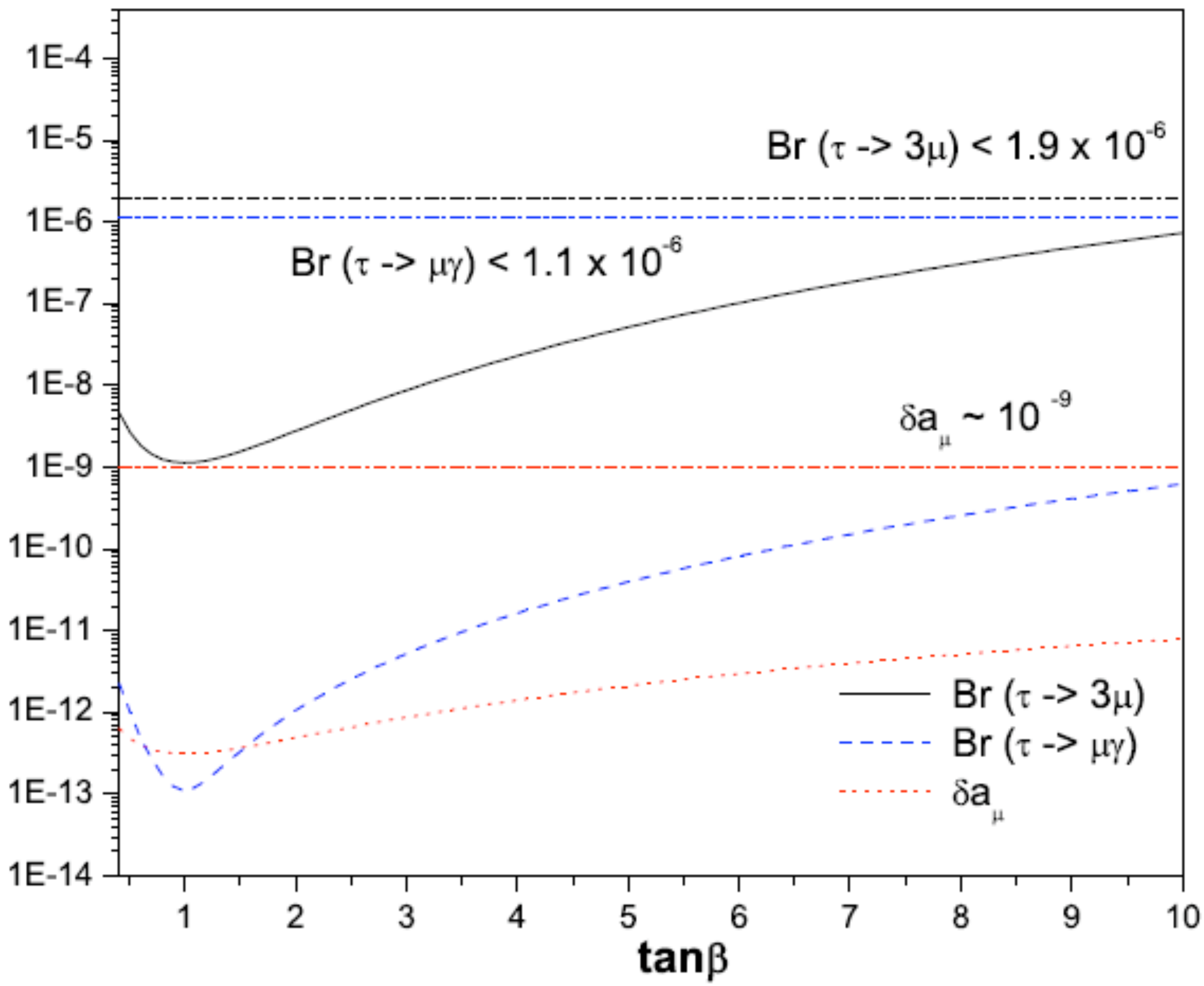
When take: $m_A = m_1 = m_2 = m_{h^-} = 100 GeV$

$$\cos 2\theta_R = \sin 2\theta_R = \sin 2\beta = 1/\sqrt{2}$$

	Branching Ratio	Experimental data
$\mu \to e\gamma$	$-$	$< 1.2 \times 10^{-11}$
$\tau \to 3\mu$	4.5×10^{-9}	$< 1.9 \times 10^{-6}$
$\tau \to \mu\gamma$	2.2×10^{-12}	$< 1.1 \times 10^{-6}$
δa_μ	6.2×10^{-13}	$\sim 10^{-9}$

But they scale with tanβ

LFV effects scale with tanβ



$$|m_3|^2 - \frac{1}{2}(|m_2|^2 + |m_1|^2) = -\frac{a^2 \tan^2 2\theta_{12}}{2}\left(1 - \frac{1}{8}\tan^2 2\theta_{12}\right) + \frac{1}{2}\Delta m^2_{sol}\left(1 - \frac{1}{2}\tan^2 2\theta_{12}\right)$$

- Small 1-3 mixing: