# Transport in Mesoscopic Devices and Quantum Dots

**A.M. CHANG**

Dept. of Physics, Duke University

Academia Sinica, Taipei

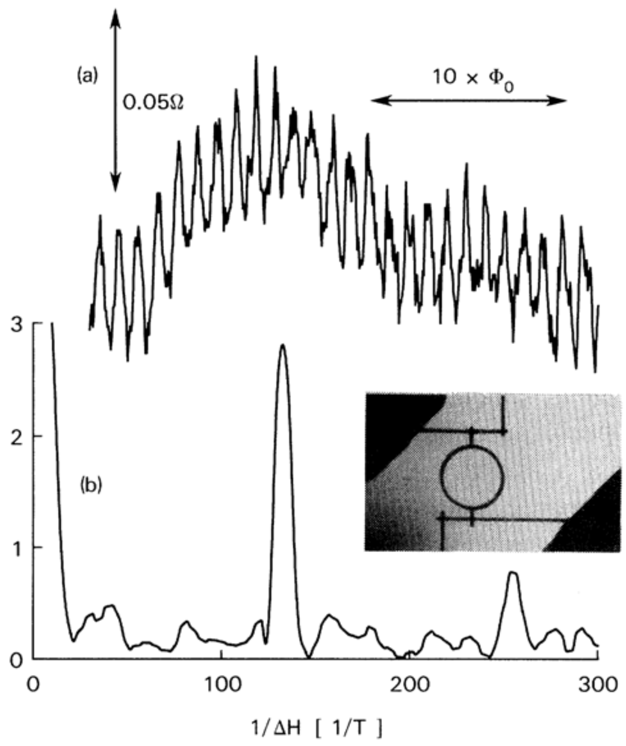
National Tsing-Hua University, Hsinchu

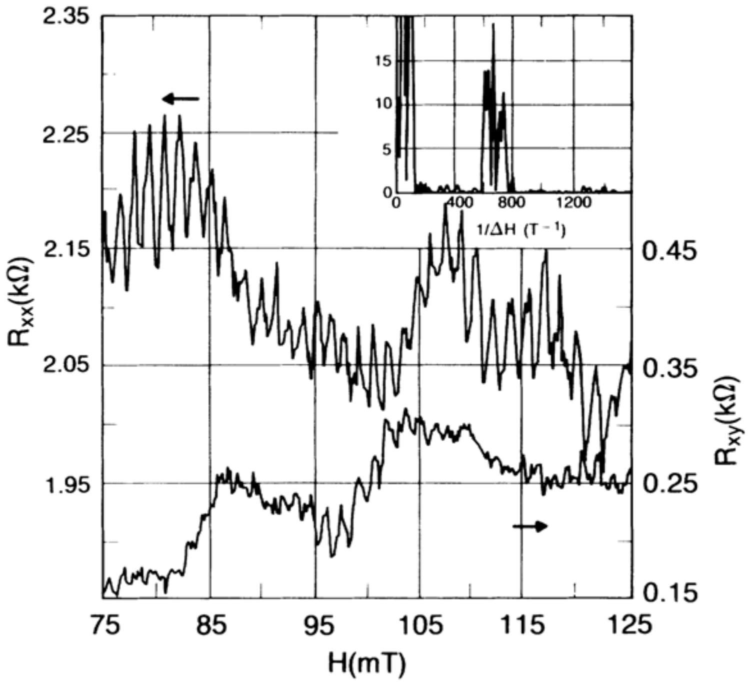
# Mesoscopic

- Between microscopic (quantum mechanics of few atoms) and Macrosopic
- Quantum Interference
- Quantum Coherence

# Metallic Rings—R.A. Webb, PRL 1985

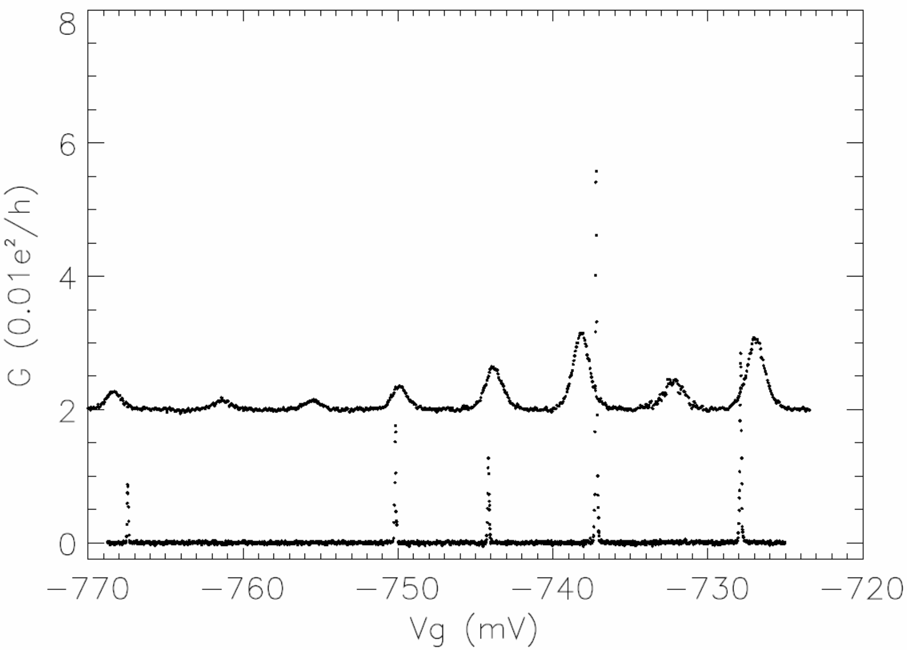
# Semiconductor Rings—Timp, Chang, PRL 1987



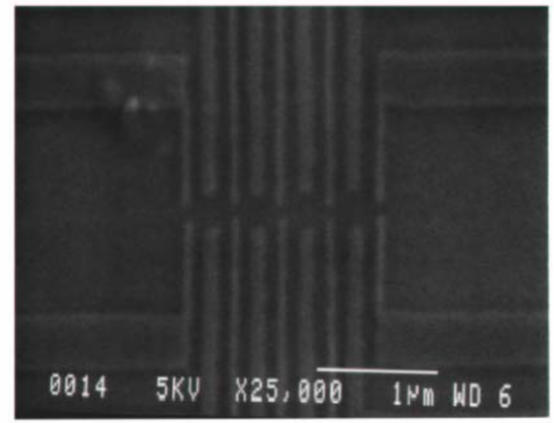



# Quantum Dot

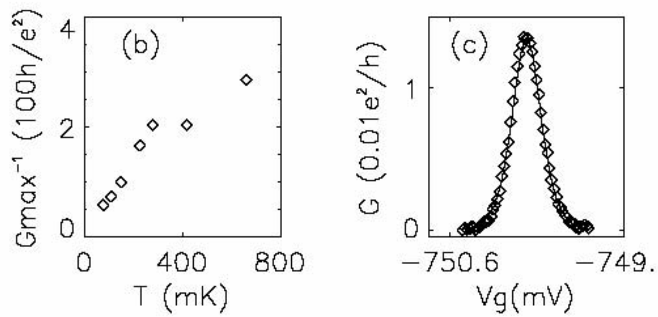
- Single Electron Transistor—an electrometer sensitive to addition of a single electron, or to electric flux from $10^{-4}$ e
- Quantum levels inside the 0-dimensional system



G (0.01e²/h)
Vg (mV)
8
6
4
2
0
−770
−760
−750
−740
−730
−720

0014 5KV X25,000 1μm WD 6

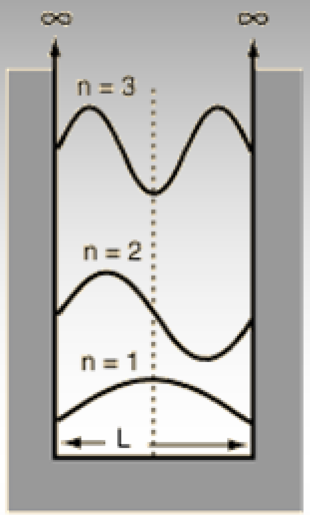
(b)
Gmax⁻¹ (100h/e²)
T (mK)
0
2
4
0
400
800
(c)
G (0.01e²/h)
Vg(mV)
0
1
−750.6
−749.8

# Key Concepts

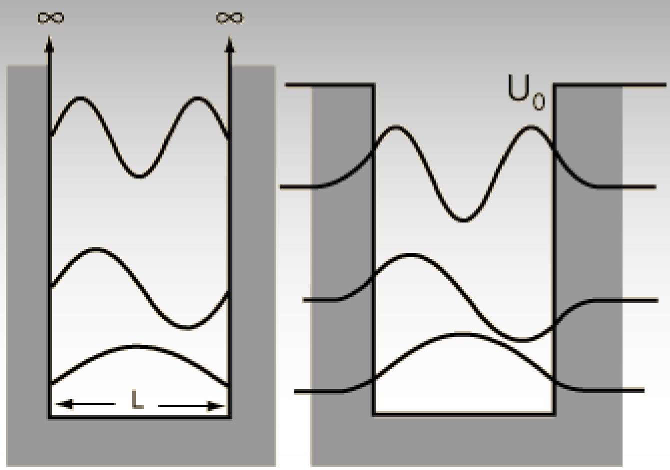
- **Size Quantization**
- **Quantum Coherence, Mesoscopics**
- **Phase Space in Reduced Dimensions**
- **Manybody effects**

**➔ NOVEL PHYSICAL PROPERTIES**

# Size Quantization




x = 0 at left wall of box.

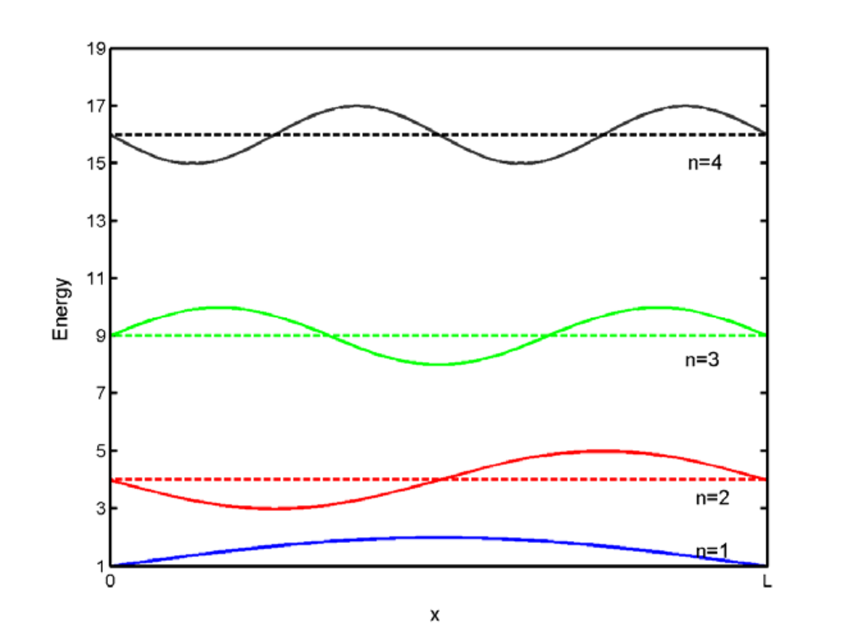



$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$E_n = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

# 0-D: Particle in a 3D Box

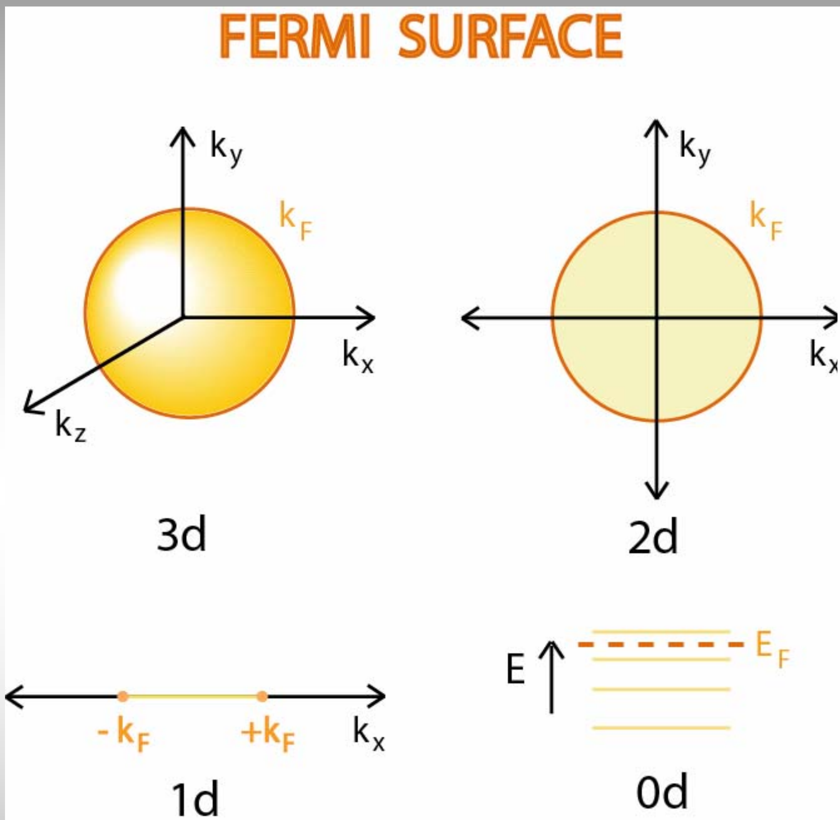
**For x, y, z, each—3D confinement:**




**What is 0D, 1D, 2D? –3D, 2D, 1D confinement.**

# Coherence

- E.g.  Laser speckle pattern—laser pointer on CD
- Holograms
- Aharonov-Bohm Effect
- Quantum Interference (Important for quantum computation)



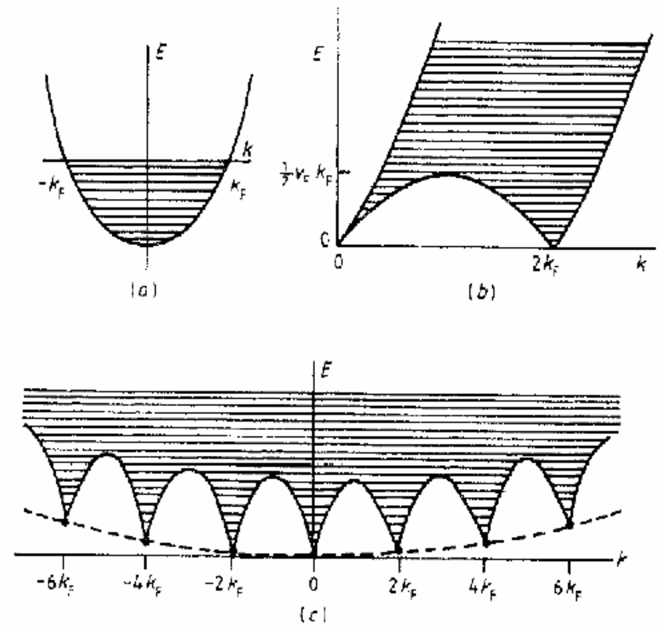



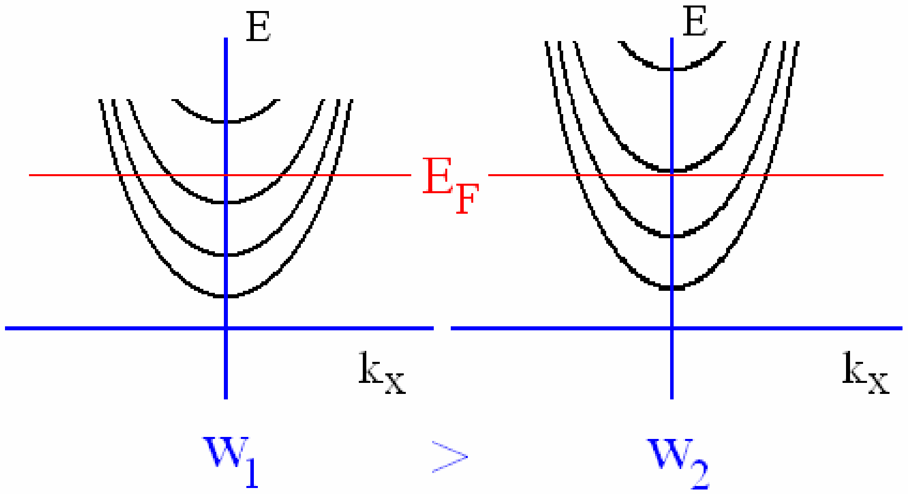

**Figure 1.** (*a*) Single-particle spectrum of the free Fermi gas in 1D; (*b*) Particle–hole spectrum; (*c*) full zero-charge (multiple particle–hole) excitation spectrum (energy differences $E(n) = 2\pi v_F n^2/L$ of extremal states at $k = 2nk_F$ greatly exaggerated).

F.D.M Haldane, J. Phys. C **14**, 2585 (1981).

# Outline

- Quantized Conductance in Quantum point contacts
- Aharonov Bohm Effect in mesoscopic rings
- 0-d: Quantum Dots
- Metallic nanowires and 1d superconductors

# Quantized Conductance




$$H = H_x + H_y = \frac{p_x^2}{2m} + \left[\frac{p_y^2}{2m} + V(y)\right]$$

$$E = \frac{\hbar^2 k_x^2}{2m} + E_n$$

y quantum levels

**$\lambda = k_x/2\pi \sim w$**

**in a quasi-1-d channel**

**$G = (e2/h)\ \mathrm{Tr}\ t\ t^+$**

# **Ballistic** Semiconductor Quantum Point Contact

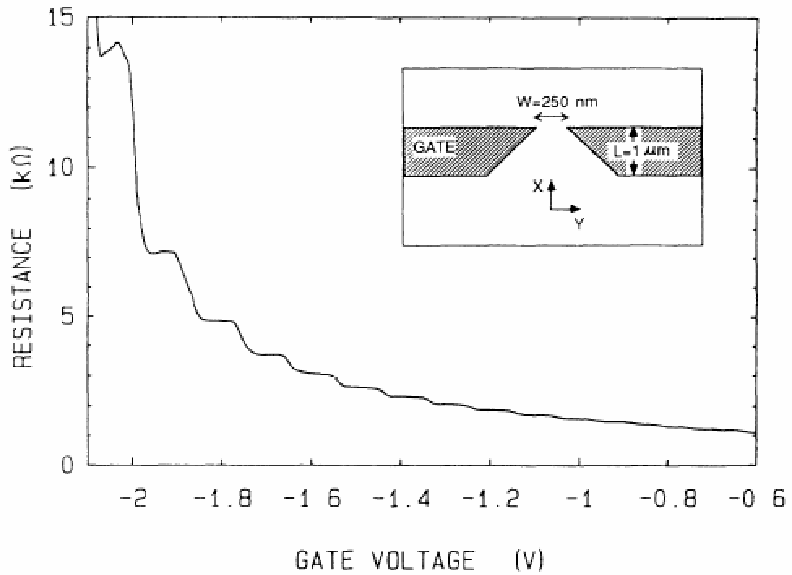



FIG. 1. Point-contact resistance as a function of gate voltage at 0.6 K. Inset: Point-contact layout.

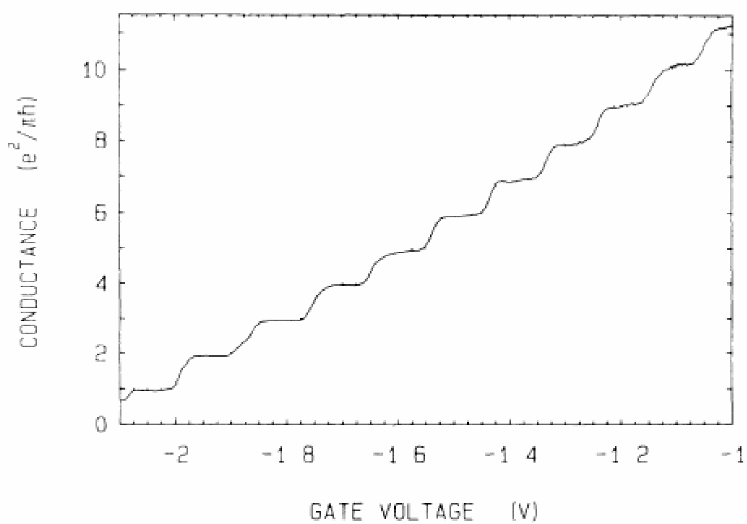
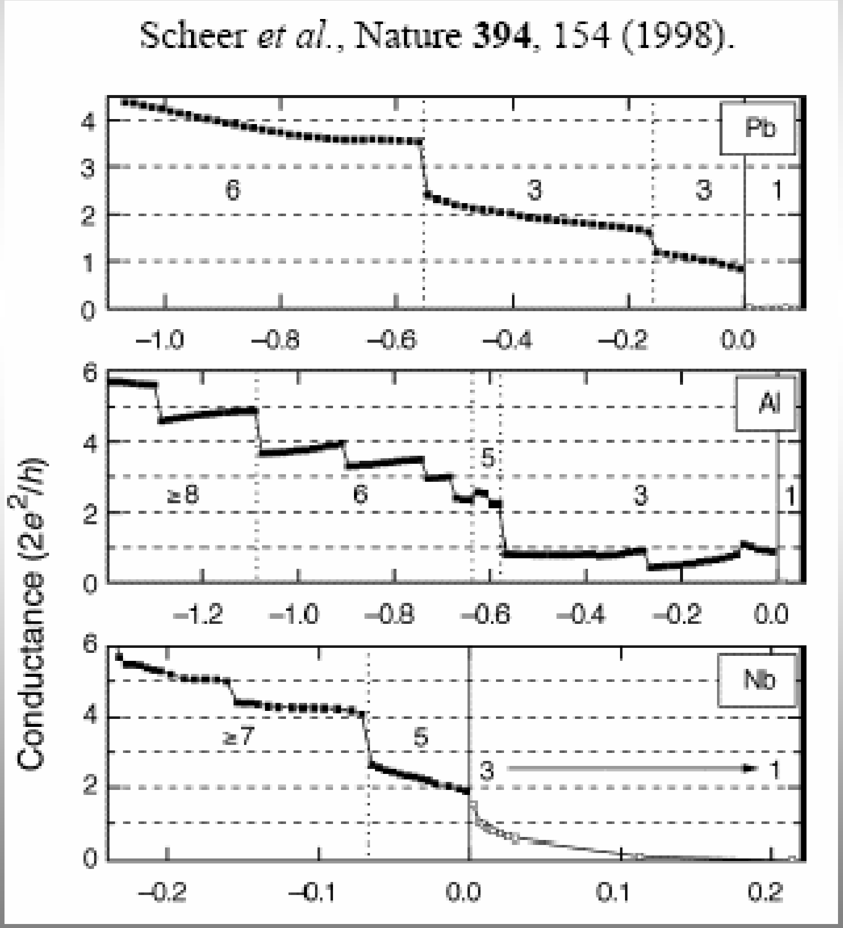



FIG. 2. Point-contact conductance as a function of gate voltage, obtained from the data of Fig. 1 after subtraction of the lead resistance. The conductance shows plateaus at multiples of $e^2/\pi\hbar$.

Quantized Conductance: $G = (e^2/h)\,\mathrm{Tr}\, t\, t^{+} = (e^2/h)\, N$

B.J. van Wees et al. (1988) PRL; Wharam et al. (1988) J. Phys. C.

# Atomic QPC



Scheer et al., Nature 394, 154 (1998).
Pb
6
3
3
1
Al
≥8
6
5
3
1
Nb
≥7
5
3
1
Conductance (2e²/h)

# Evidence for Saturation of Channel Transmission from Conductance Fluctuations in Atomic-Size Point Contacts

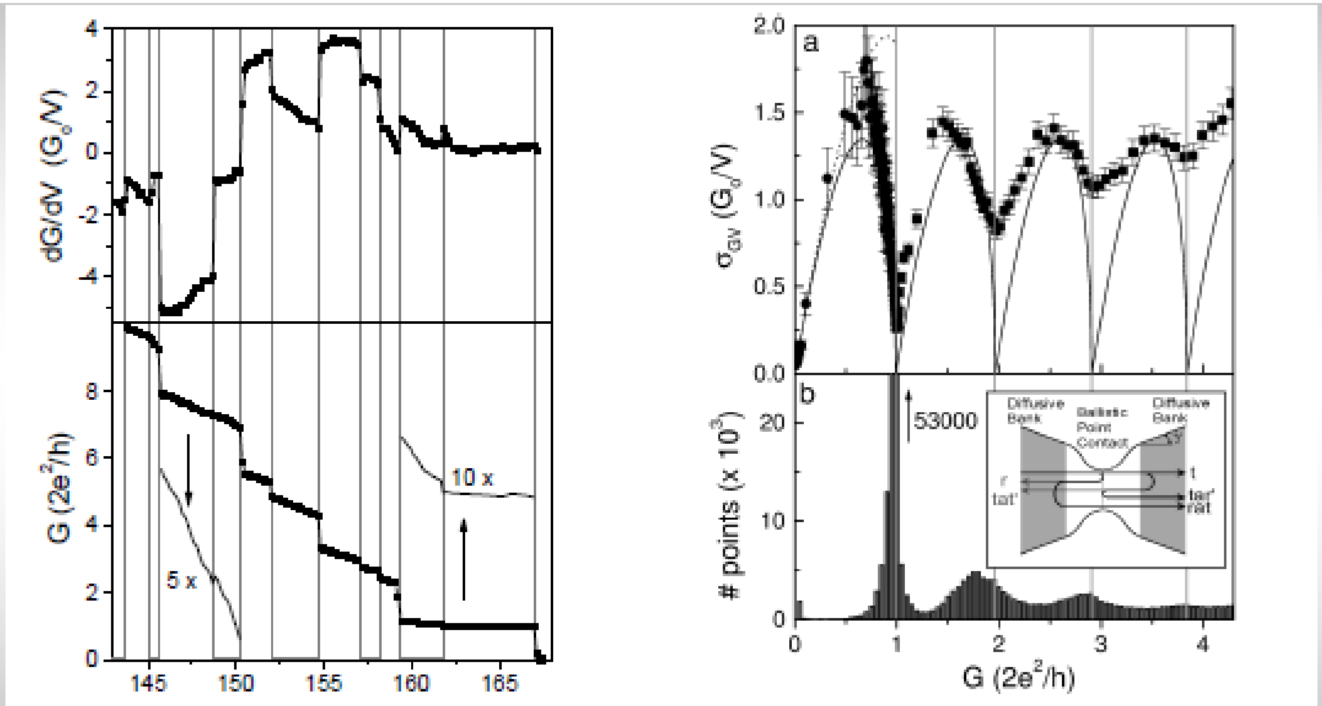
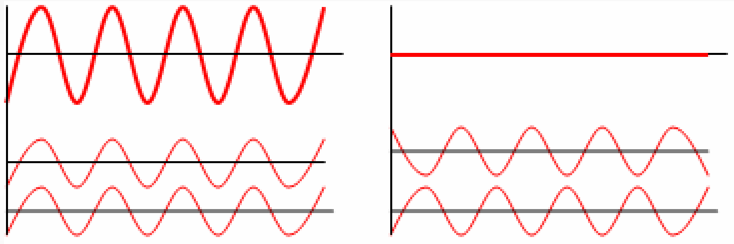


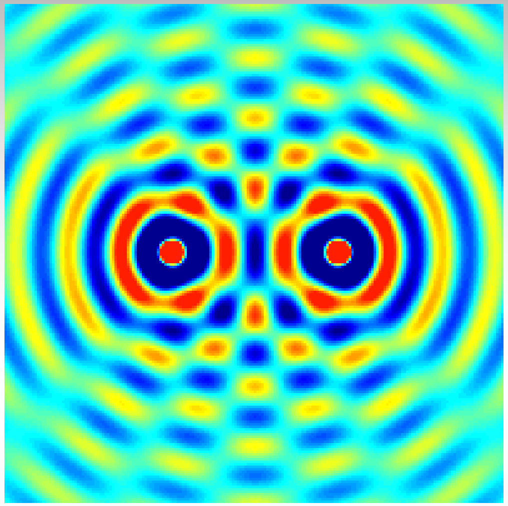

FIG. 3. (a) Standard deviation of the voltage dependence of the conductance versus conductance for 3500 curves. The

Au break junctions: Ludolph et al., PRL 1999.

# Aharono-Bohm Effect

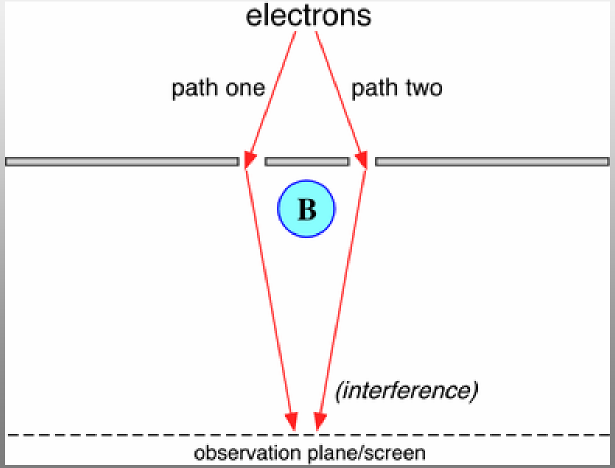
Wave Interference



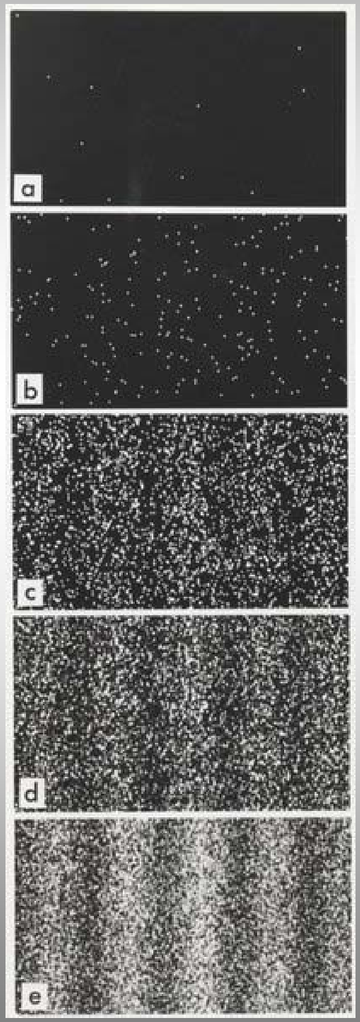


$I = I_o \cos^2(\Delta\phi/2)$

$$\varphi = \frac{q}{\hbar}\int_P \mathbf{A}\cdot d\mathbf{x},$$



# Electron Interference

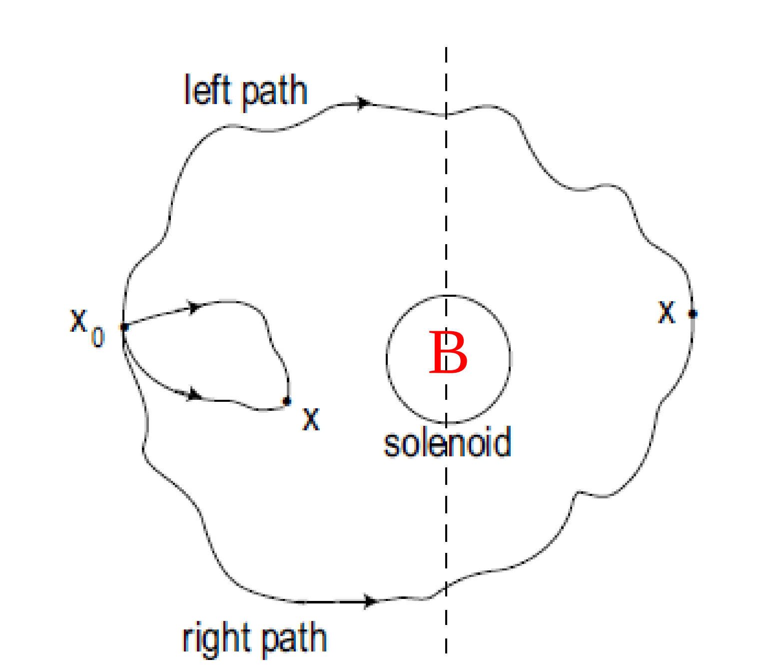


$$H = \frac{1}{2m}(\mathbf{p} - q\mathbf{A})^2$$

$$\psi_\Phi(\mathbf{x}, t) = \psi_\ell(\mathbf{x}, t)e^{iqs_\ell(\mathbf{x})} + \psi_r(\mathbf{x}, t)e^{iqs_r(\mathbf{x})}$$ ; q = 2πe/h

$$s_\ell(\mathbf{x}) \equiv \int_{\substack{\mathbf{x}_i \\ \text{left path}}}^{\mathbf{x}} \mathbf{A}(\mathbf{x}') \cdot d\mathbf{x}',$$

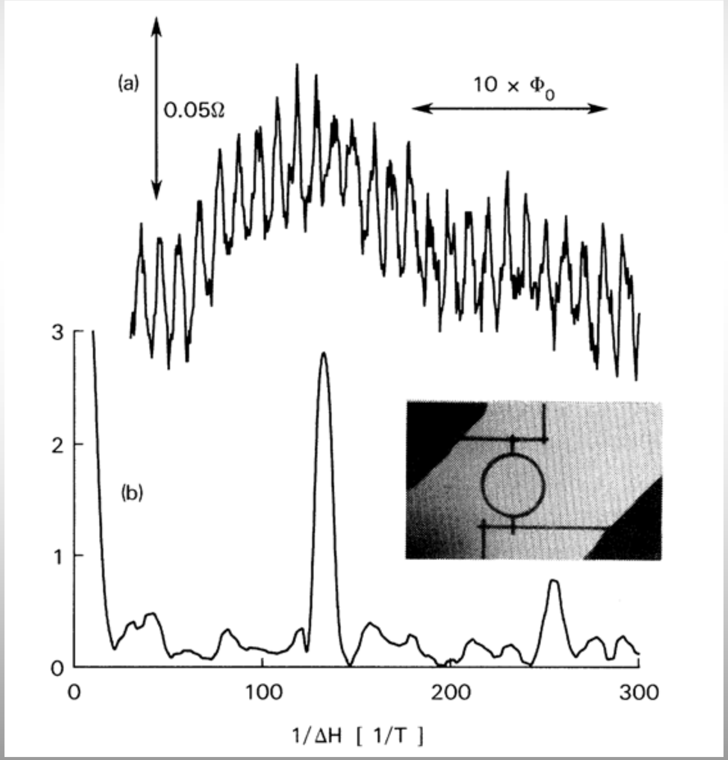
$$s_r(\mathbf{x}) \equiv \int_{\substack{\mathbf{x}_i \\ \text{right path}}}^{\mathbf{x}} \mathbf{A}(\mathbf{x}') \cdot d\mathbf{x}'$$

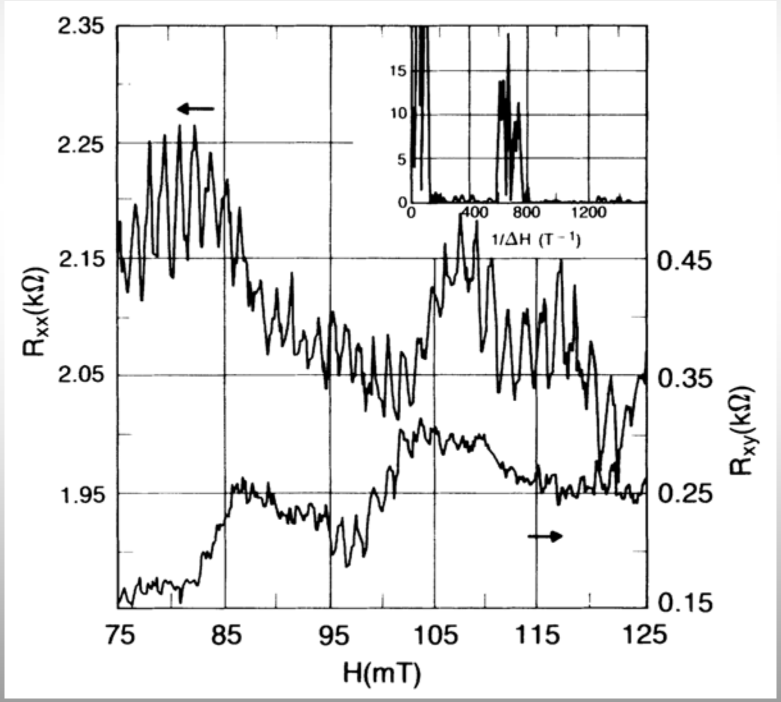



$$\oint_{\substack{-\text{left path} \\ +\text{right path}}} \mathbf{A}(\mathbf{x}') \cdot d\mathbf{x}' = \begin{cases} 0 & \mathbf{x} \text{ in front of solenoid} \\ \Phi & \mathbf{x} \text{ behind solenoid.} \end{cases}$$ = B x (Area)

Metallic Rings—
R.A. Webb, PRL 1985

Semiconductor Rings—Timp, Chang, PRL 1987

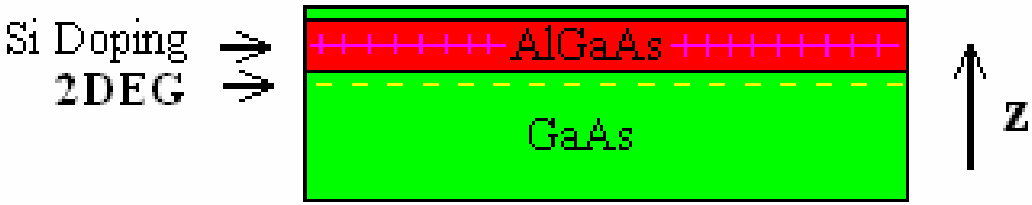


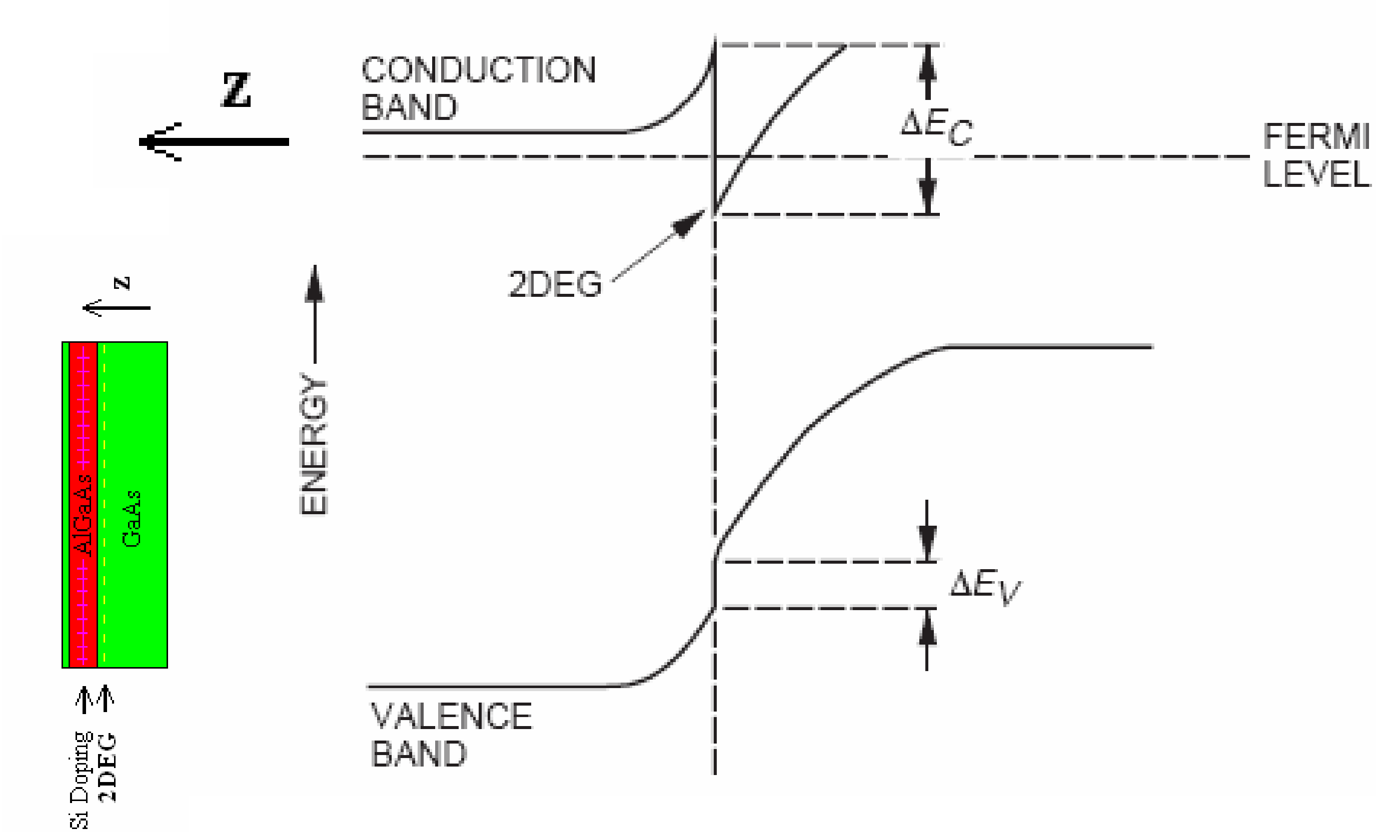



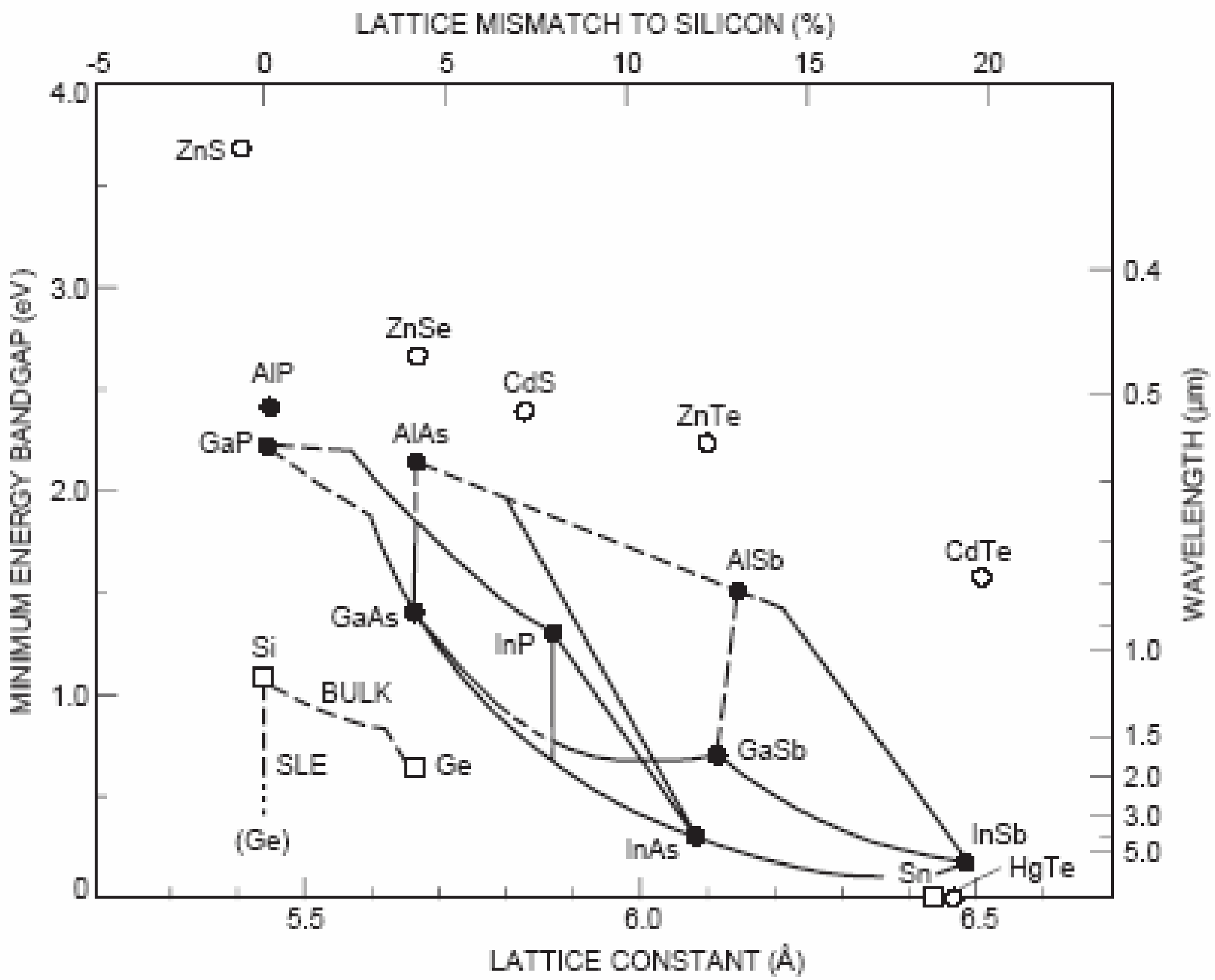
# Systems

- Semiconductor—2DEG, 1DEG, Quantum Dots
- Metallic—Nanowires and rings, Quantum Dots,
- Molecular and atomic systems
- New Materials: e.g. Carbon nanotubes, graphene sheets

# 2-Dimensional Electron Gas in GaAs-AlGaAs Heterostructure





LATTICE MISMATCH TO SILICON (%)
-5
0
5
10
15
20
4.0
3.0
2.0
1.0
0
MINIMUM ENERGY BANDGAP (eV)
WAVELENGTH (µm)
0.4
0.5
1.0
1.5
2.0
3.0
5.0
ZnS
ZnSe
CdS
ZnTe
CdTe
AlP
GaP
AlAs
AlSb
GaAs
InP
Si
BULK
SLE
Ge
(Ge)
GaSb
InAs
InSb
Sn
HgTe
5.5
6.0
6.5
LATTICE CONSTANT (Å)

# The Hall Effect

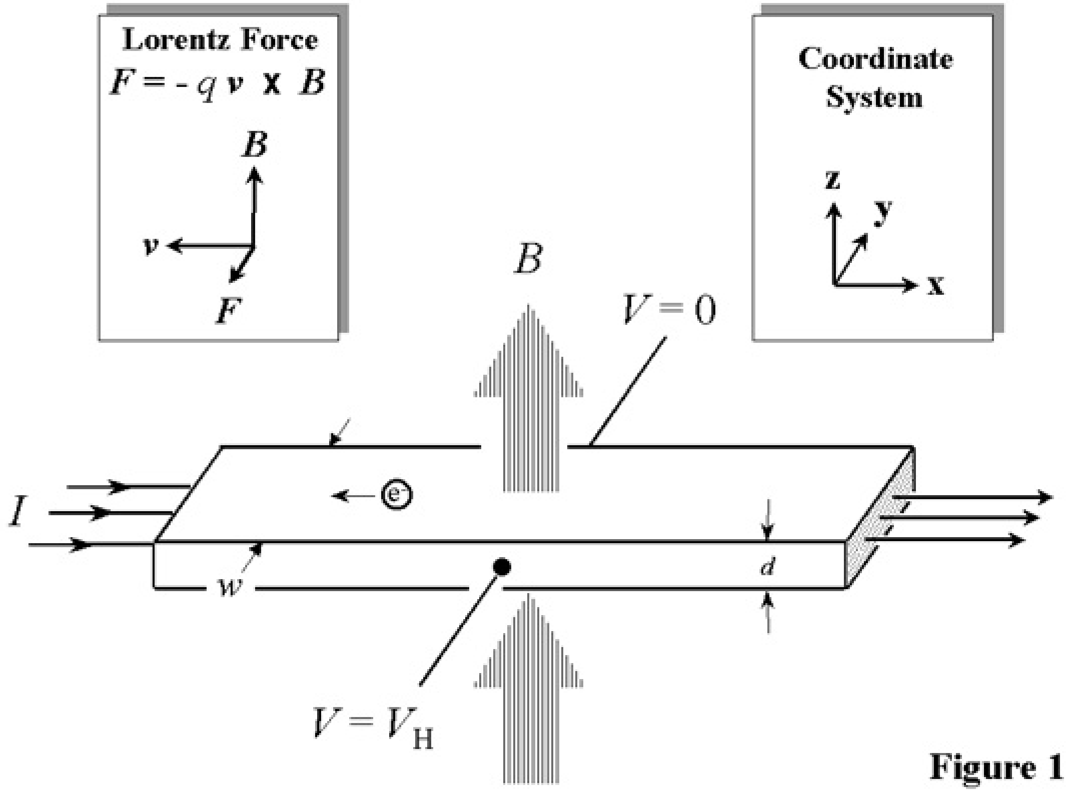



Figure 1

# Classical Drude Conductivity

$$\mathbf{v_d} = -e\mathbf{E}\tau/m \qquad (1)$$

$$\mathbf{j} = -ne\mathbf{v_d} = \sigma_0 \mathbf{E} \qquad \sigma_0 = ne^2\tau/m \qquad (2)$$

$$\begin{aligned}\mathbf{j} &= \sigma \cdot \mathbf{E} \\ \mathbf{E} &= \rho \cdot \mathbf{j}\end{aligned} \qquad \sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix},\ \rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \qquad (4)$$

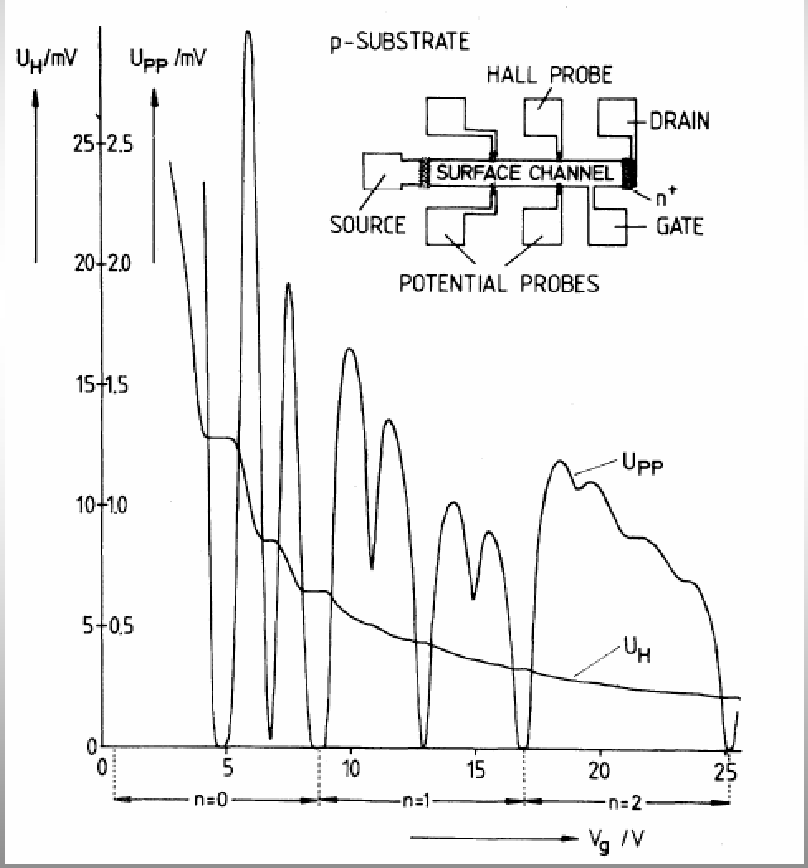
$$\mathbf{v_d} = -e\left(\mathbf{E} + \frac{\mathbf{v_d} \times \mathbf{B}}{c}\right)\frac{\tau}{m} \qquad (5)$$

$$\begin{aligned}\sigma_0 E_x &= \omega_c \tau j_y + j_x \\ \sigma_0 E_y &= -\omega_c \tau j_x + j_y\end{aligned} \qquad \omega_c = \frac{eB}{mc} \qquad (6)$$

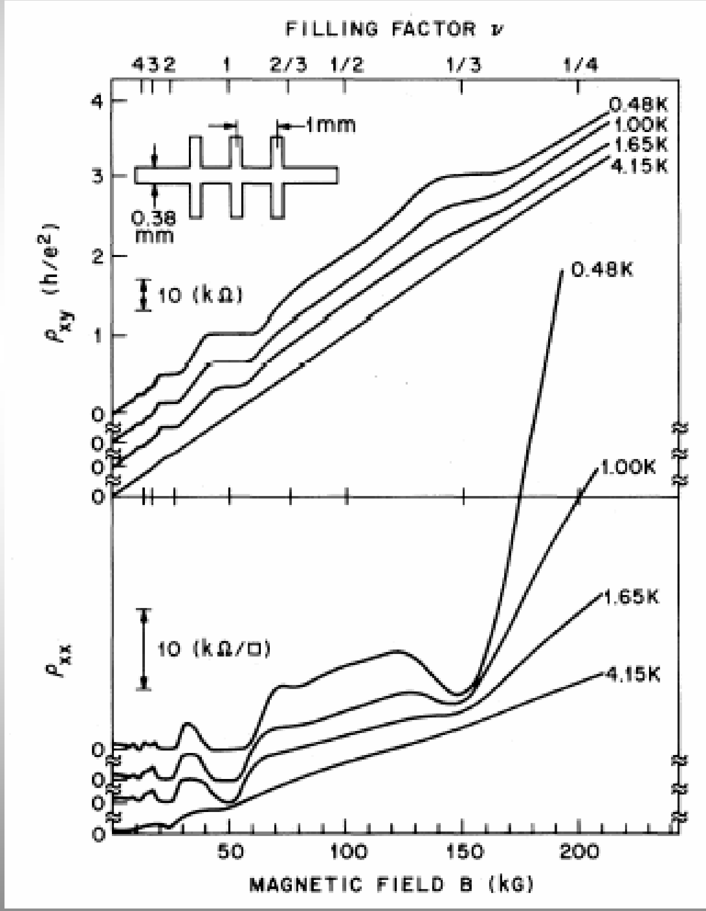
$$\rho_{xx} = \rho_{yy} = 1/\sigma_0,\ \rho_{xy} = -\rho_{yx} = \omega_c \tau/\sigma_0$$

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1+(\omega_c\tau)^2},\ \sigma_{xy} = -\sigma_{yx} = \frac{-\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2}$$

# 2DEG-Quantized Hall Effect




Von Klitzing, PRL 1980

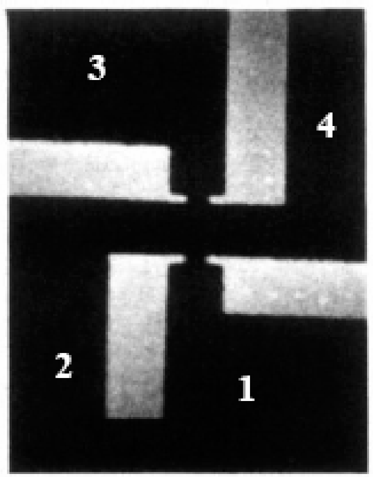


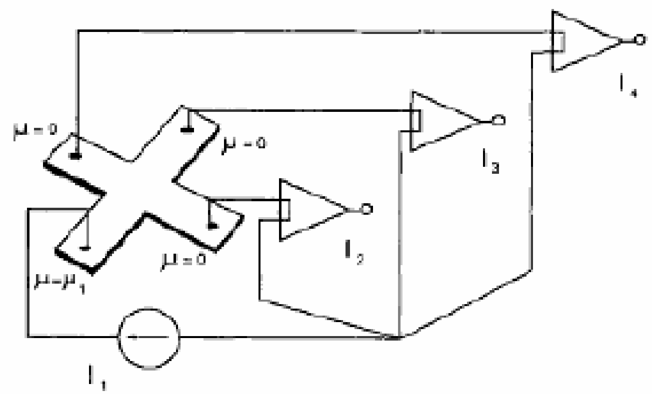

Tsui, Stormer, PRL 1982

# Reciprocity—Buttiker, Landauer

Cross geometry

Shephard *et al.*, PRB **46**, 9648 (1992).

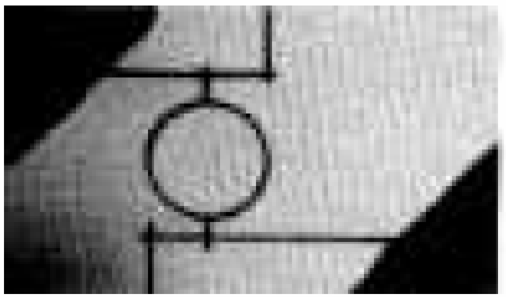


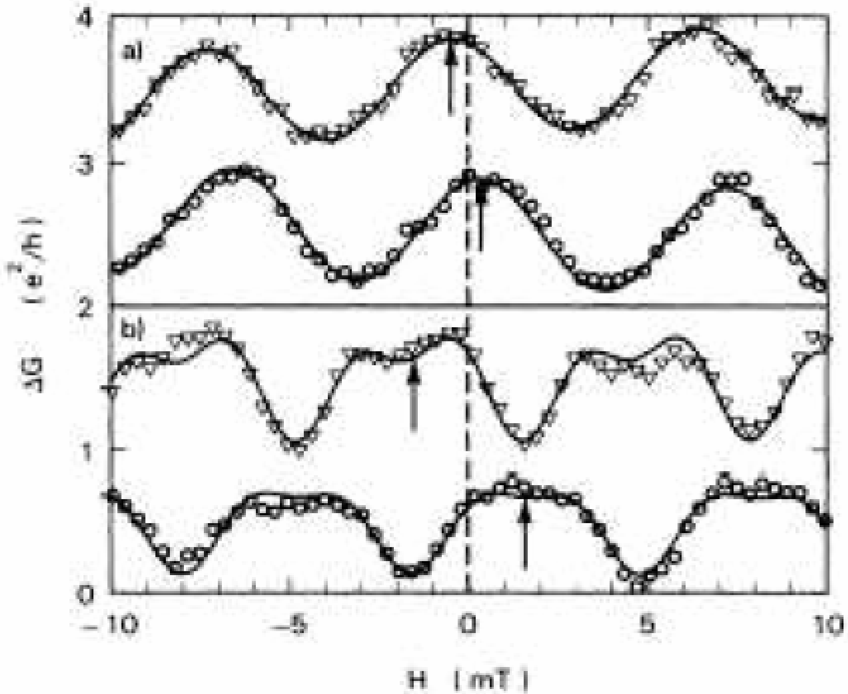




- Usual GaAs 2deg + Au gates.

# Reciprocity: Benoit et al.

Benoit, Washburn, Umbach, Laibowitz, Webb, PRL 57, 1765 (1986)

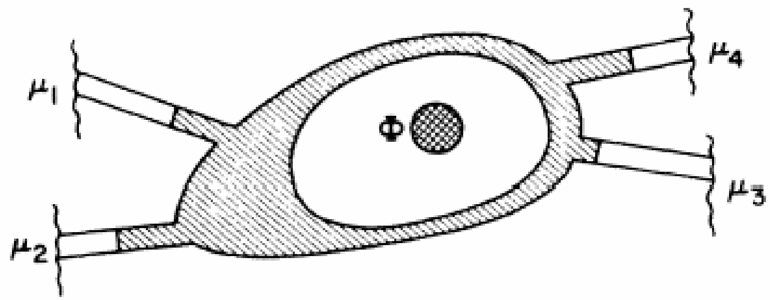




VOLUME 57, NUMBER 14 PHYSICAL REVIEW LETTERS 6 OCTOBER 1986

# Four-Terminal Phase-Coherent Conductance

M. Büttiker
*IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598*




$$I_i = \frac{e}{h}\left[(1-R_{ii})\mu_i - \sum_{j\neq i} T_{ij}\mu_j\right]$$

$$\mathcal{R}_{mn,kl} = (h/e^2)(T_{km}T_{ln} - T_{kn}T_{lm})/D$$

$$R_{ii}(\Phi)=R_{ii}(-\Phi), \quad T_{ij}(\Phi)=T_{ji}(-\Phi).$$

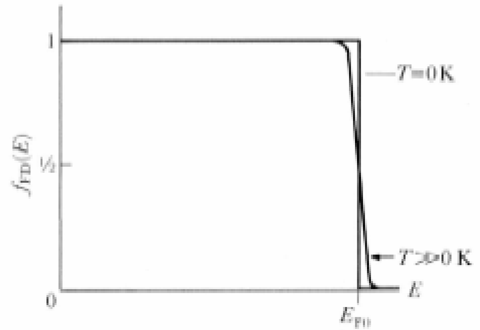
# Fermi-Dirac; Fermi Level

$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$

$$E = \frac{p^2}{2m} = \left(\frac{h}{\lambda}\right)^2 \frac{1}{2m} = \left(\frac{h}{2\pi}\frac{2\pi}{\lambda}\right)^2 \frac{1}{2m} = \frac{\left(\hbar k\right)^2}{2m}$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

k = n(2π/L); L = size of system



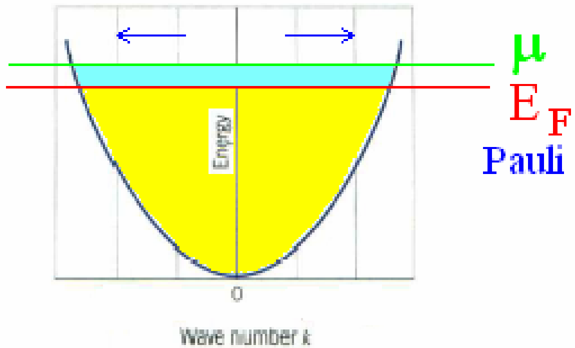




**FIGURE 11.25** The parabolic relationship between energy and wave number for a free particle.

# Buttiker

as a function of differences of voltages, $V_i = \mu_i / e$,

$$I_1 = \alpha_{11}(V_1 - V_3) - \alpha_{12}(V_2 - V_4), \tag{3a}$$

$$I_2 = -\alpha_{21}(V_1 - V_3) + \alpha_{22}(V_2 - V_4). \tag{3b}$$

I find the following expressions for the conductances of Eq. (3):

$$\alpha_{11} = (e^2/h)[(1 - R_{11})S - (T_{14} + T_{12})(T_{41} + T_{21})]/S, \tag{4a}$$

$$\alpha_{12} = (e^2/h)(T_{12}T_{34} - T_{14}T_{32})/S, \tag{4b}$$

$$\alpha_{21} = (e^2/h)(T_{21}T_{43} - T_{23}T_{41})/S, \tag{4c}$$

$$\alpha_{22} = (e^2/h)[(1 - R_{22})S - (T_{21} + T_{23})(T_{32} + T_{12})]/S, \tag{4d}$$

where

$$S = T_{12} + T_{14} + T_{32} + T_{34} = T_{21} + T_{41} + T_{23} + T_{43}. \tag{5}$$

$$\mathcal{R}_{13,24} = (V_2 - V_4)/I_1 = \alpha_{21}/(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}).$$

# The Hall Effect

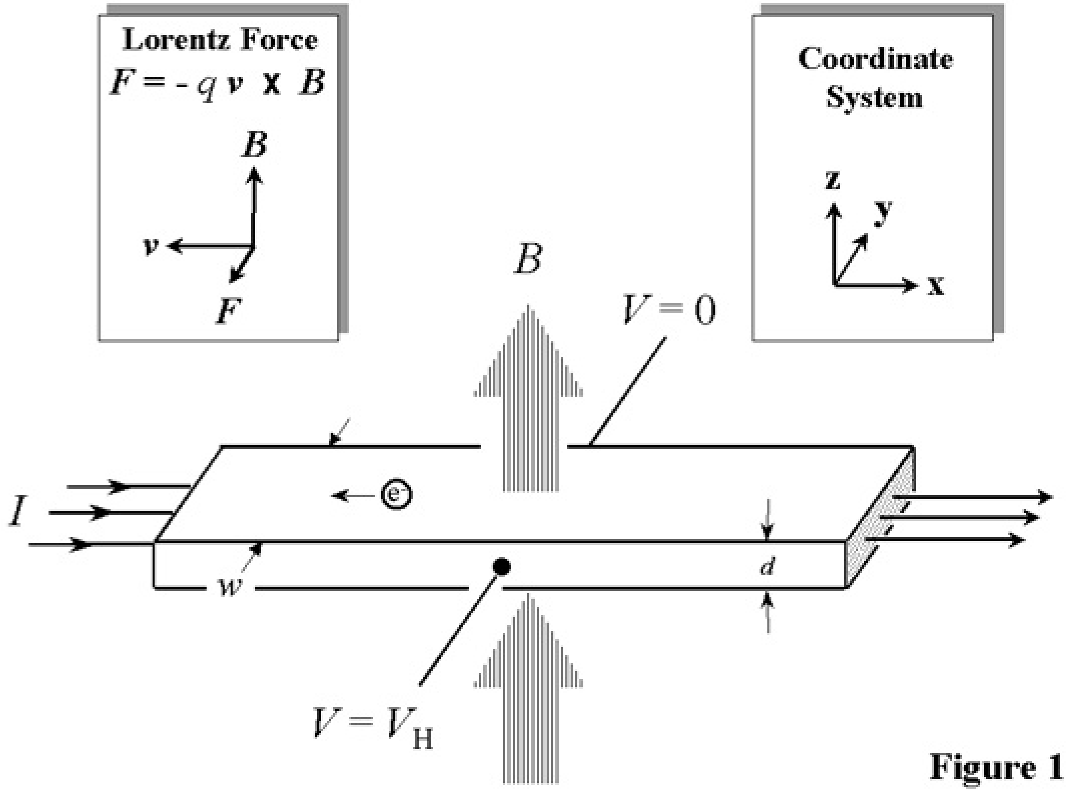



Figure 1

# Classical Drude Conductivity

$$\mathbf{v_d} = -e\mathbf{E}\tau/m \tag{1}$$

$$\mathbf{j} = -ne\mathbf{v_d} = \sigma_0 \mathbf{E} \qquad \sigma_0 = ne^2\tau/m \tag{2}$$

$$\mathbf{j} = \sigma \cdot \mathbf{E}$$
$$\mathbf{E} = \rho \cdot \mathbf{j}$$

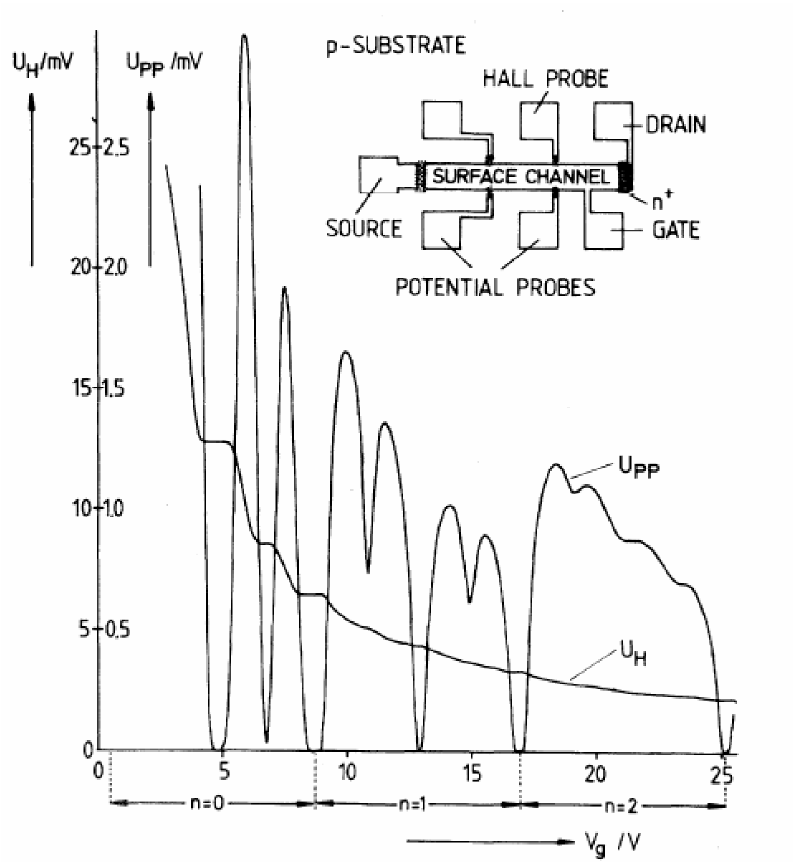
$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix},\ \rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \tag{4}$$

$$\mathbf{v_d} = -e\left(\mathbf{E} + \frac{\mathbf{v_d} \times \mathbf{B}}{c}\right)\frac{\tau}{m} \tag{5}$$

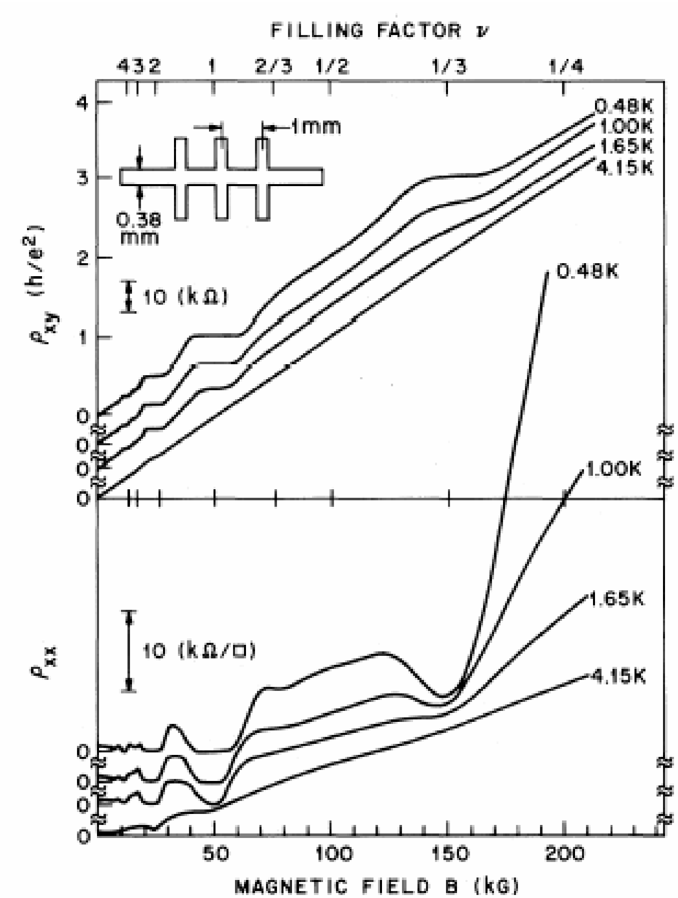
$$\begin{aligned} \sigma_0 E_x &= \omega_c \tau j_y + j_x \\ \sigma_0 E_y &= -\omega_c \tau j_x + j_y \end{aligned} \qquad \omega_c = \frac{eB}{mc} \tag{6}$$

$$\rho_{xx} = \rho_{yy} = 1/\sigma_0,\ \rho_{xy} = -\rho_{yx} = \omega_c\tau/\sigma_0$$
$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1+(\omega_c\tau)^2},\ \sigma_{xy} = -\sigma_{yx} = \frac{-\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2}$$

# 2DEG-Quantized Hall Effect

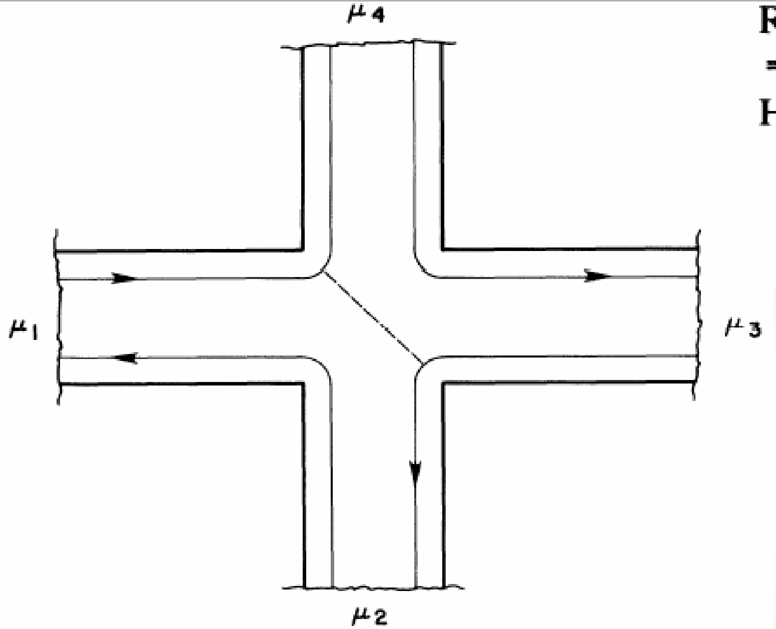



Von Klitzing, PRL 1980




Tsui, Stormer, PRL 1982

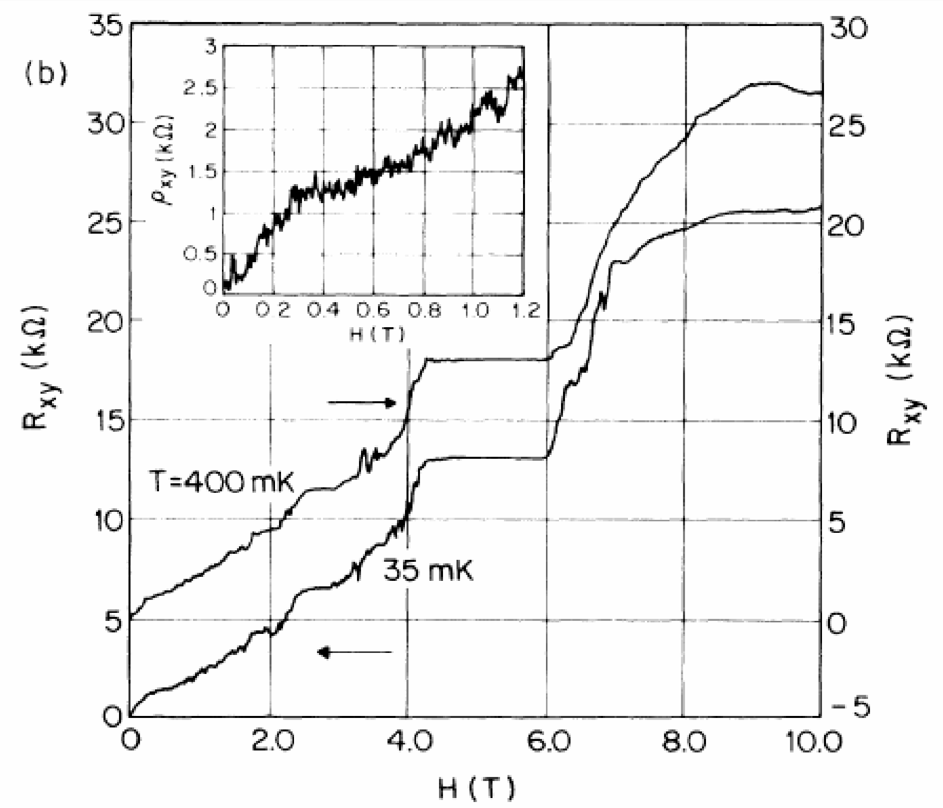
# Buttiker Formalism




Refs. 2 and 6.) We have $T_{12}=T_{34}=N$, and $T_{21}=T_{43}\equiv\mathcal{T}$, $T_{23}=T_{41}=N-\mathcal{T}$. Using Eq. (1) we find a Hall resistance

$$\mathcal{R}_H=\mathcal{R}_{13,42}=\frac{h}{e^2}\left(\frac{1}{N}-\frac{\mathcal{T}}{N(N-\mathcal{T})}\right). \quad (2)$$

Buttiker, PRL 1989




Timp, Chang, PRL 1987

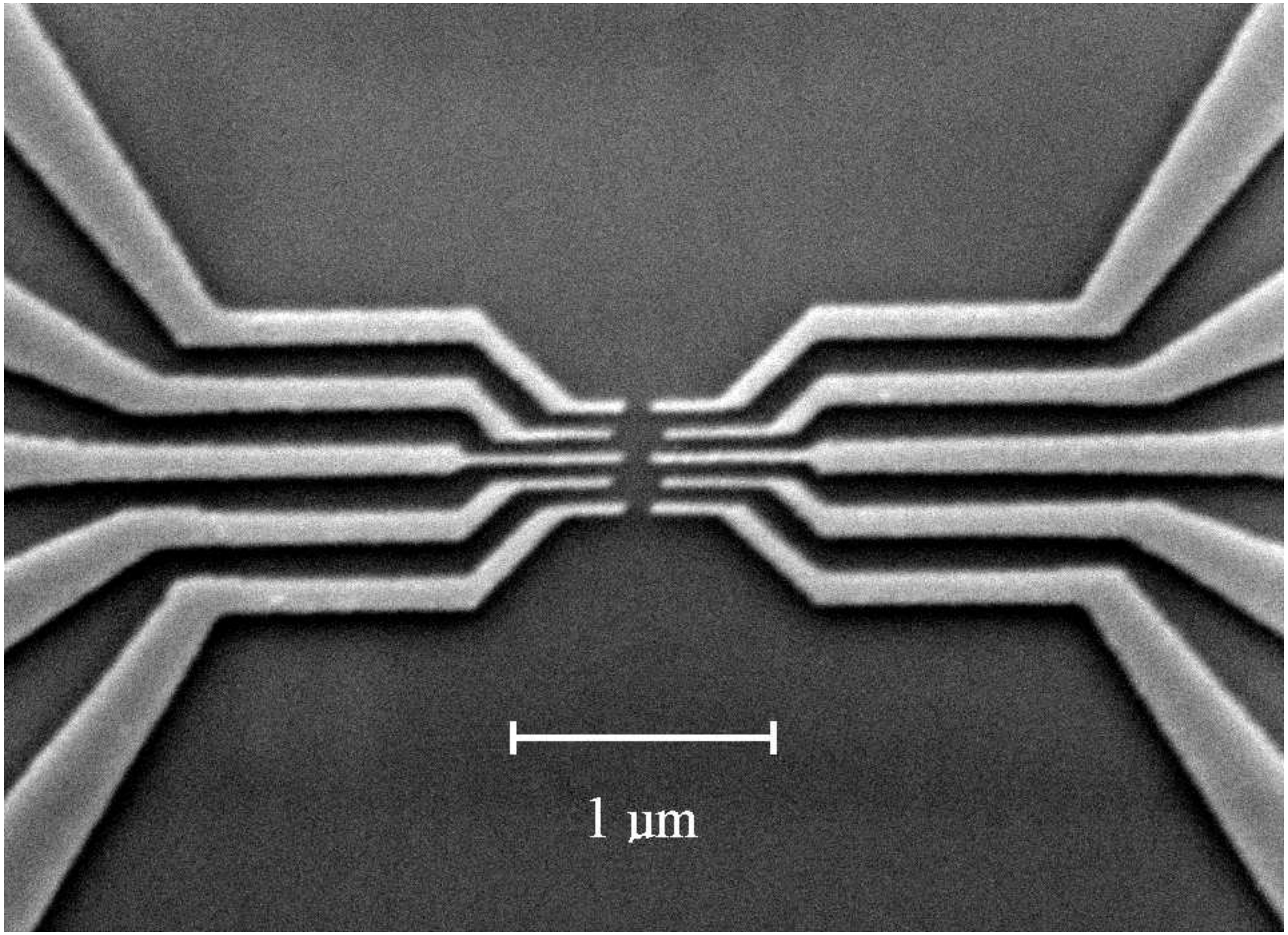
# KONDO EFFECT AND SPIN - ENTANGLEMENT IN DOUBLE QUANTUM DOTS

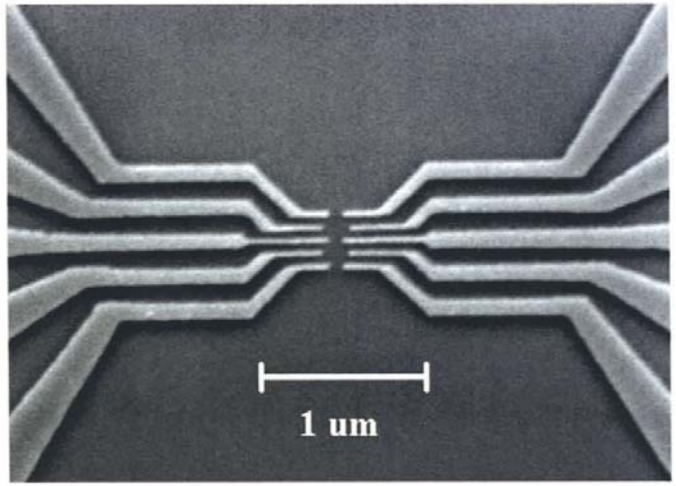
**ALBERT M. CHANG**

**Department of Physics**

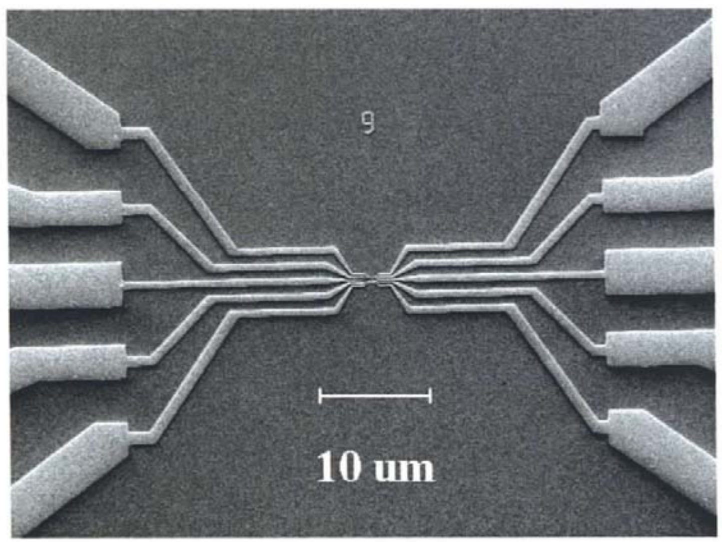
**Duke University, Durham, NC 27708**

# Double Quantum Dot Molecule



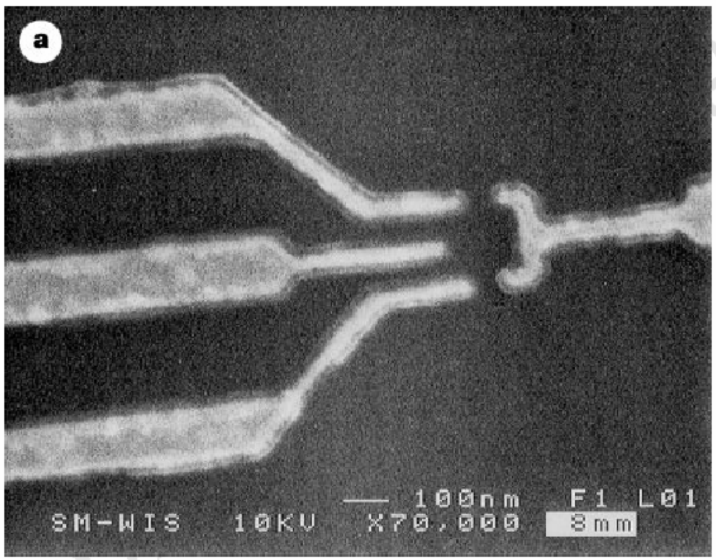


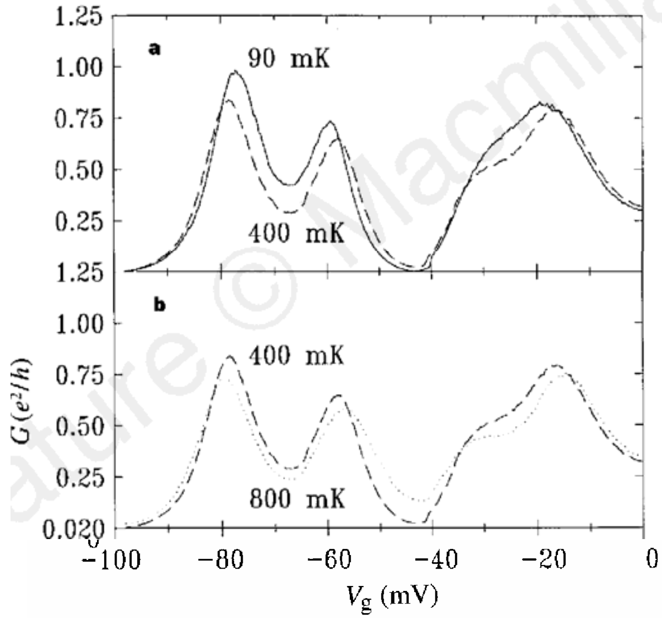

The size is 180 nm. Designed for `the Kondo effect in artificial molecules'.

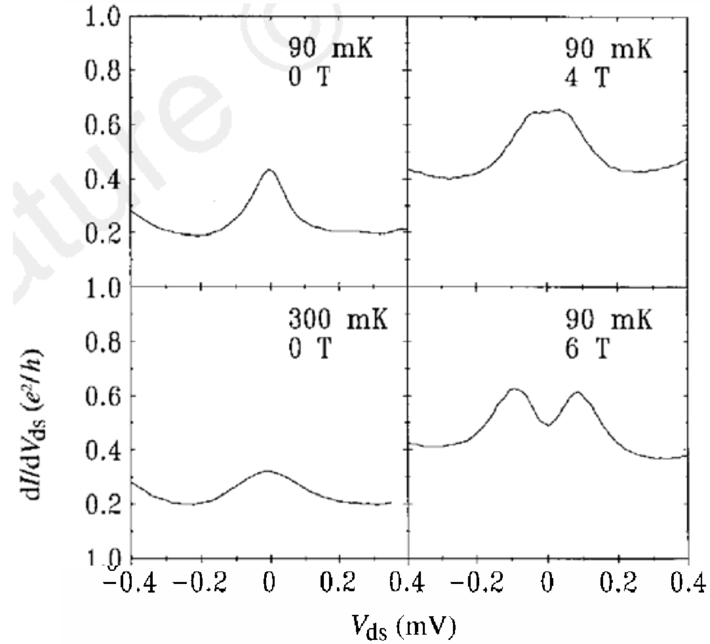



Bigger picture of the above device showing outer fan-out.

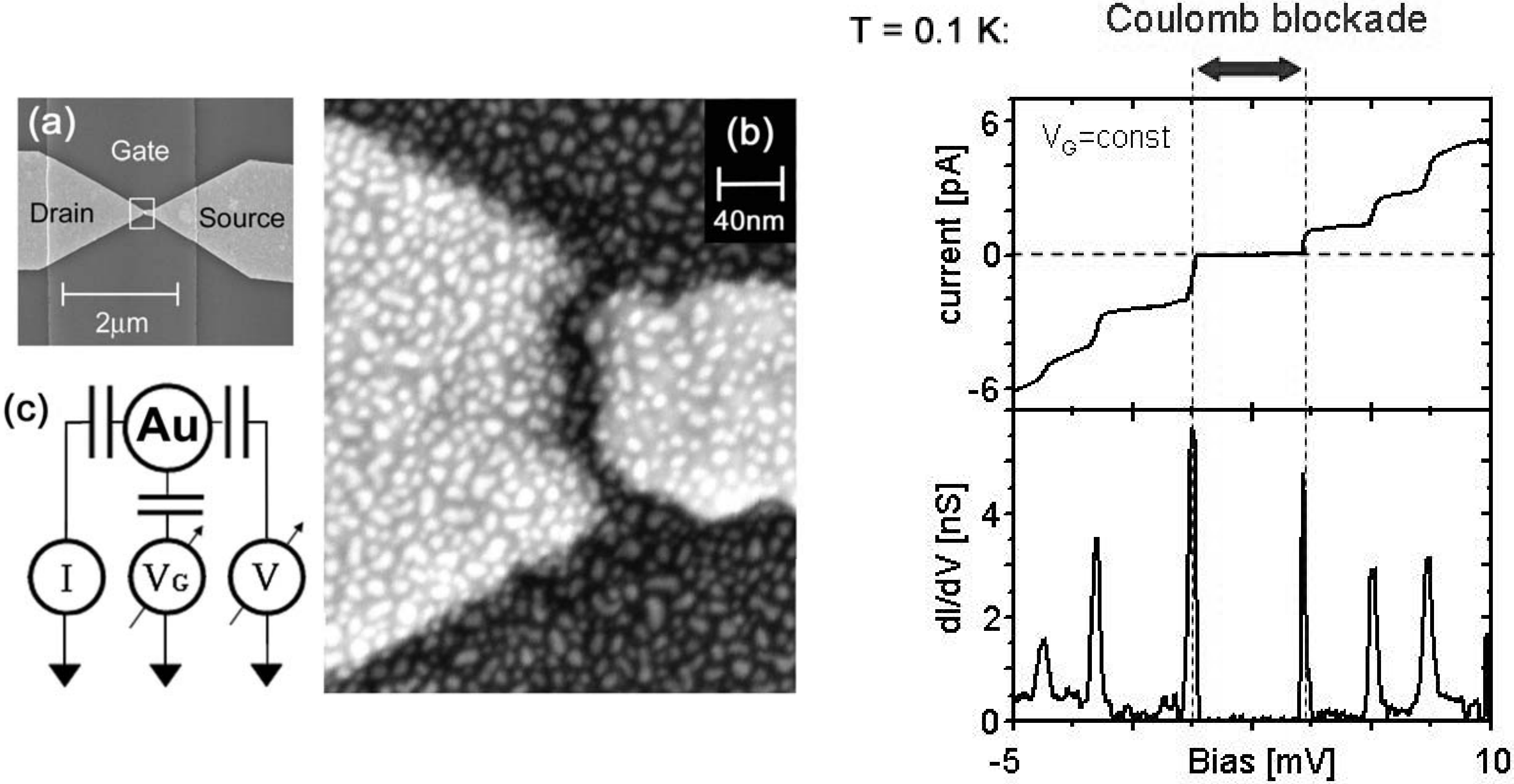
**D. Goldhaber-Gordon,**
**M.A. Kastner et al.,**
**Nature 391, 156-159 (1998).**



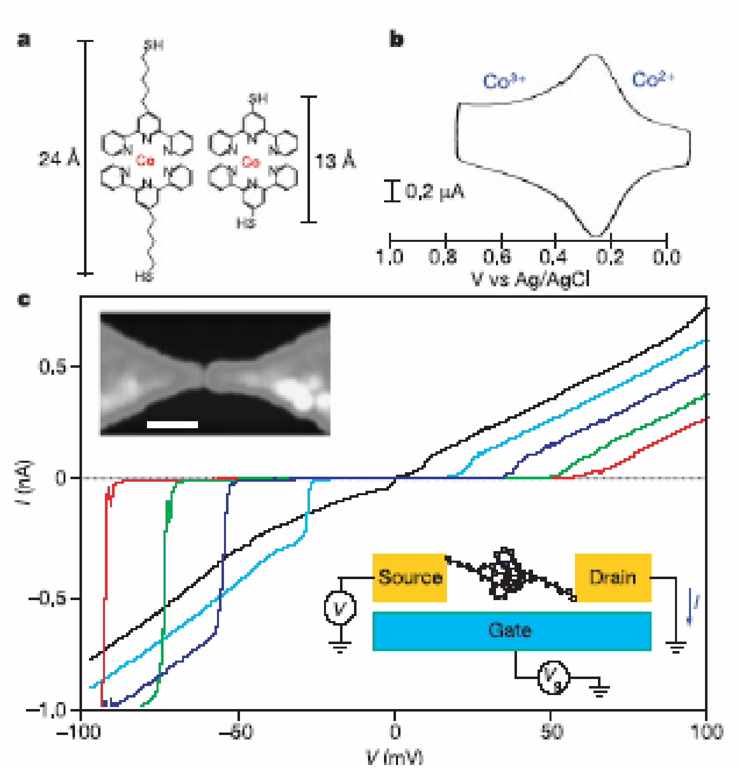



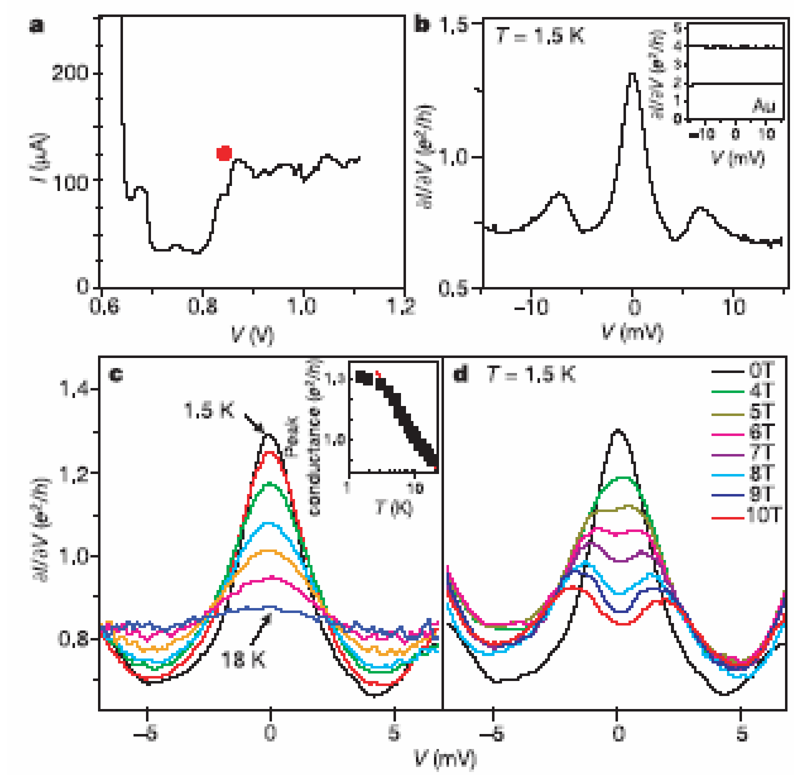
# Gold Nanoparticles




Ralph (Cornell)

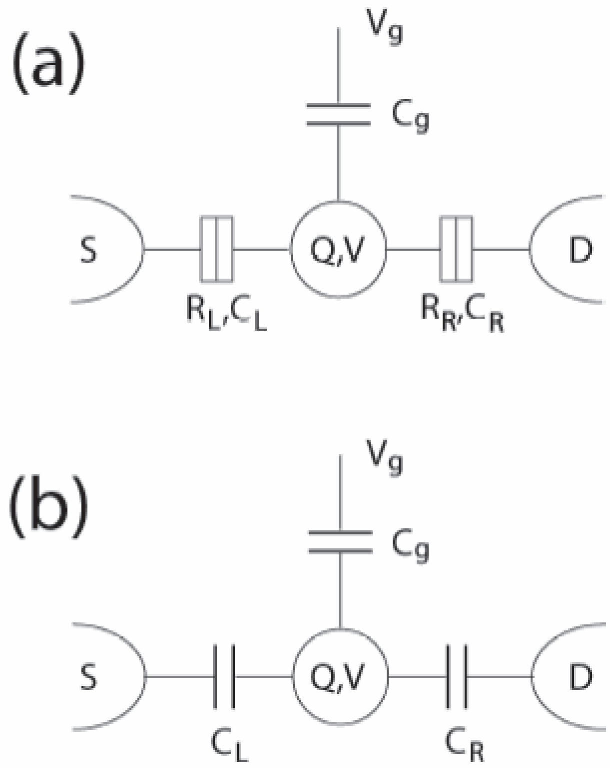
# Single Atom Transistor

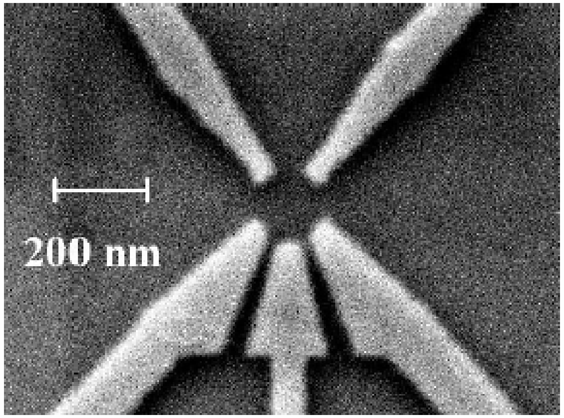


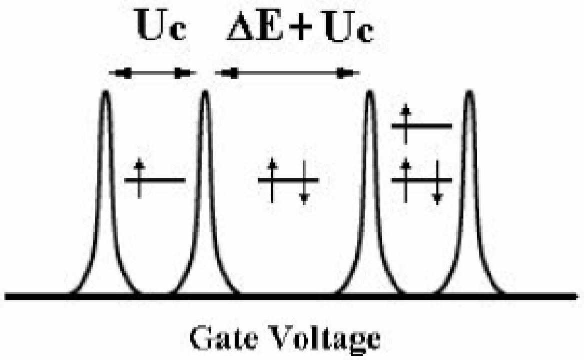



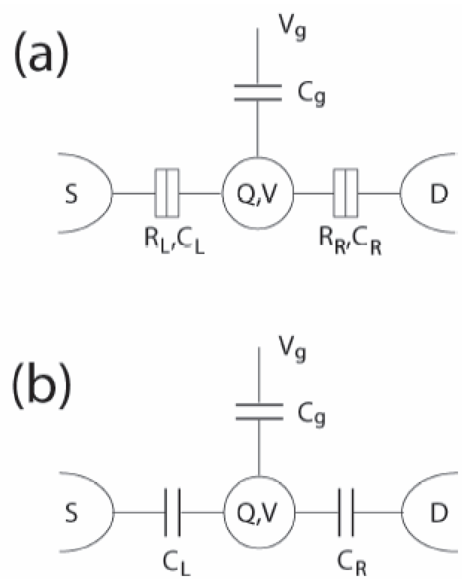

McEuen, Ralph, et al., Nature 2002.

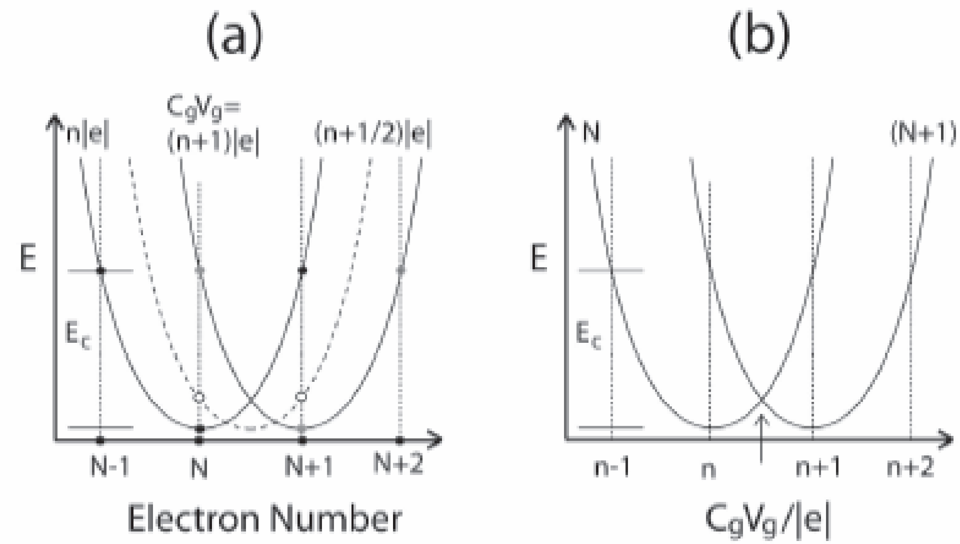
# Quantum Dots



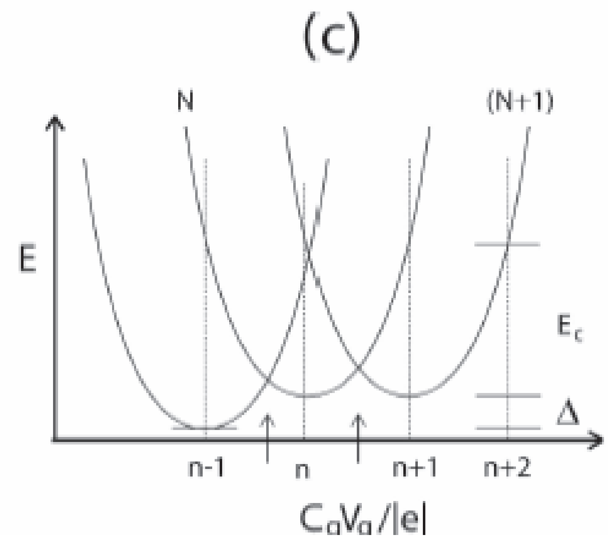






$$E = \frac{Q^2}{2C} - \frac{C_g}{C} Q V_g,$$

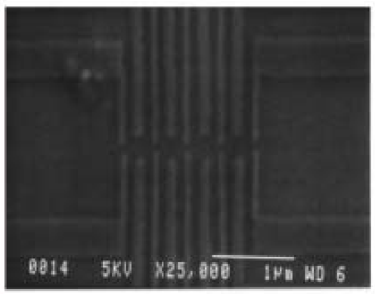
$$E = \frac{(-N|e| + C_g V_g)^2}{2C} - \frac{C_g^2}{2C} V_g^2,$$

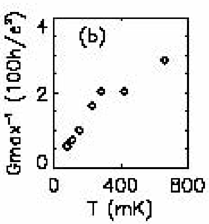


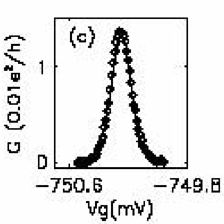
$$I(kT, eV_{bias}) = \frac{e^2}{h} \int dE[f(E + eV_{bias}) - f(E)] \frac{\Gamma_L \Gamma_R}{\Gamma^2 + (E - E_o)^2}. \tag{4}$$

$$G_{\max} = \frac{e^2}{h} \frac{\pi}{2kT} \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}$$

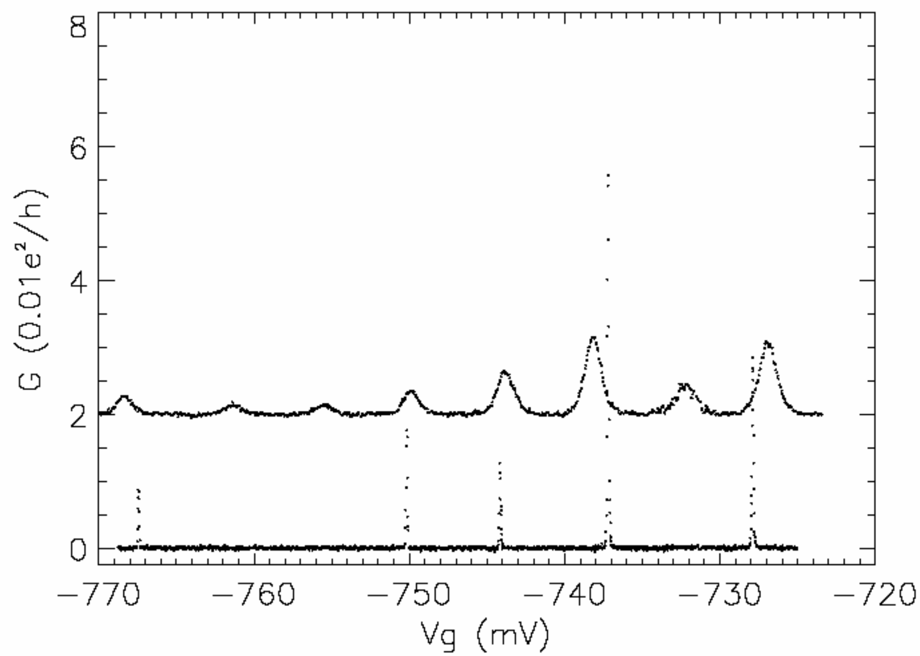
$$\Gamma \ll kT \ll \Delta$$



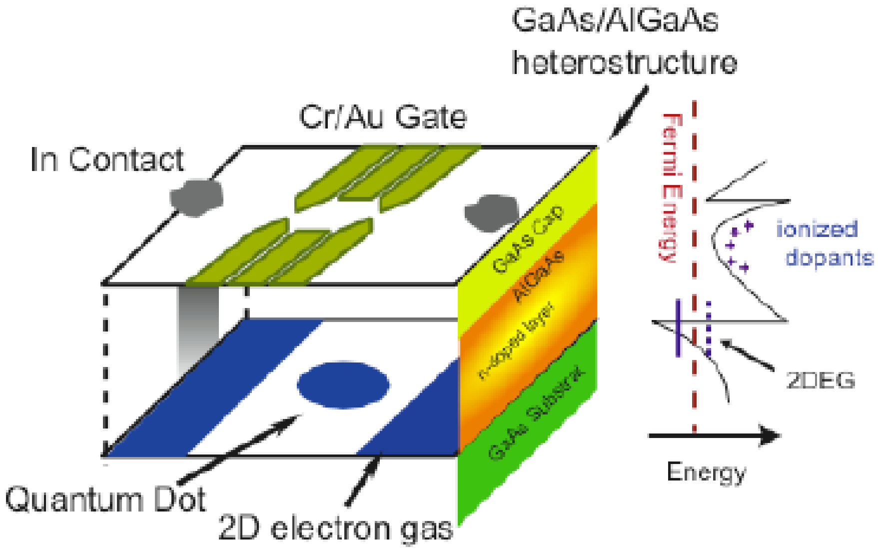

$$-\partial f / \partial \epsilon$$

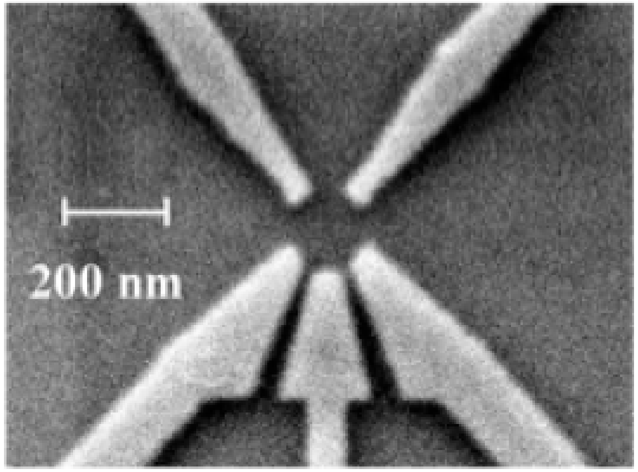


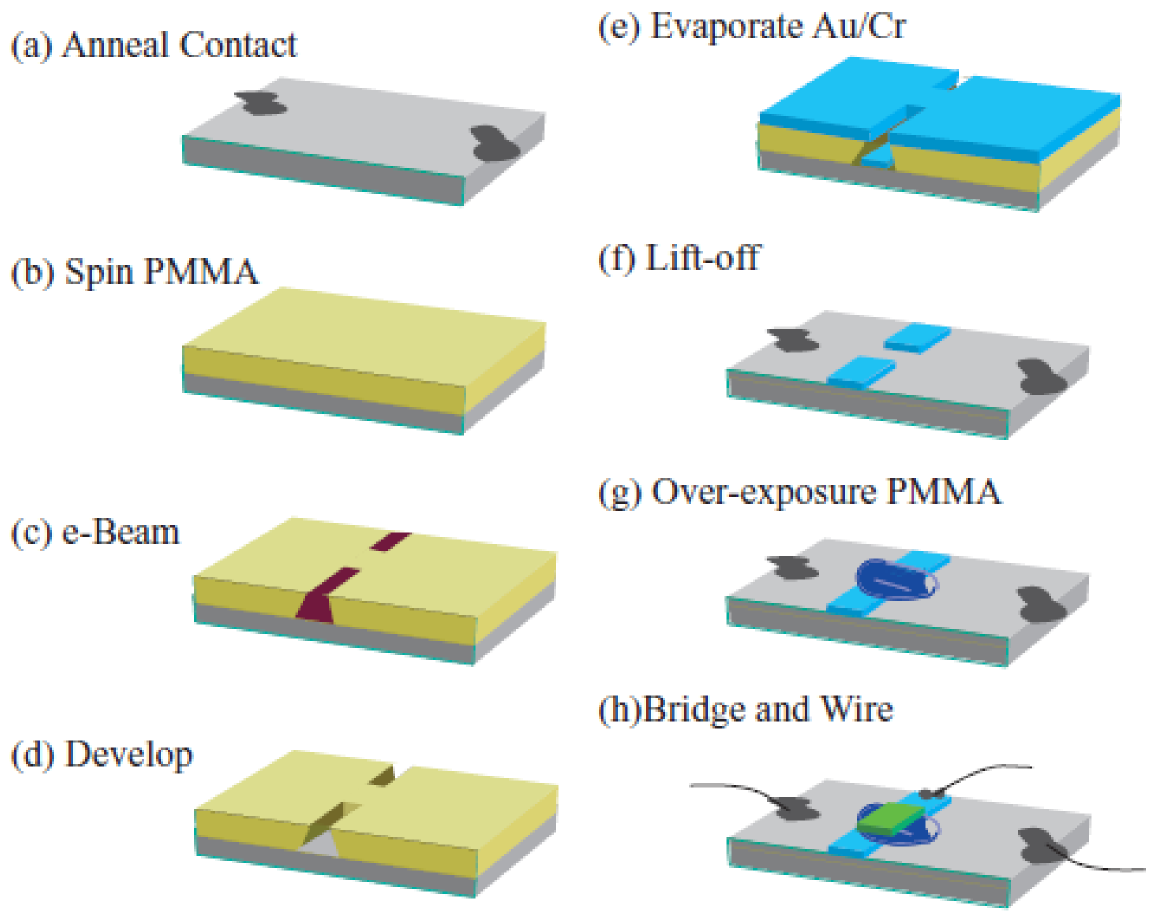
# Fabrication Techniques

- Advanced Electron Beam Lithography
- Self-Assembly/chemical growth
- Template
- NanoImprint
- Exfoliation--Graphene

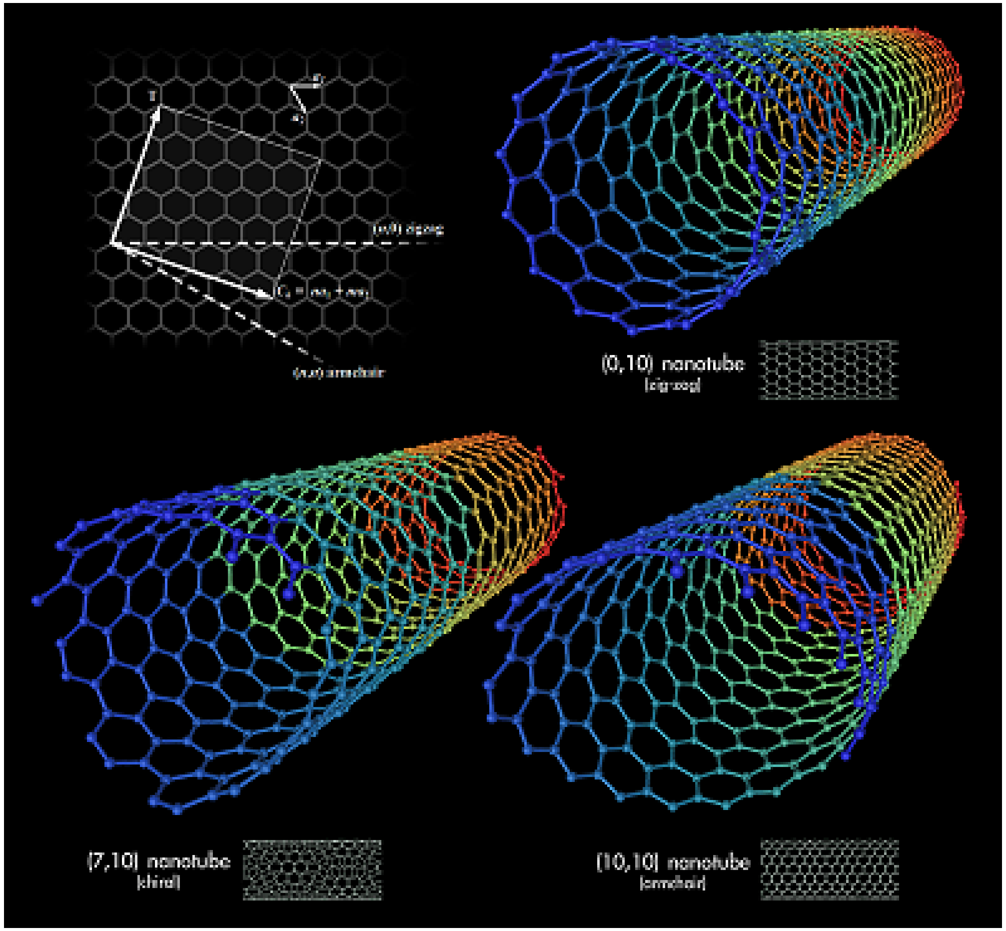
# Quantum Dots! (Robust Qubits)




(a) Anneal Contact
(b) Spin PMMA
(c) e-Beam
(d) Develop
(e) Evaporate Au/Cr
(f) Lift-off
(g) Over-exposure PMMA
(h)Bridge and Wire

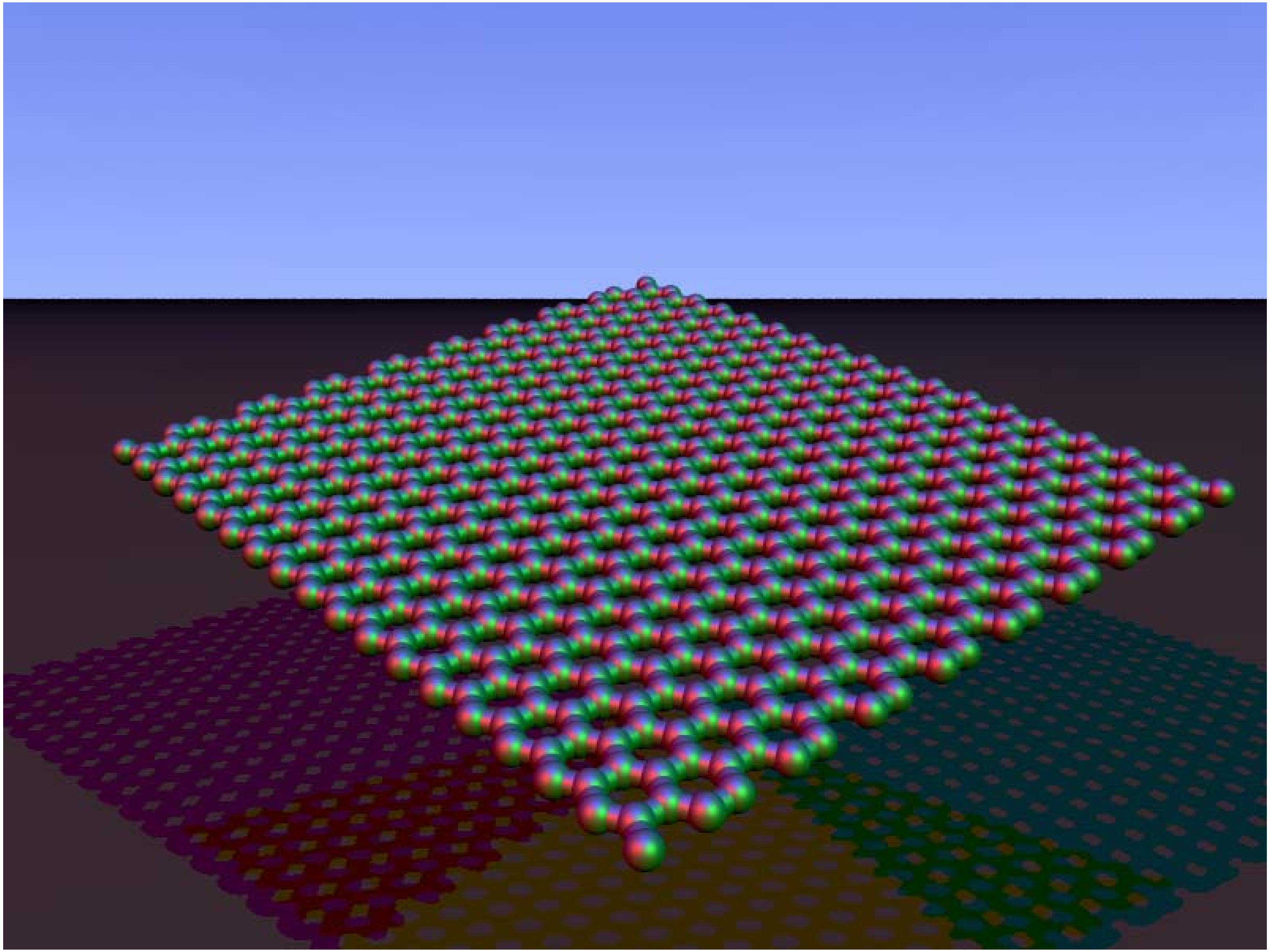
# Self-Assembly

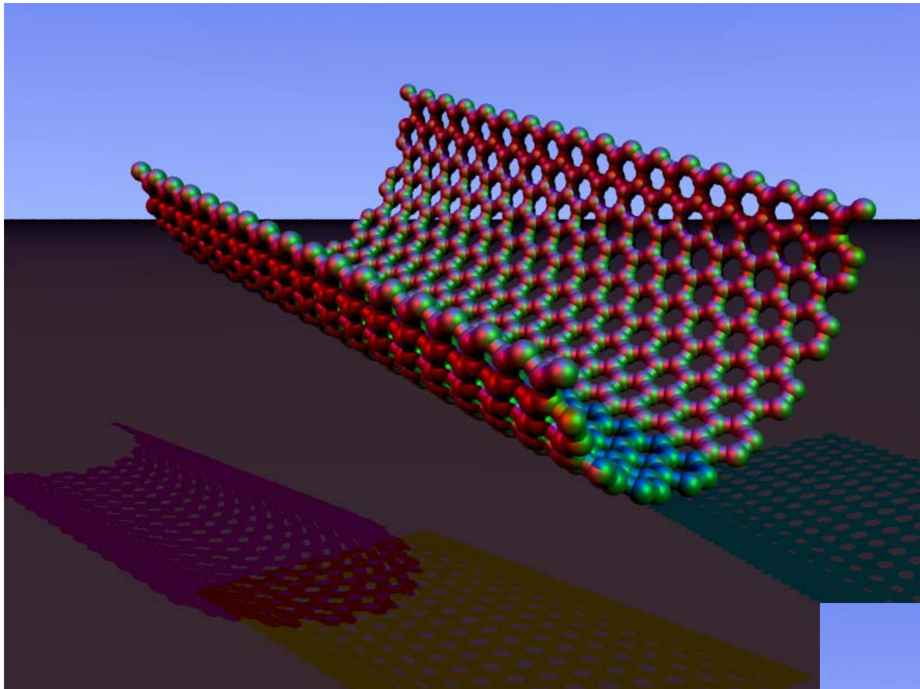



Ijima, 1991

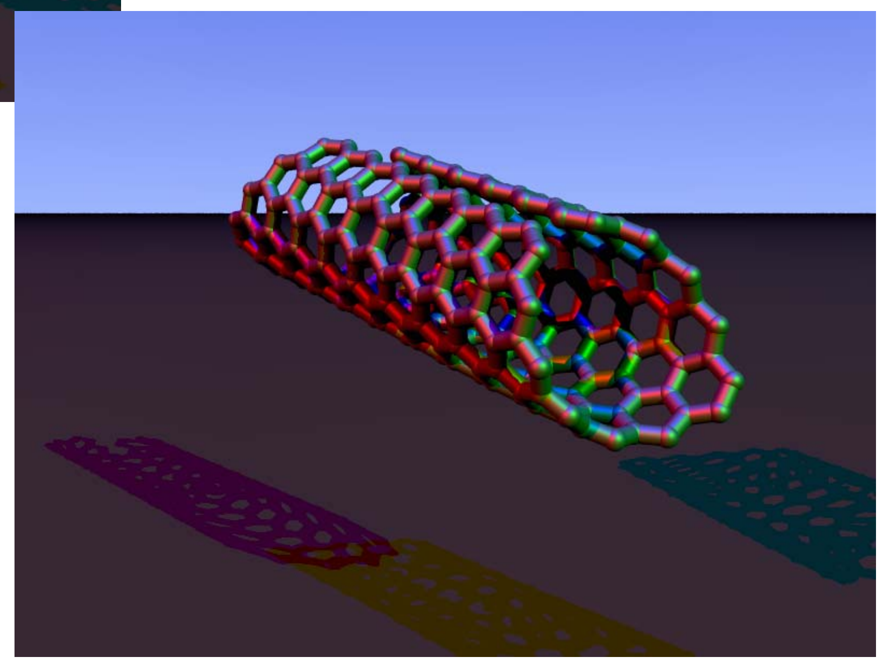
# Graphene Sheet

AK Geim; P. Kim





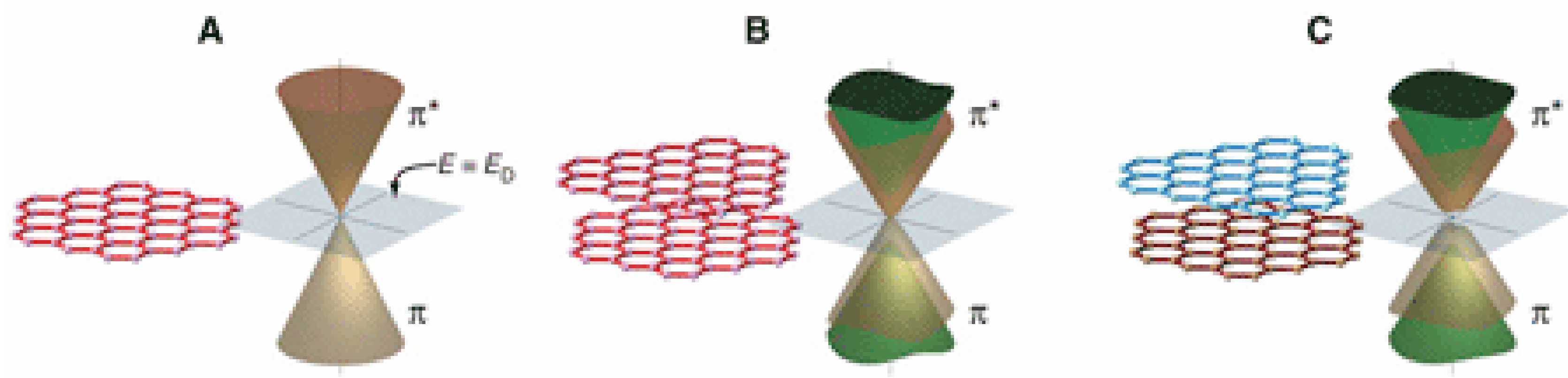
Armchair--Metallic

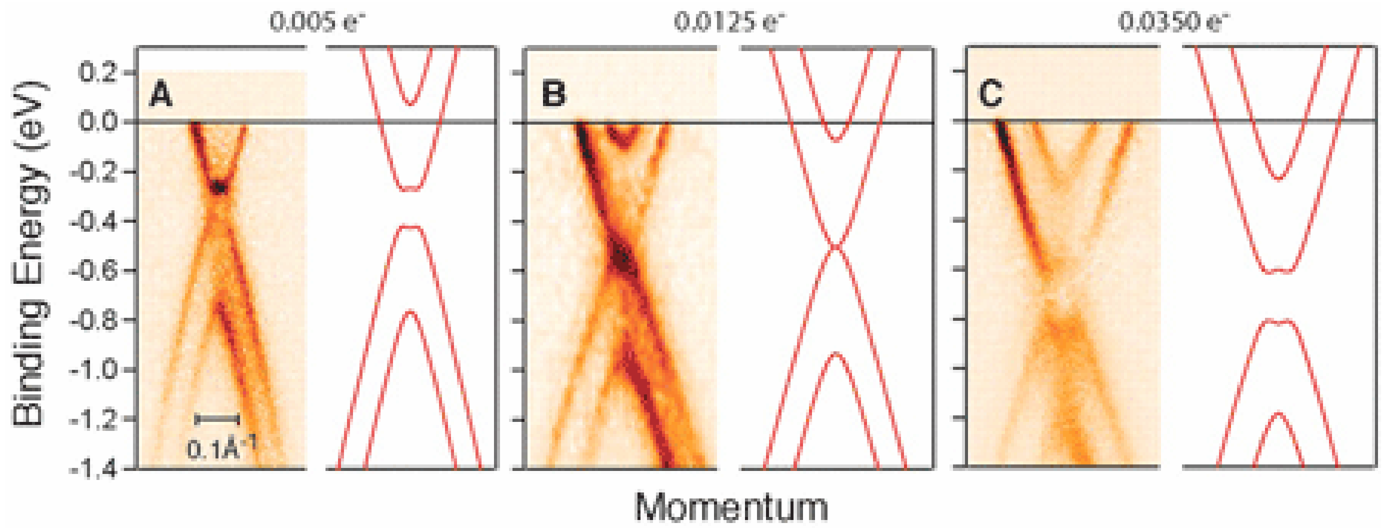


Zigzag--
Semiconducting

| Comparison of Mechanical Properties[26][27][28][29][30][31][32] | | | |
|---|---|---|---|
| **Material** | **Young's Modulus (TPa)** | **Tensile Strength (GPa)** | **Elongation at Break (%)** |
| SWNT | ~1 (from 1 to 5) | 13-53[E] | 16 |
| Armchair SWNT | 0.94[T] | 126.2[T] | 23.1 |
| Zigzag SWNT | 0.94[T] | 94.5[T] | 15.6-17.5 |
| Chiral SWNT | 0.92 | | |
| MWNT | 0.8-0.9[E] | 150 | |
| Stainless Steel | ~0.2 | ~0.65-1 | 15-50 |
| Kevlar | ~0.15 | ~3.5 | ~2 |
| Kevlar[T] | 0.25 | 29.6 | |

# Single and Bi-layer Graphene



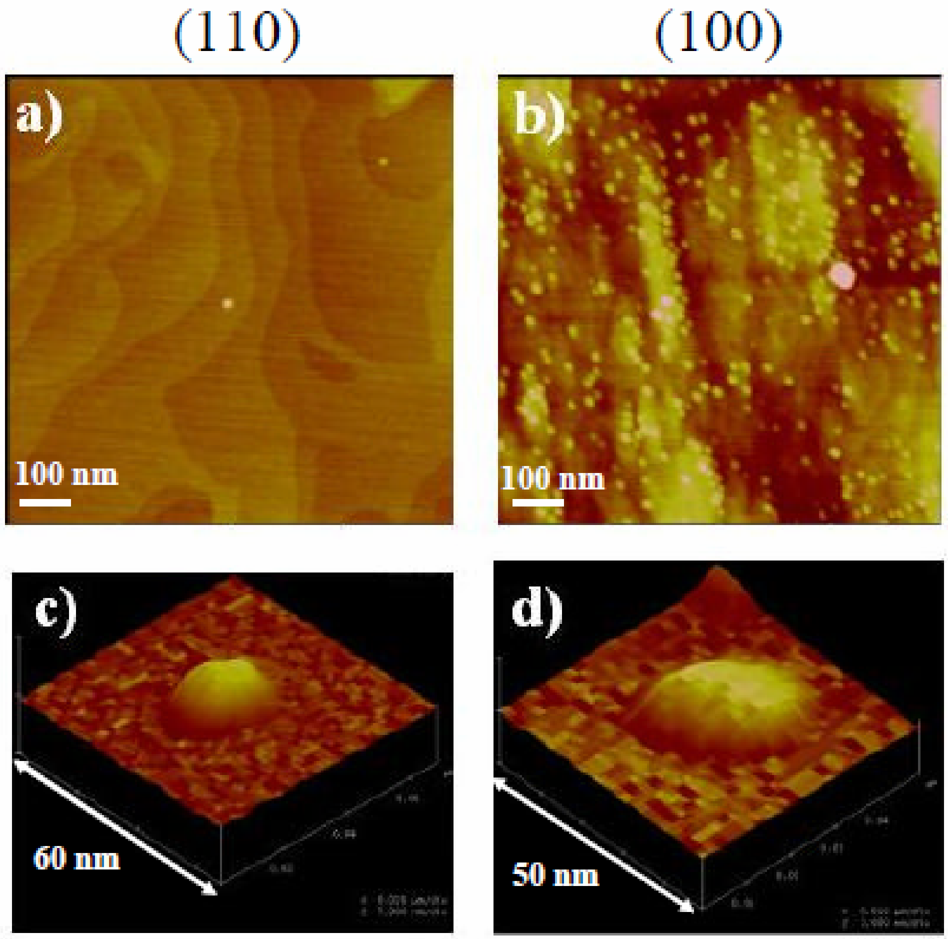



# Self-Assembled Quantum Dots

E.g. Gold nanoparticles,
ZnS claded CdSe NP

Size-Sorted Colloidal Nanocrystals

# AFM of (110) InAs QDs




S. Lyons, Princeton University

# Template
# Superconducting Al Nanowire

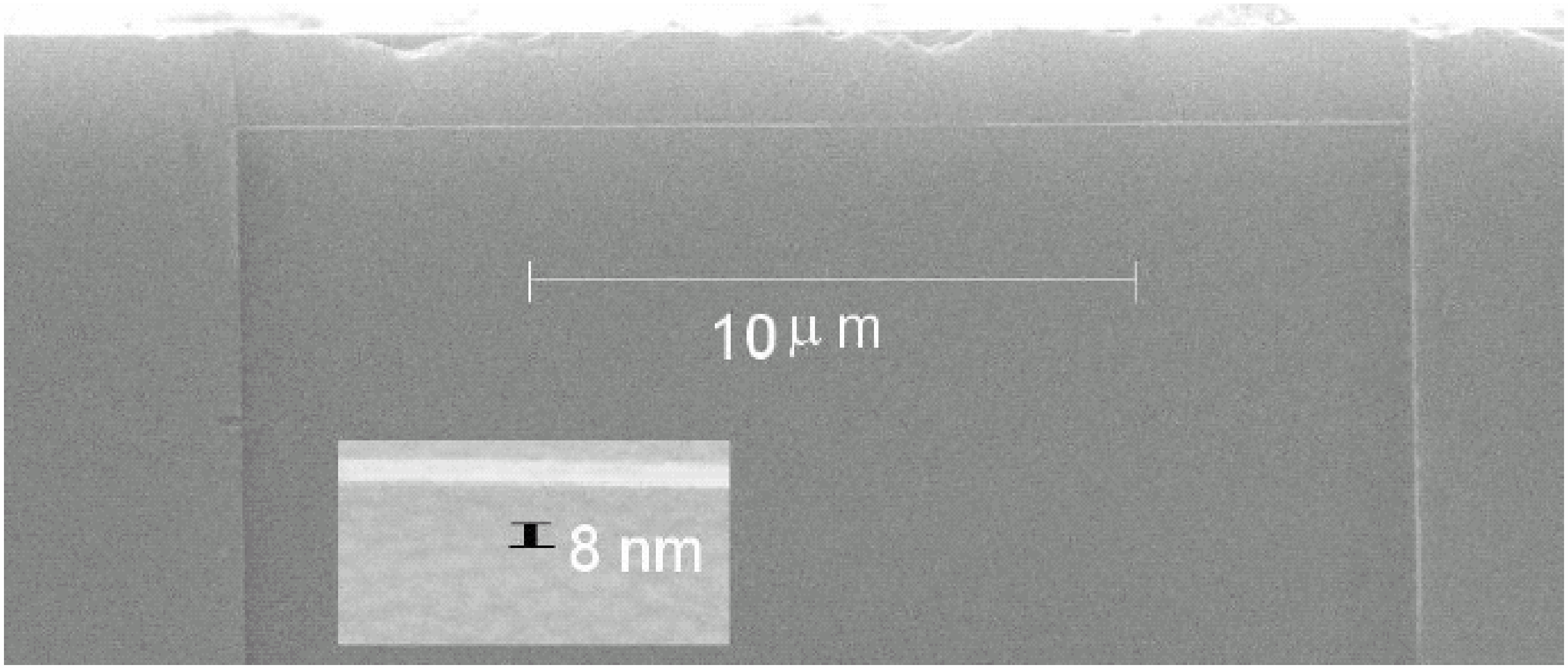
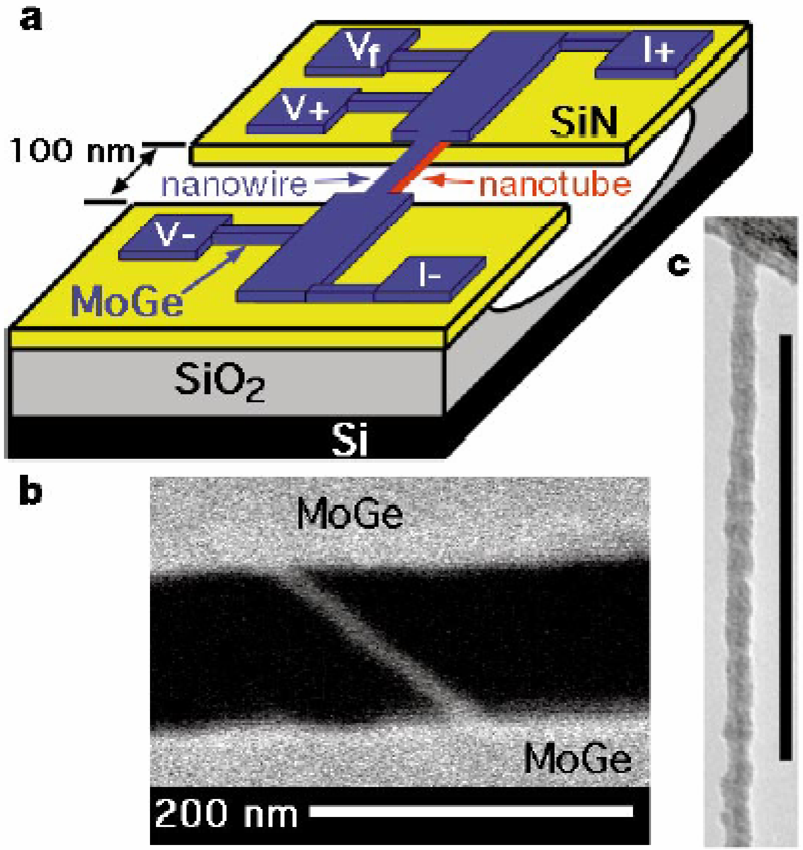


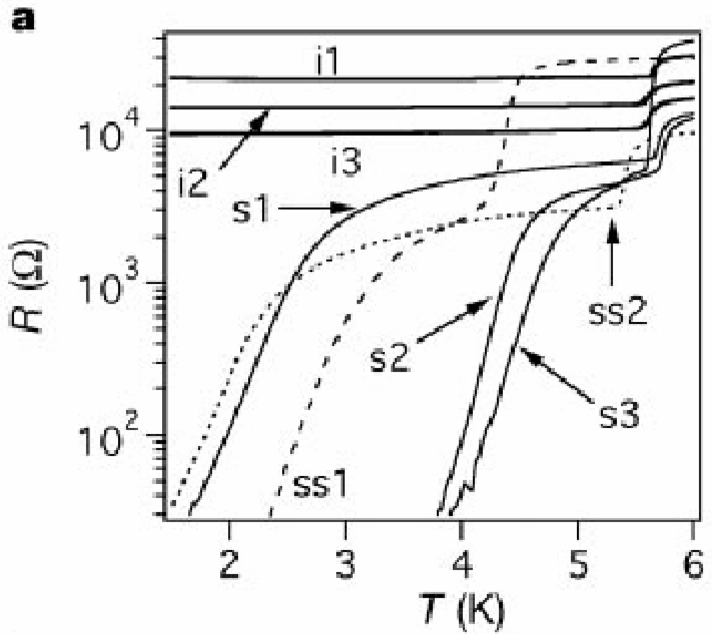

FIG. 2: SEM image of an 8 nm wide wire 20 $\mu$m long.

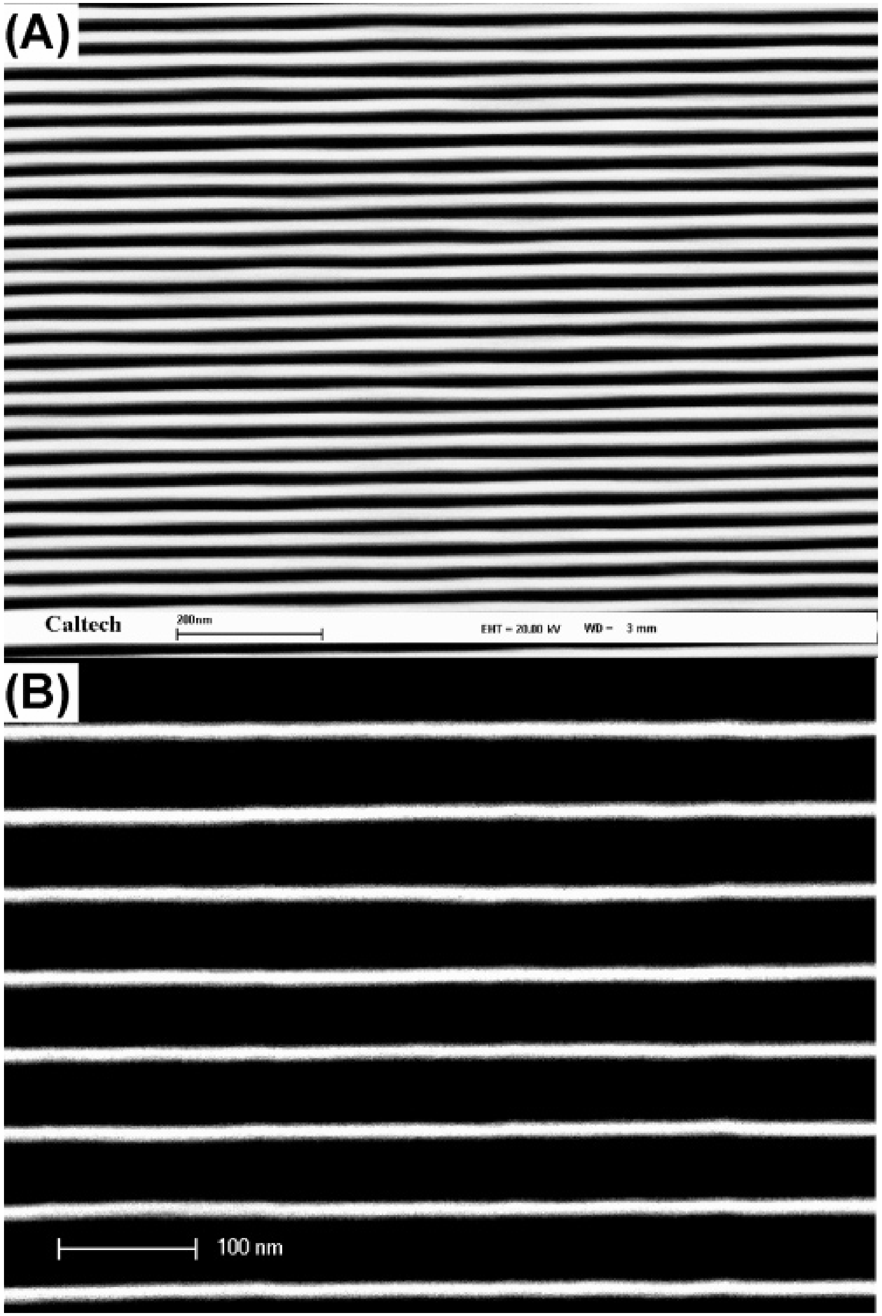
**F. Altomare, A.M. Chang et al., APL (2005)**

**A. Bezryadin, C.N Lau, and M Tinkham, Nature 404, 971 (2000).**

# Mo_{0.79}Ge_{0.21}

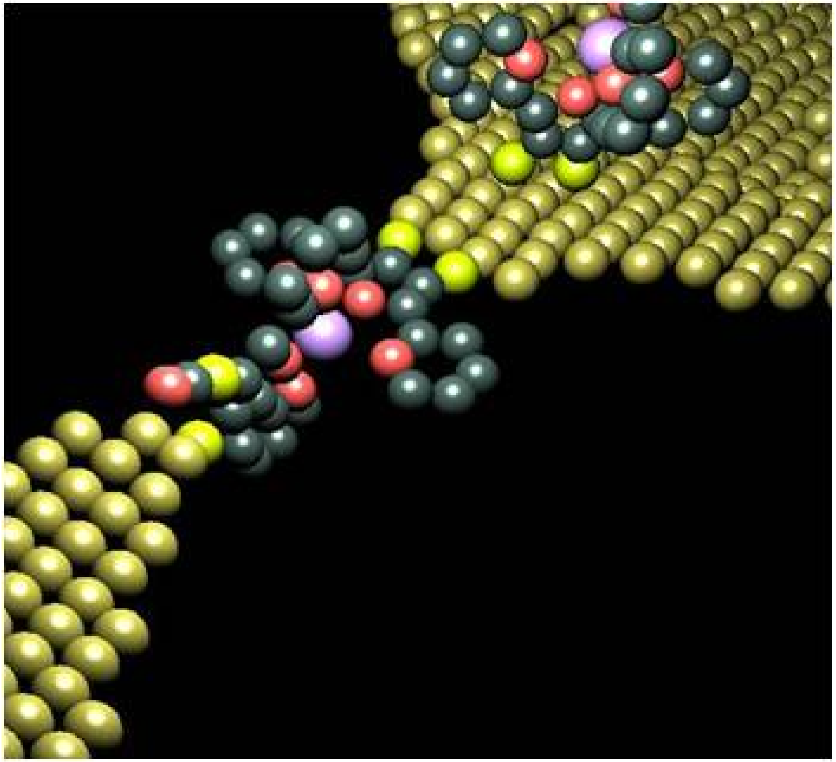


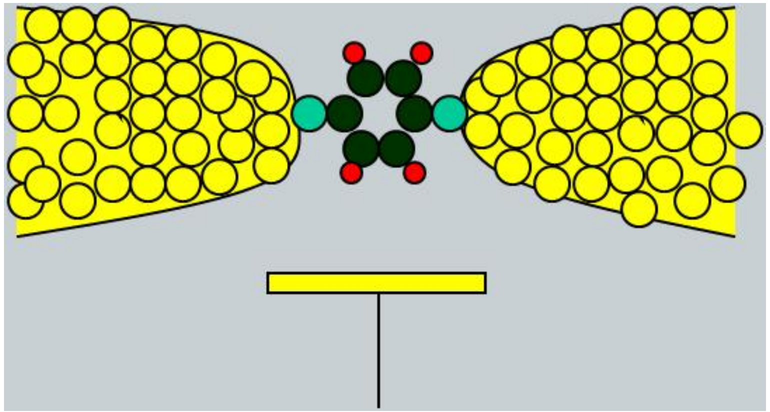






J. Heath, 2008

# Combination—e.g. Molecular Devices

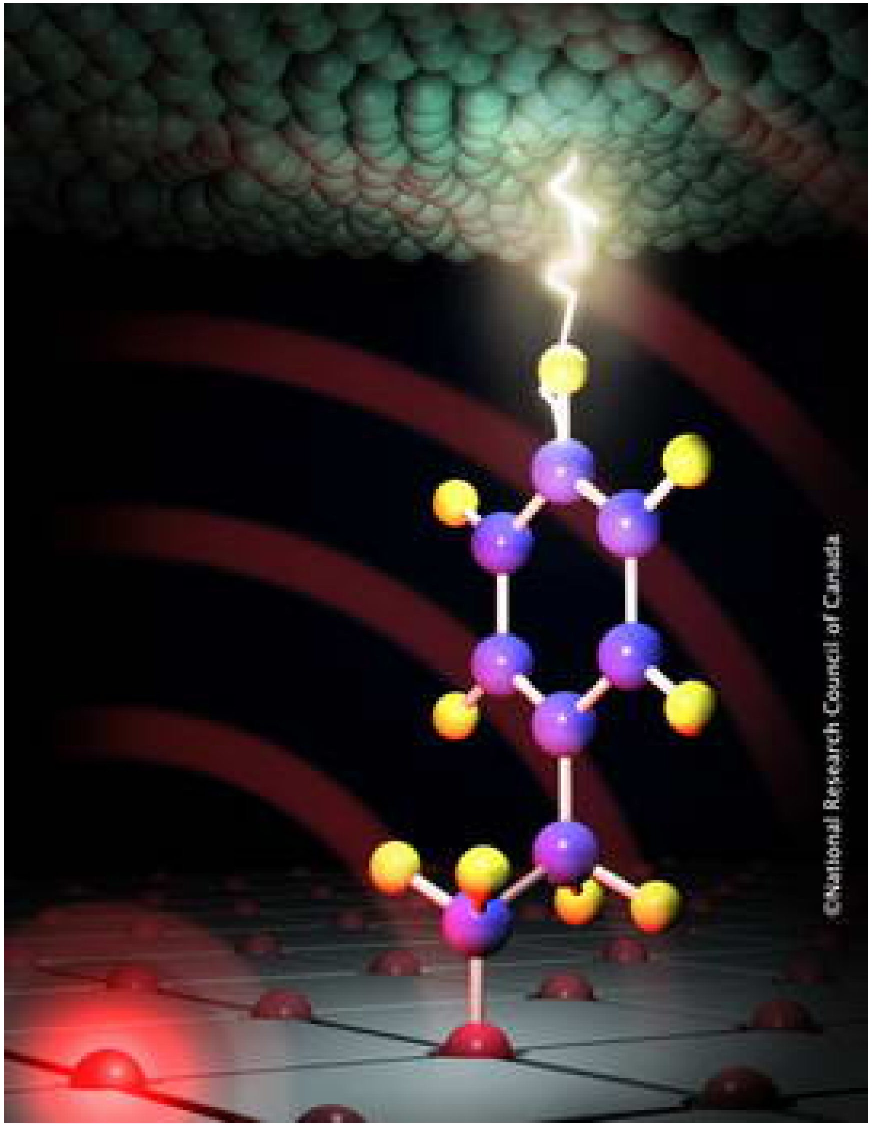




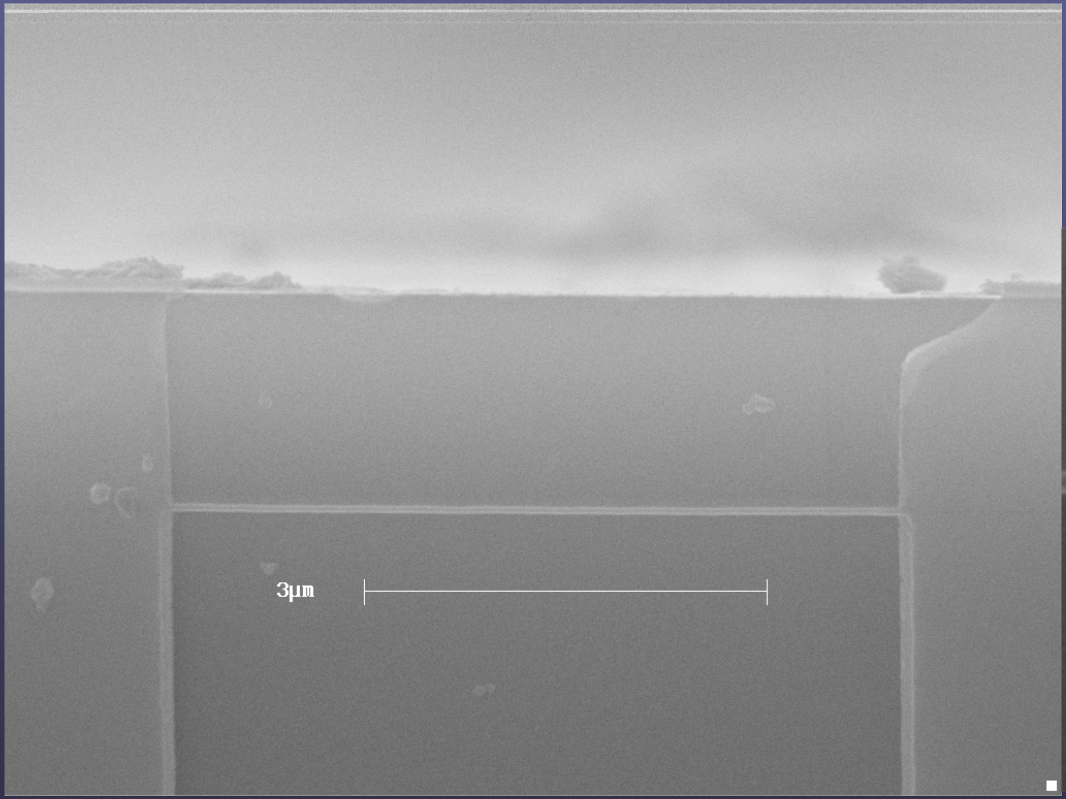
Tetrahydrofuran (THF)

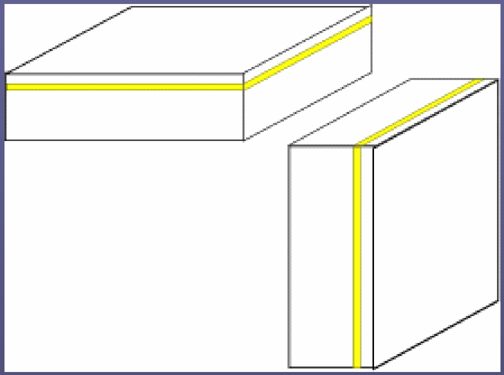
Natelson (Rice);

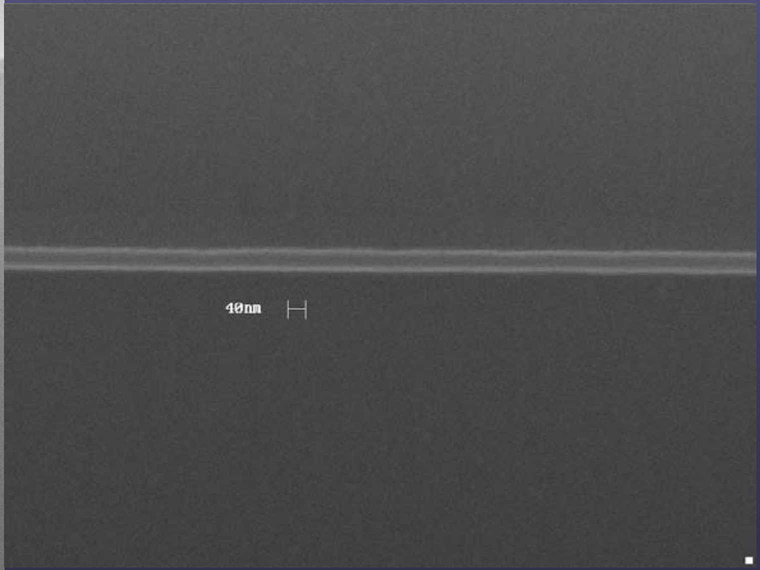
image: J.W. Ciszek


©National Research Council of Canada

# Metallic Nanowire





# Superconducting Al Nanowire

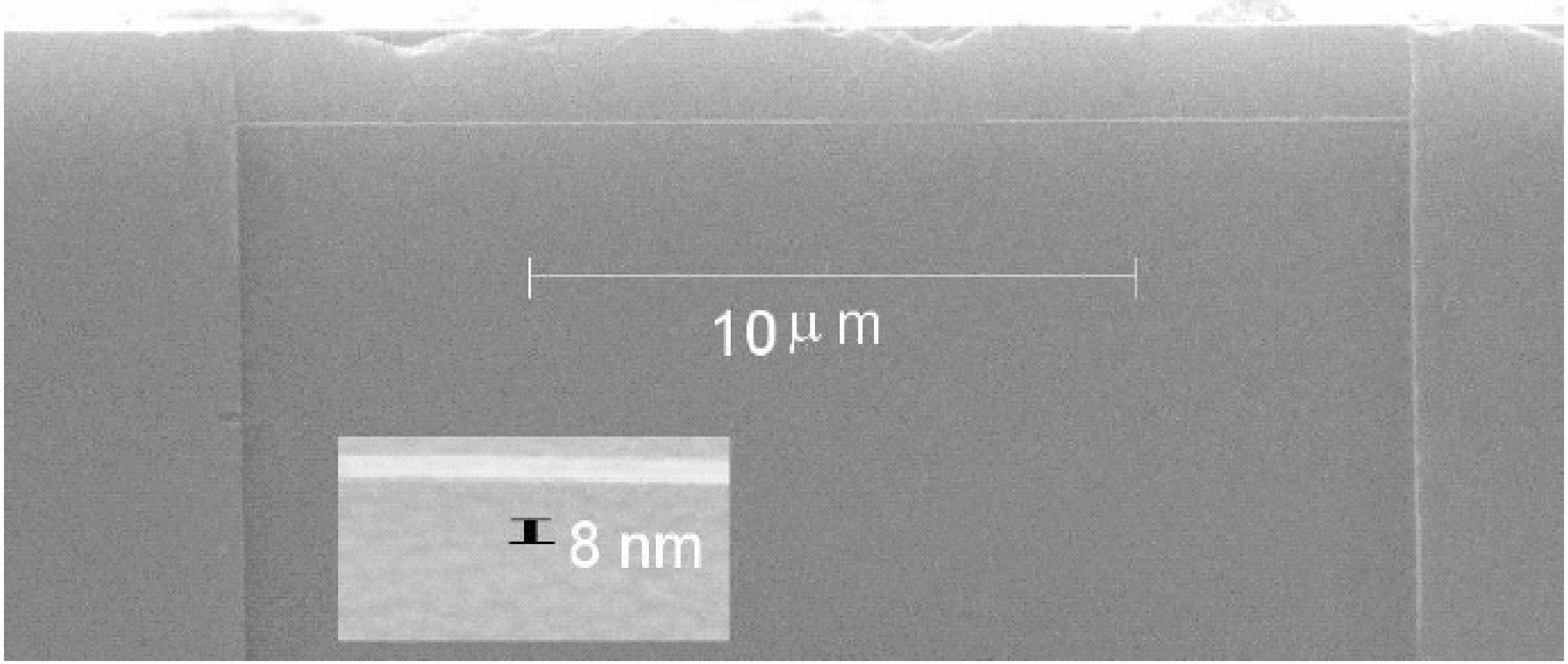
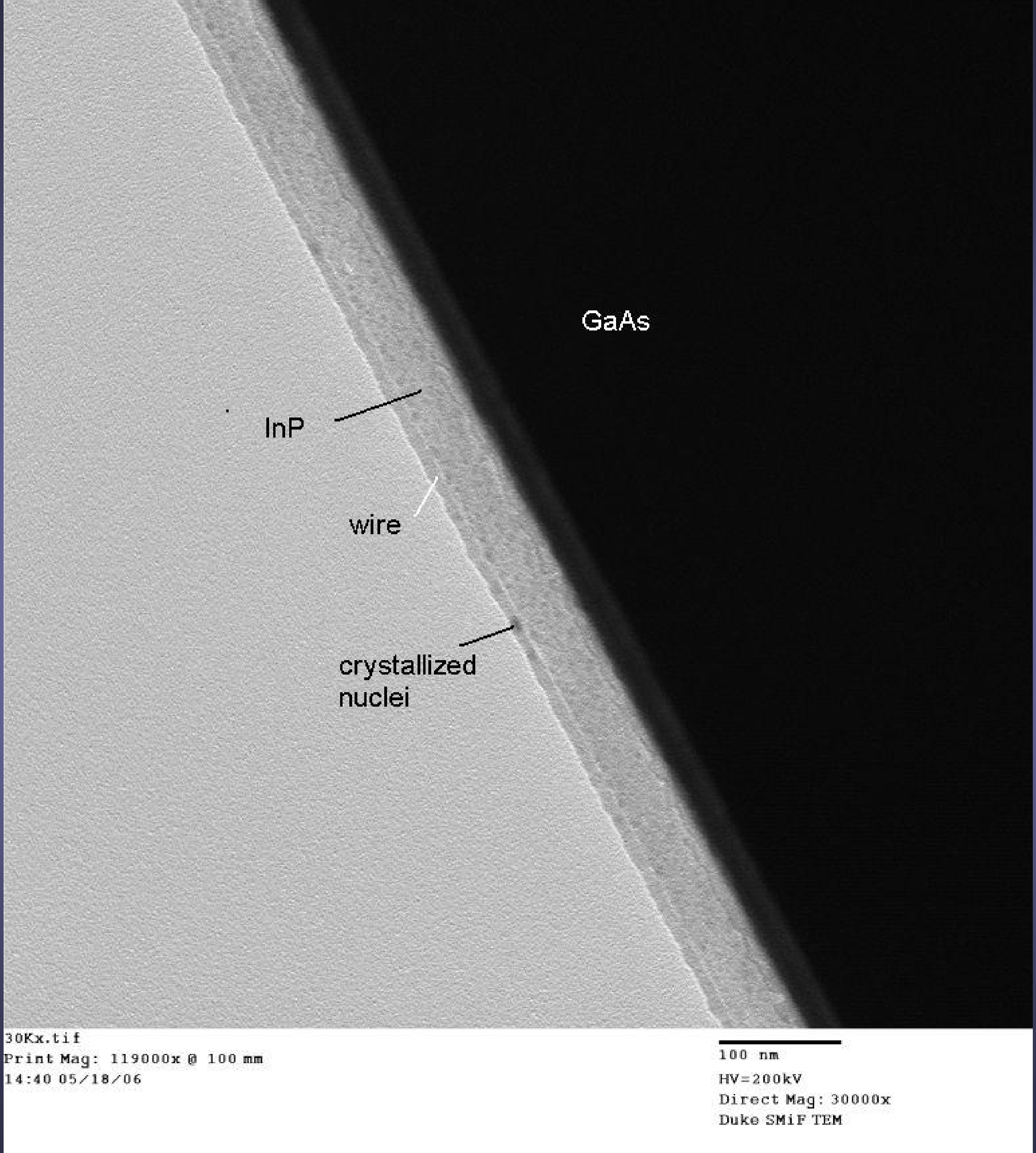
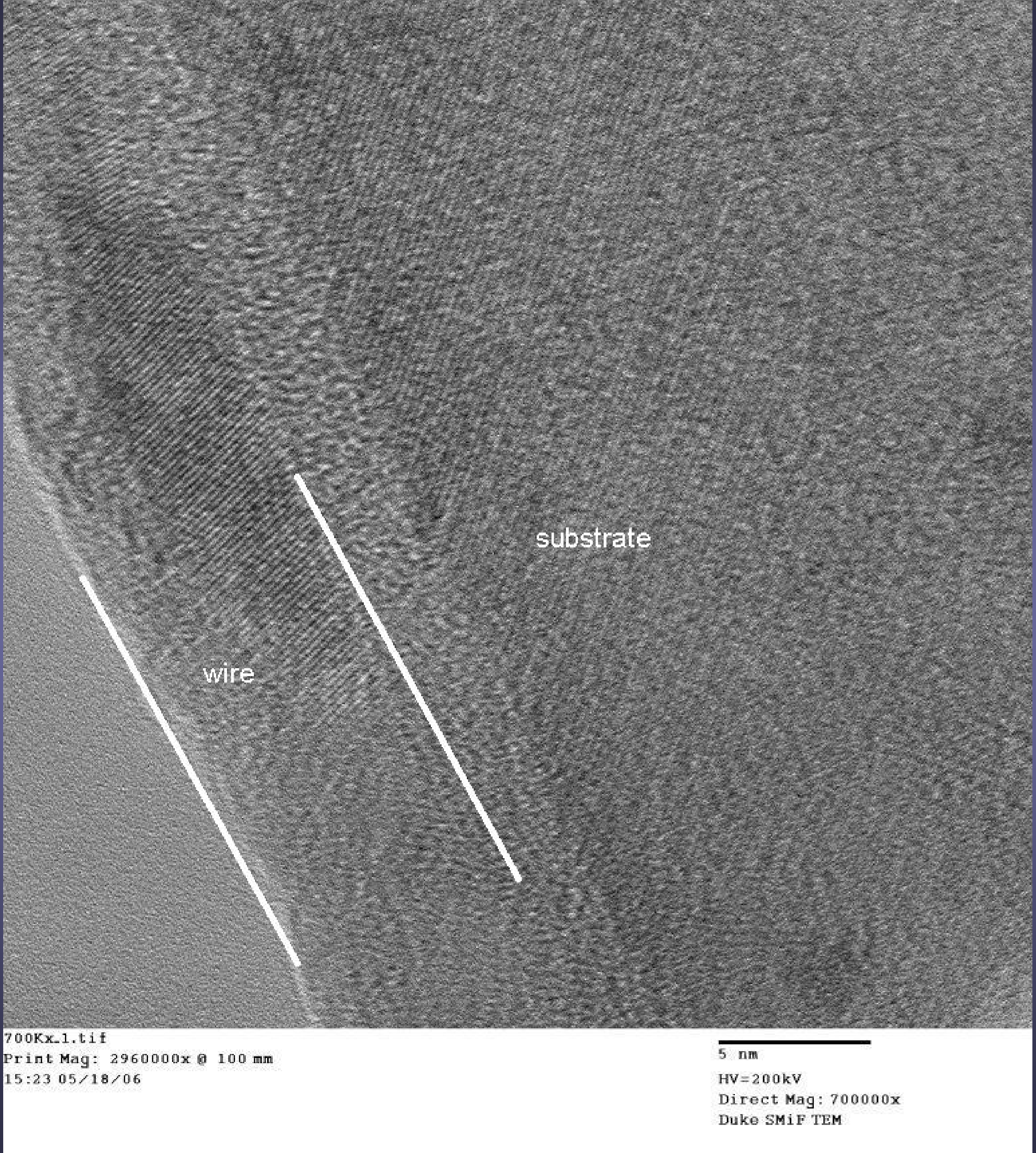



FIG. 2: SEM image of an 8 nm wide wire 20 $\mu$m long.

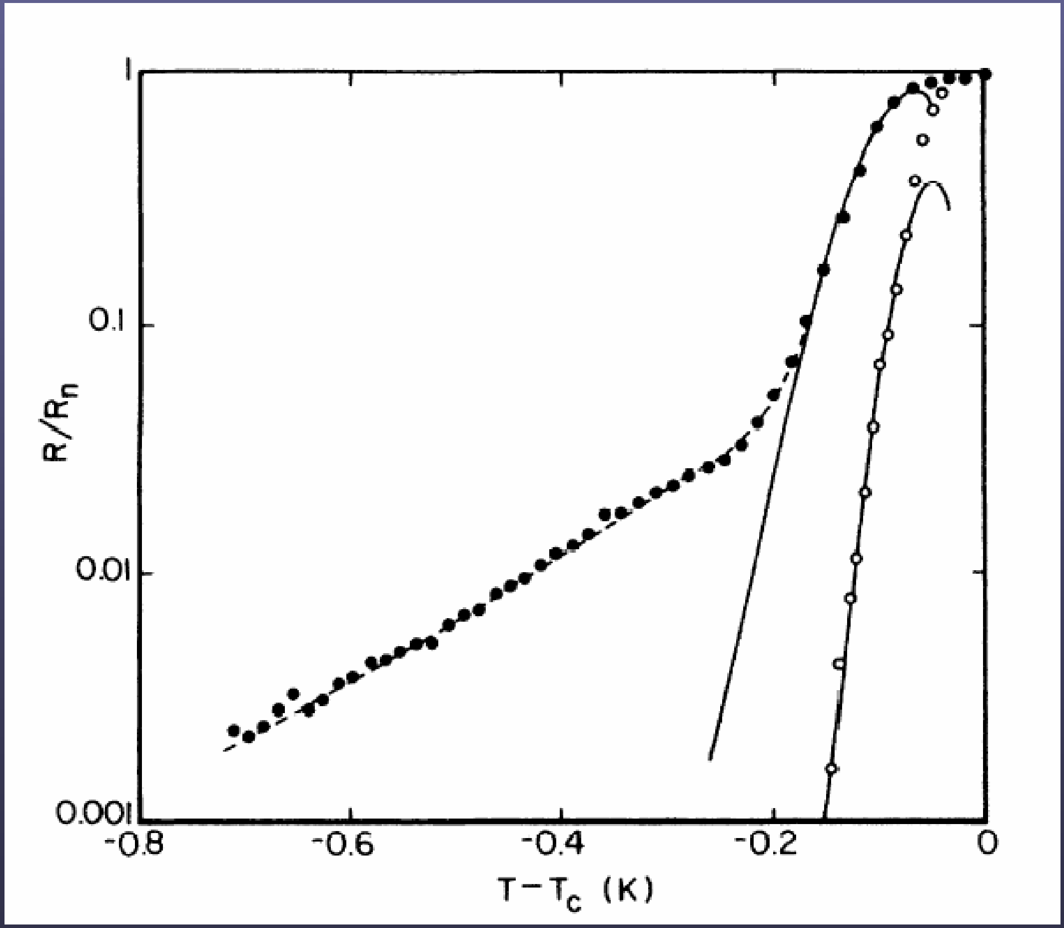
F. Altomare, A.M. Chang et al., APL (2005) and Cond-Mat (Sub. to PRL)


GaAs
InP
wire
crystallized
nuclei
30Kx.tif
Print Mag: 119000x @ 100 mm
14:40 05/18/06
100 nm
HV=200kV
Direct Mag: 30000x
Duke SMiF TEM

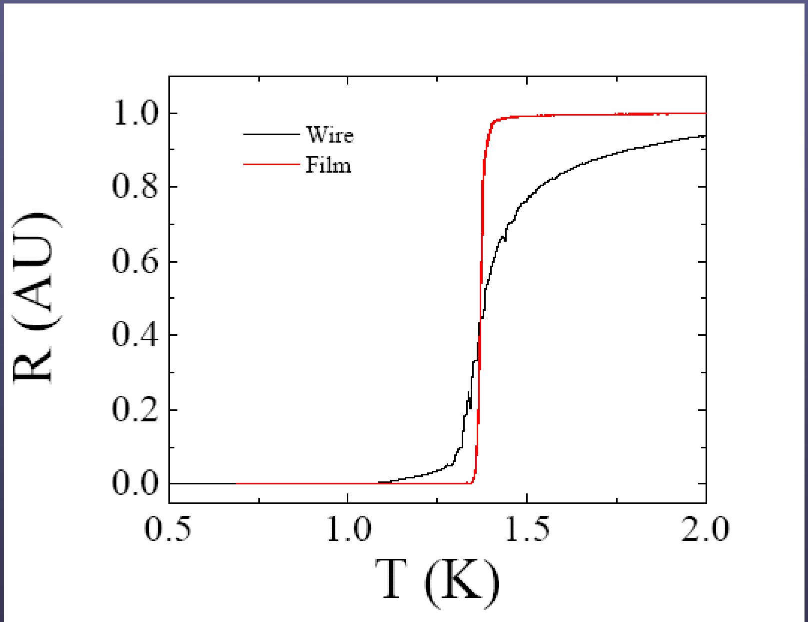

substrate
wire
700Kx_1.tif
Print Mag: 2960000x @ 100 mm
15:23 05/18/06
5 nm
HV=200kV
Direct Mag: 700000x
Duke SMiF TEM

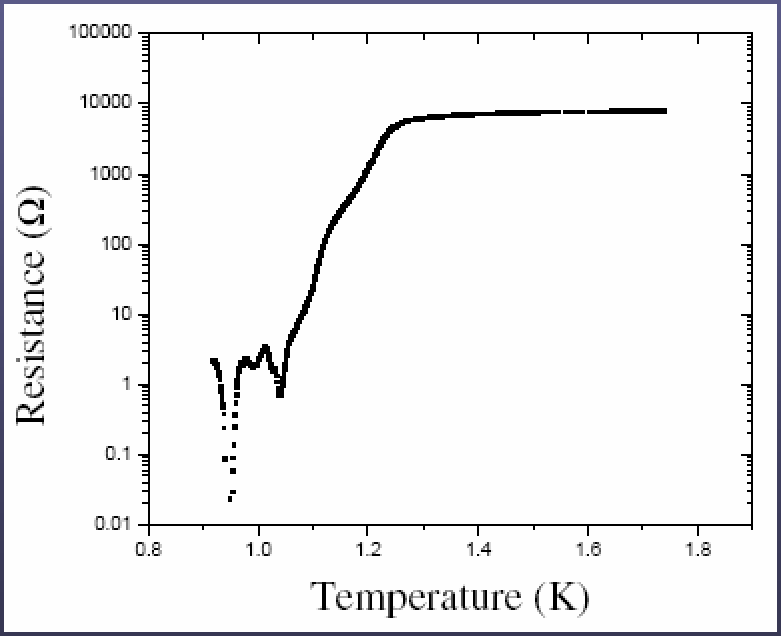
**N. Giordano**
**Phys. Rev. Lett. 61, 2137-2140 OCT 31 1988; and Phys. Rev. B 41, 6350 (1990).**

## 41 nm diameter In wire



# 1D versus 2D Superconductivity



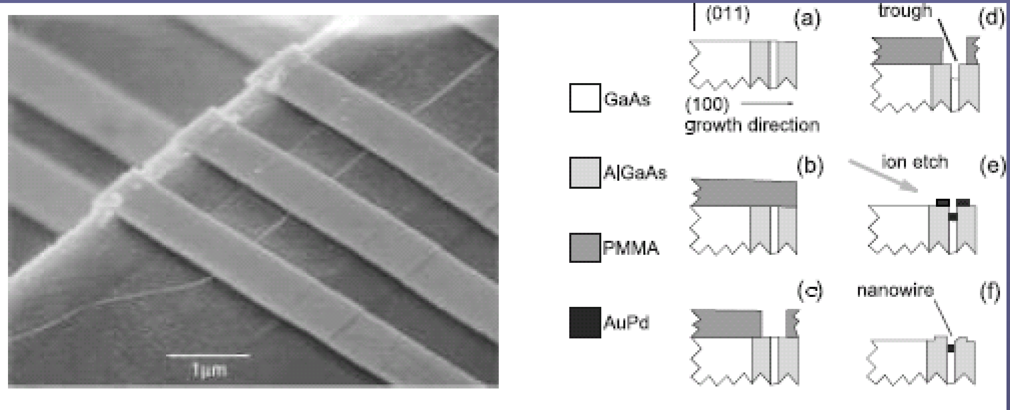
# Superconducting ultra-narrow Al nanowires

Fabio Altomare[1,2], Albert M. Chang[1,2],
Michael R. Melloch[3],
Yuguong Yang[4], Charles W. Tu[4]

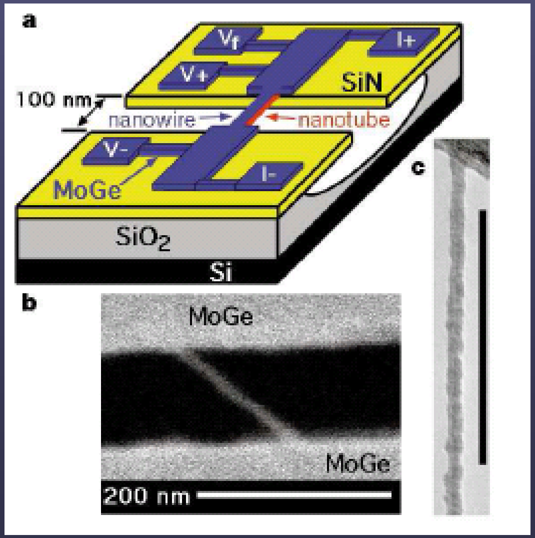
1) Dept. of Physics, Purdue University
2) Dept. of Physics, Duke University
3) School of Electrical and Computer Engineering, Purdue University
4) Dept. of Computer and Electrical Engineering, UCSD

Partially supported by NSF DMR-0135931 DMR-0401648

# State of the art techniques

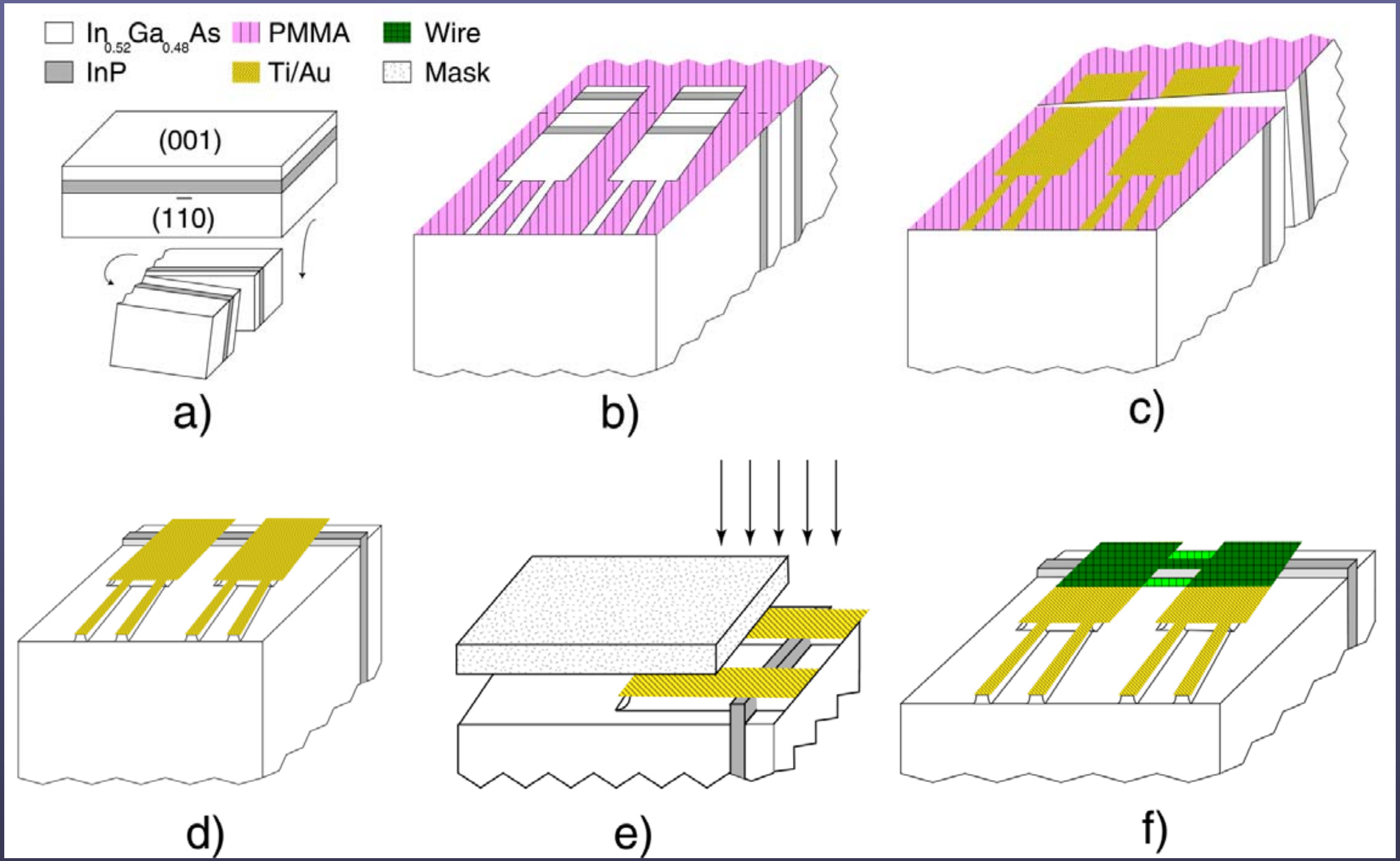



Natelson *et al.*,
Appl. Phys. Lett. **77**,
1991-1993 (2000)




Bezryadin *et al.*,
Nature **404**,
971-973 (2000)

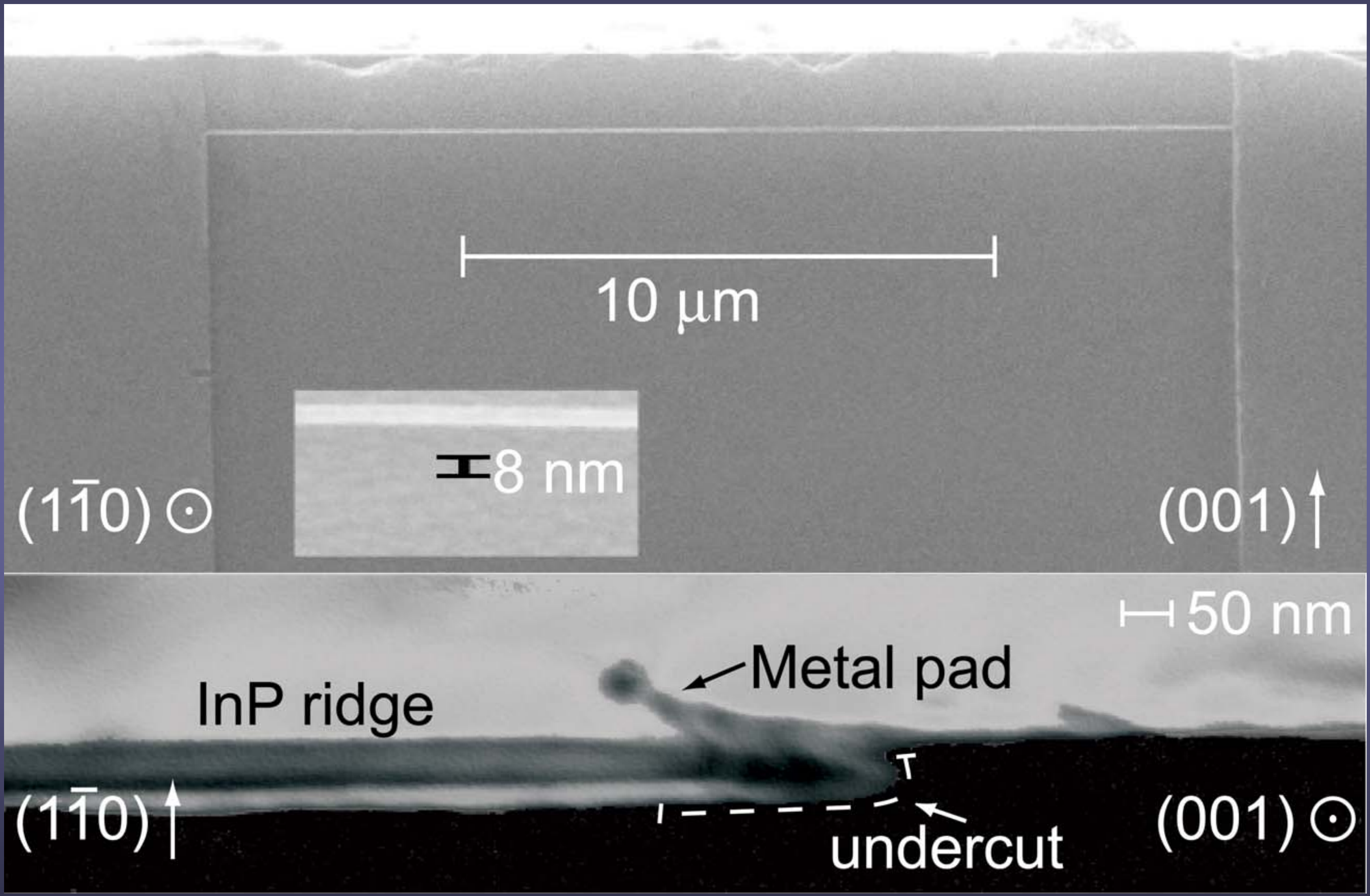
# Fabrication scheme




a) Sample preparation
b) Lithography
c) Evaporation
d) Wet etching
e) Final evaporation
f) Final result

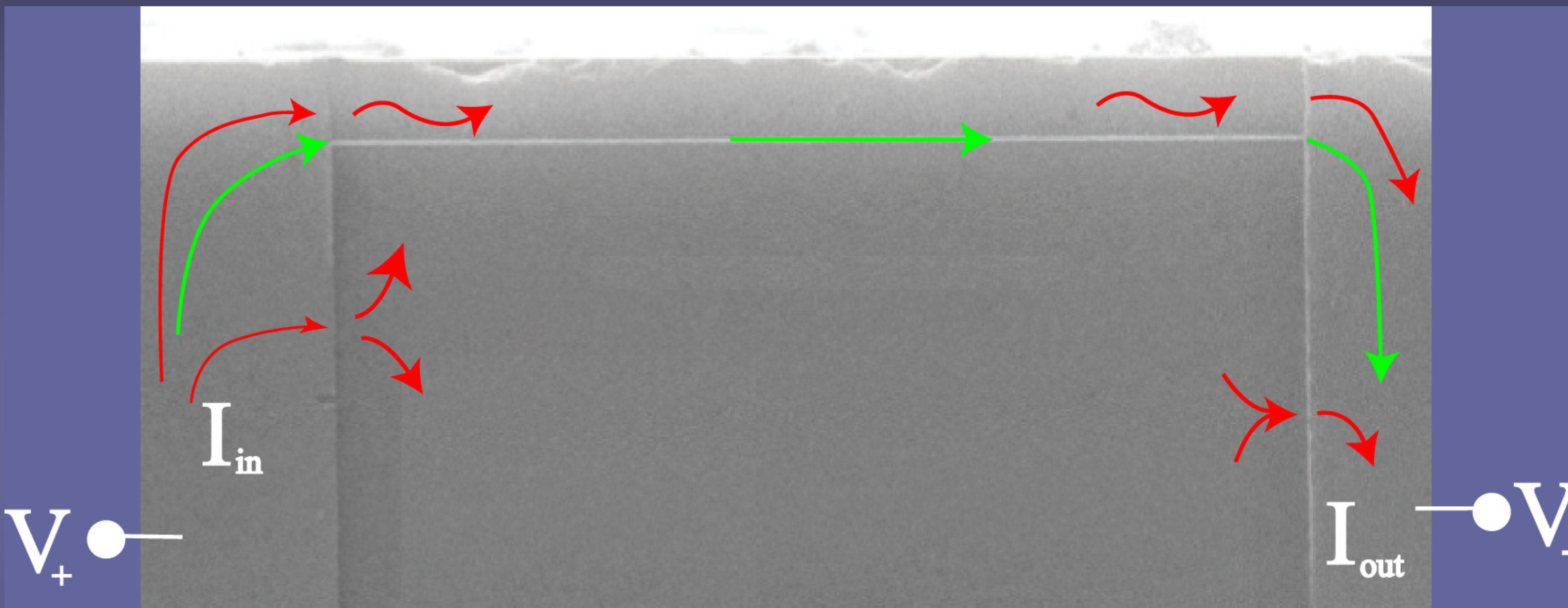
cond-mat/0412210

# SEM image



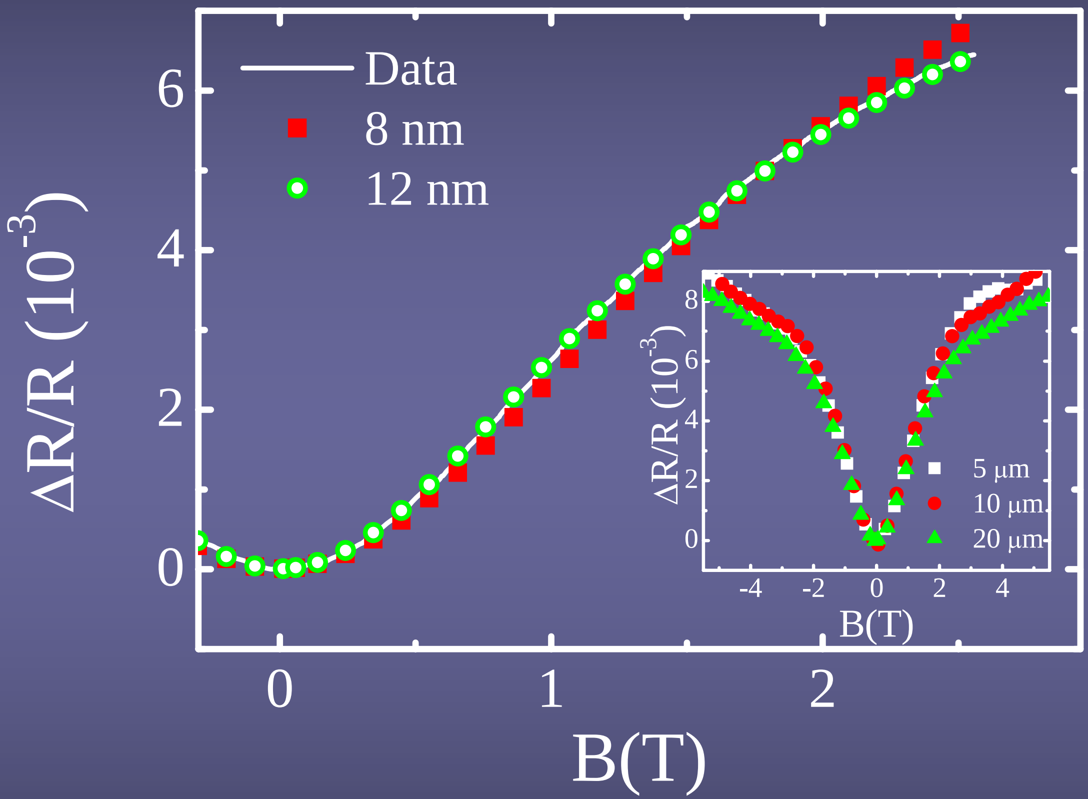
10 μm
8 nm
(1$\bar{1}$0)
(001)
50 nm
Metal pad
InP ridge
(1$\bar{1}$0)
(001)
undercut

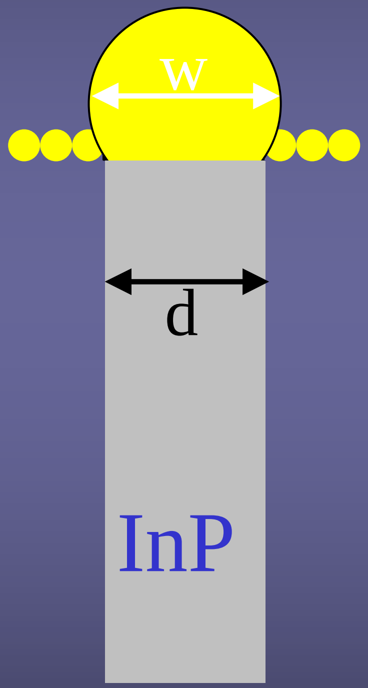
# Weak Localization Magnetoresistance




$$\frac{\Delta R}{R} = \frac{e^2}{\pi\hbar}\frac{R}{L}\left[\frac{3}{2}\left(\frac{1}{L_\phi^2} + \frac{4}{3L_{so}^2} + \frac{w^2e^2}{3\hbar^2}B^2\right)^{-1/2} - \frac{1}{2}\left(\frac{1}{L_\phi^2} + \frac{w^2e^2}{3\hbar^2}B^2\right)^{-1/2}\right]$$

# AuPd wires



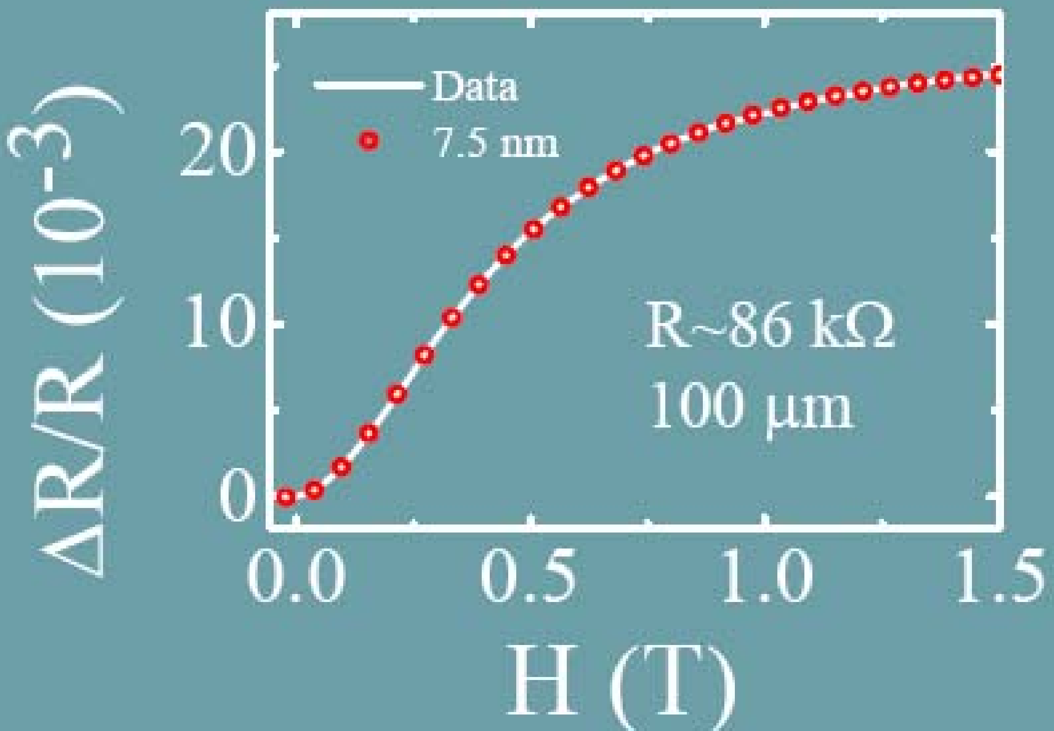

L=5, 10, 20 µm
R= 20, 35, 70 kΩ

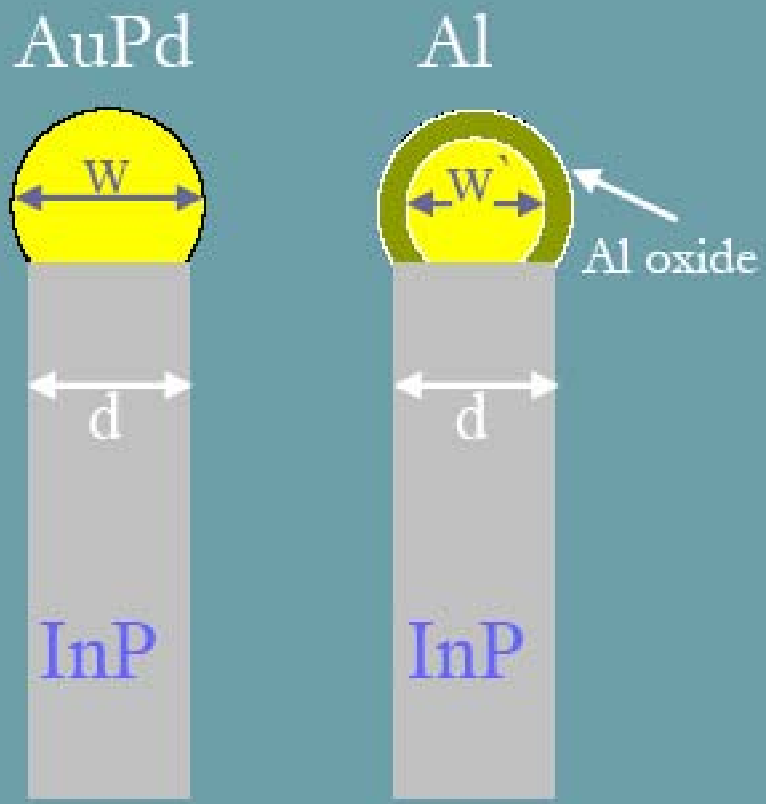
w = 8 nm
$L_\phi$ = 99 nm

w = 12 nm
$L_\phi$ = 81 nm

cond-mat/0412210

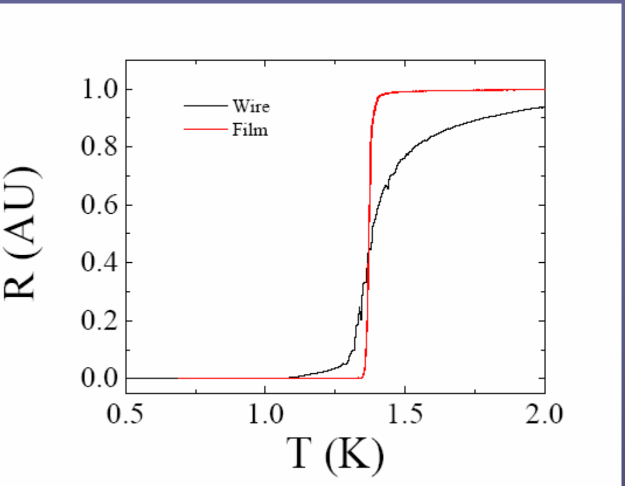
# Al nanowires

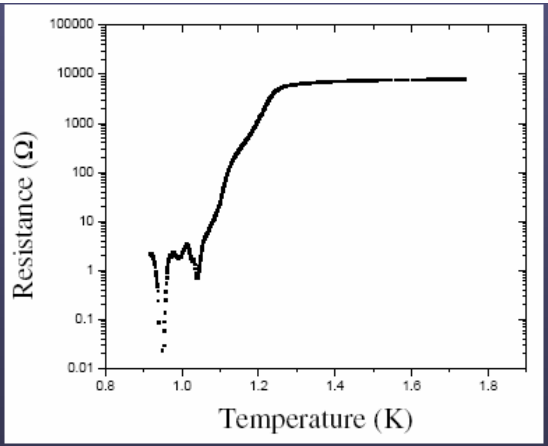


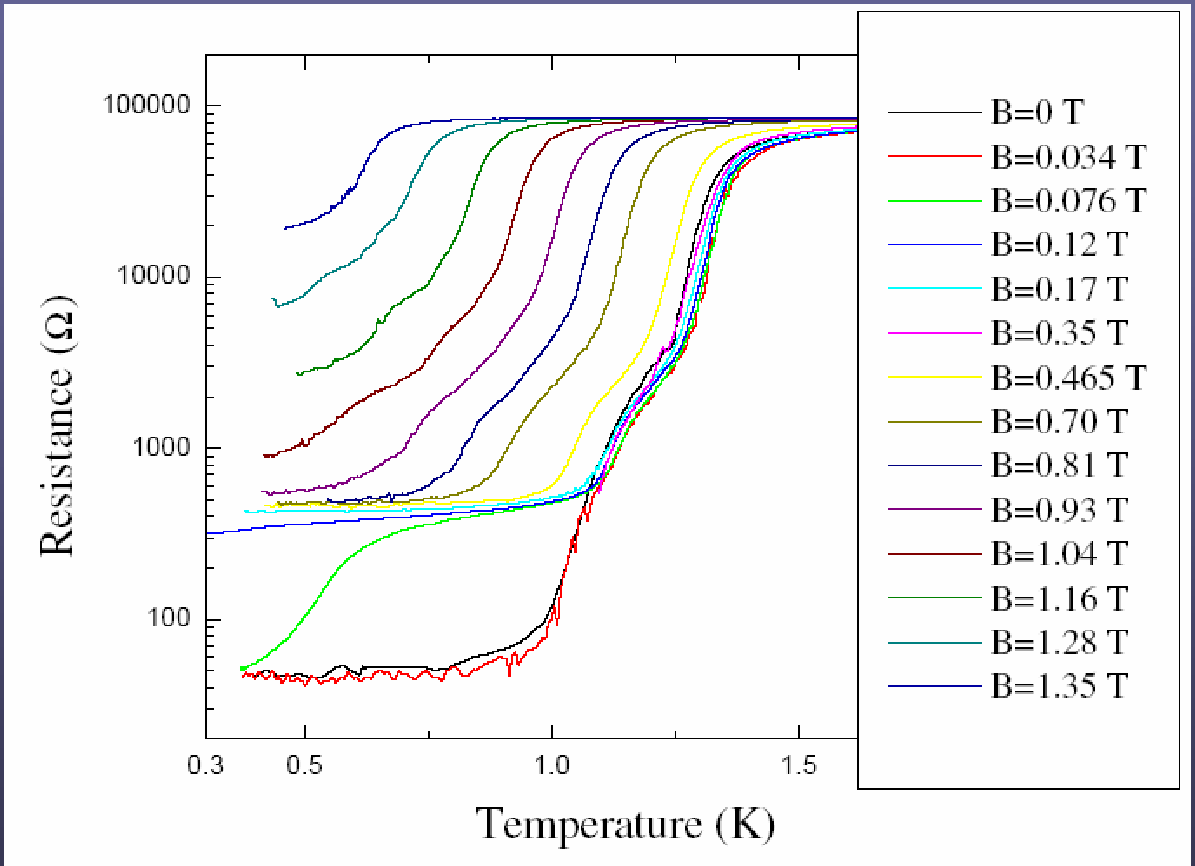




$w = .5$ nm

Physical Review Letters **97**, 017001 (2006)


Wire
Film
R (AU)
T (K)
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.0
1.5
2.0

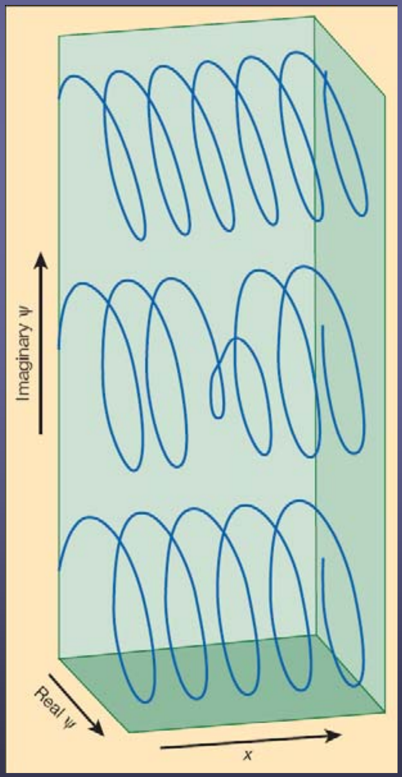

Resistance (Ω)
Temperature (K)
100000
10000
1000
100
10
1
0.1
0.01
0.8
1.0
1.2
1.4
1.6
1.8

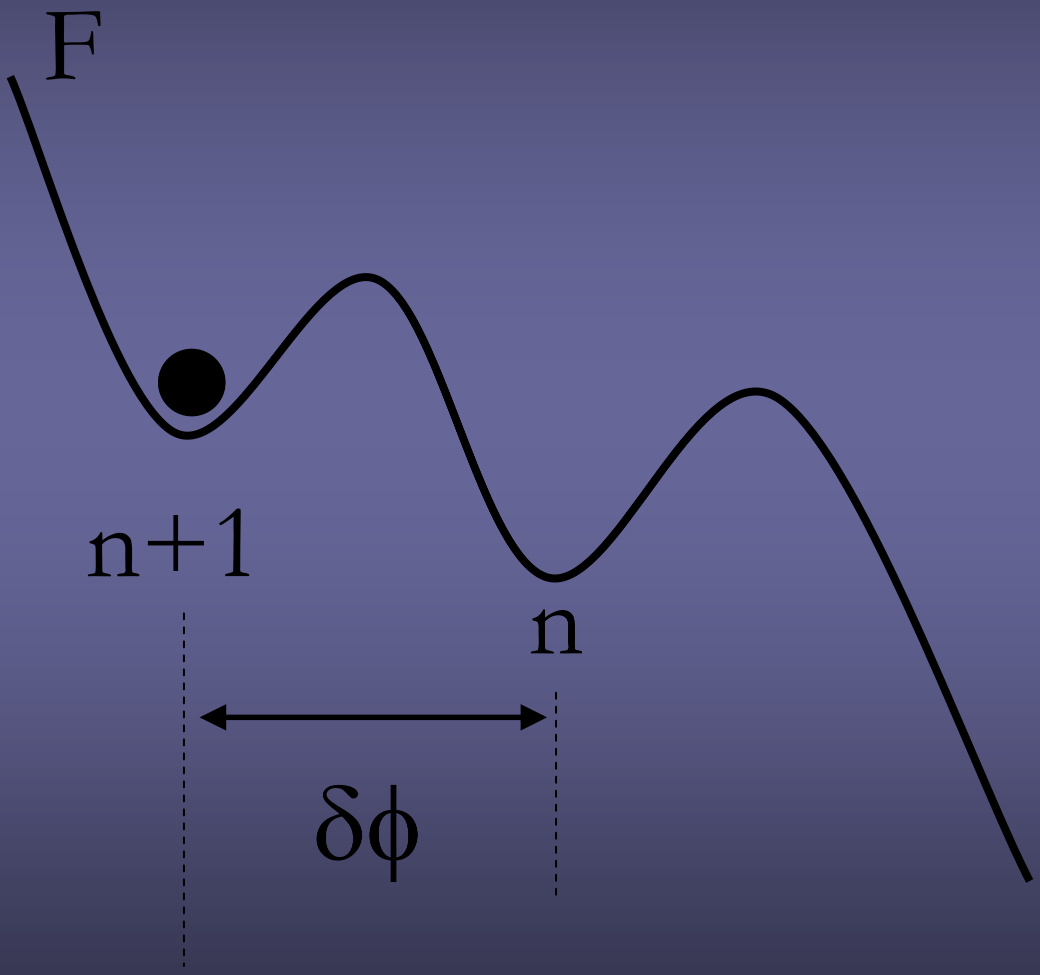

Resistance (Ω)
Temperature (K)
100000
10000
1000
100
0.3
0.5
1.0
1.5
B=0 T
B=0.034 T
B=0.076 T
B=0.12 T
B=0.17 T
B=0.35 T
B=0.465 T
B=0.70 T
B=0.81 T
B=0.93 T
B=1.04 T
B=1.16 T
B=1.28 T
B=1.35 T

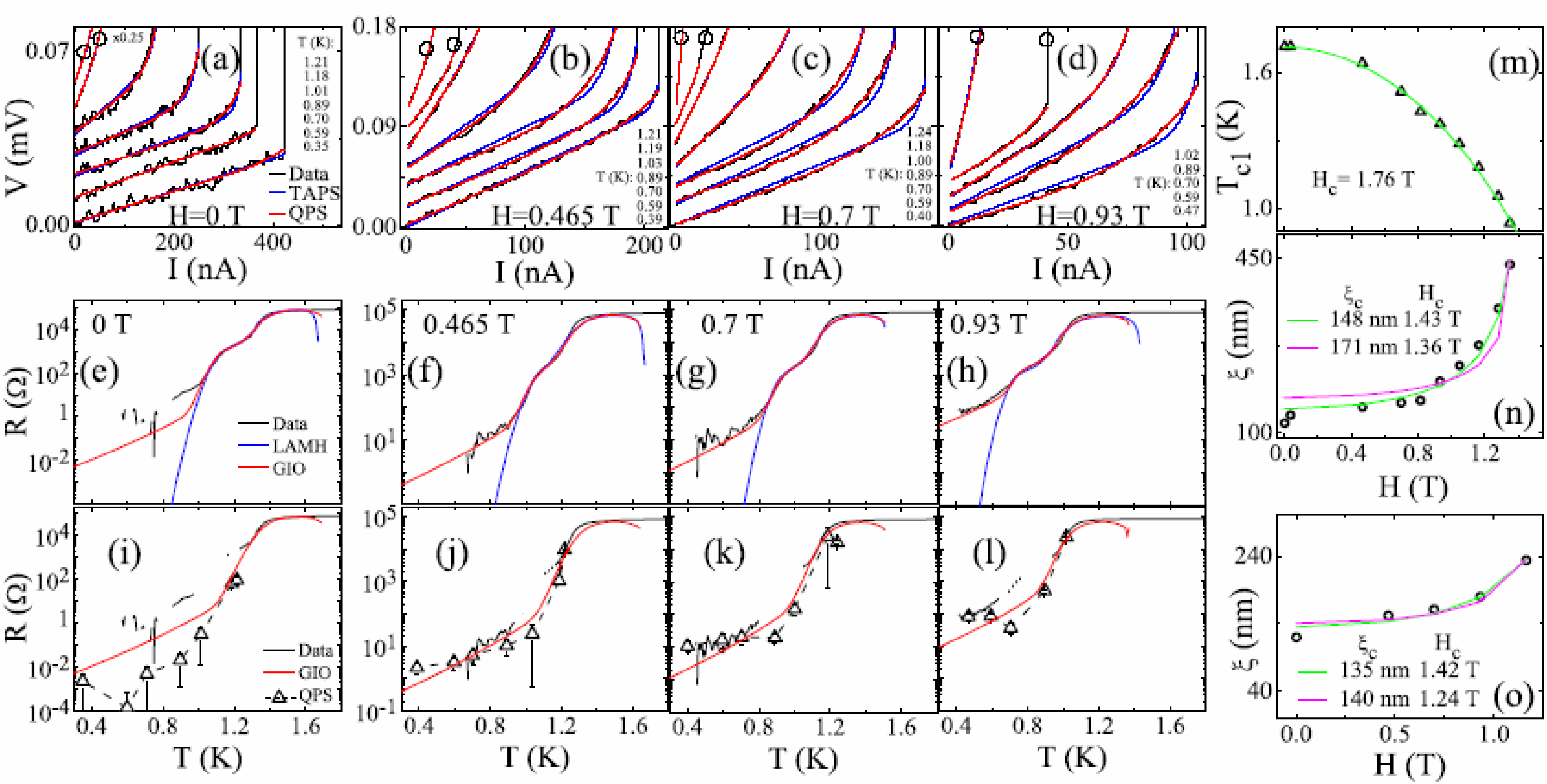
# Model of a superconducting wire

$$\psi = fe^{ikx}$$

$$k = 2n\pi / L$$





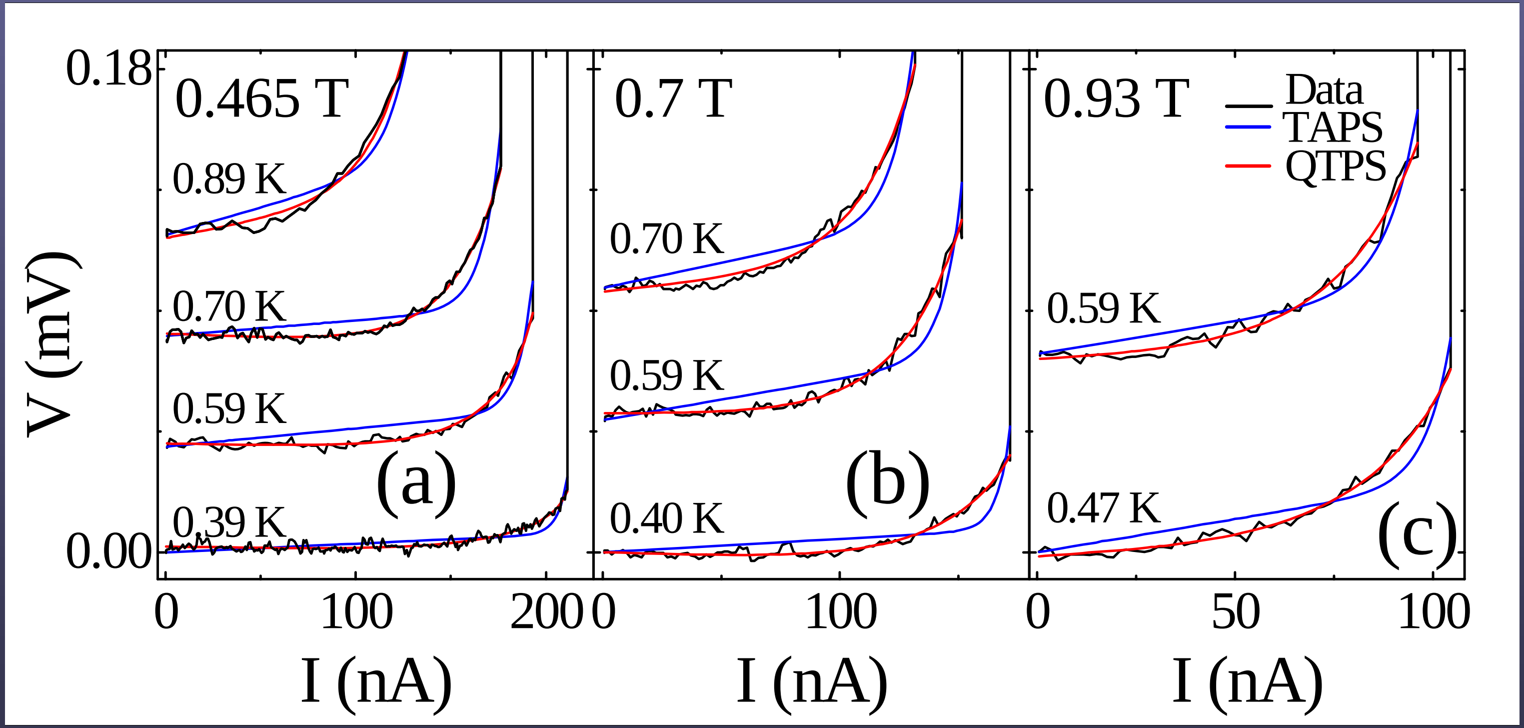

$$R_{LAMH} = R_q \frac{\Omega}{kT} \exp\left(-\frac{\Delta F}{k_B T}\right) \qquad R_{GZ} = B_{GIO} R_q \frac{l}{\xi} \sqrt{\frac{\Delta F}{h/\tau_{GL}}} \exp\left(-a_{GIO} \frac{\Delta F}{h/\tau_{GL}}\right)$$

$$\Omega = \frac{L}{\xi}\left(\frac{\Delta F}{kT}\right)^{\frac{1}{2}} \frac{1}{\tau_{GL}}, \qquad \Delta F = \frac{8\sqrt{2}}{3}\frac{H_c^2}{8\pi} A\xi$$

# Non-linear V-I

$$V_{LAMH} = R_{TAPS} I_0 \sinh\left(I/I_0\right)$$

$$V_{GIO} = R_{QTPS} I_{GIO} \sinh\left(I/I_{GIO}\right)$$




Physical Review Letters **97**, 017001 (2006)

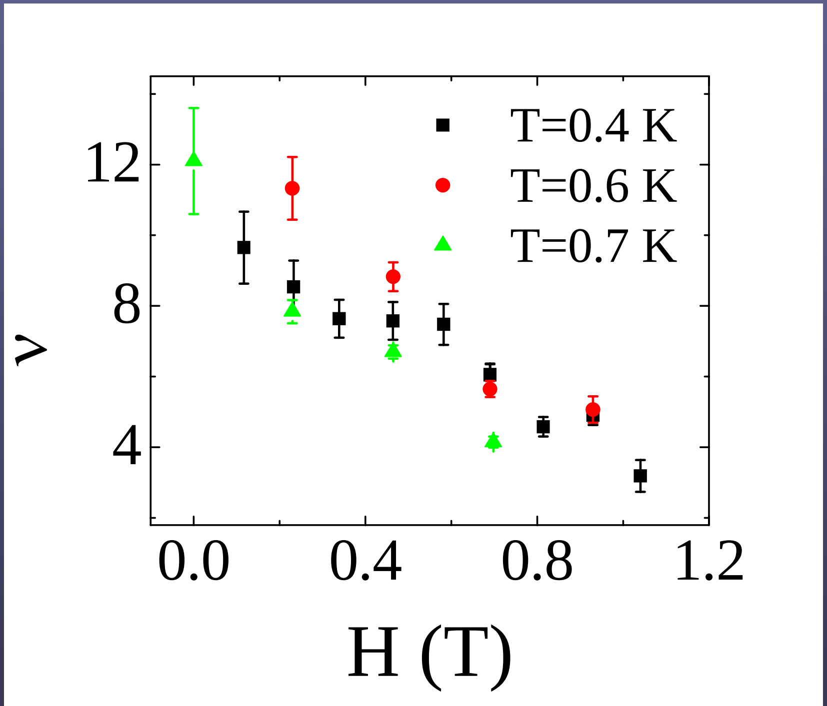
# Power Law dependence

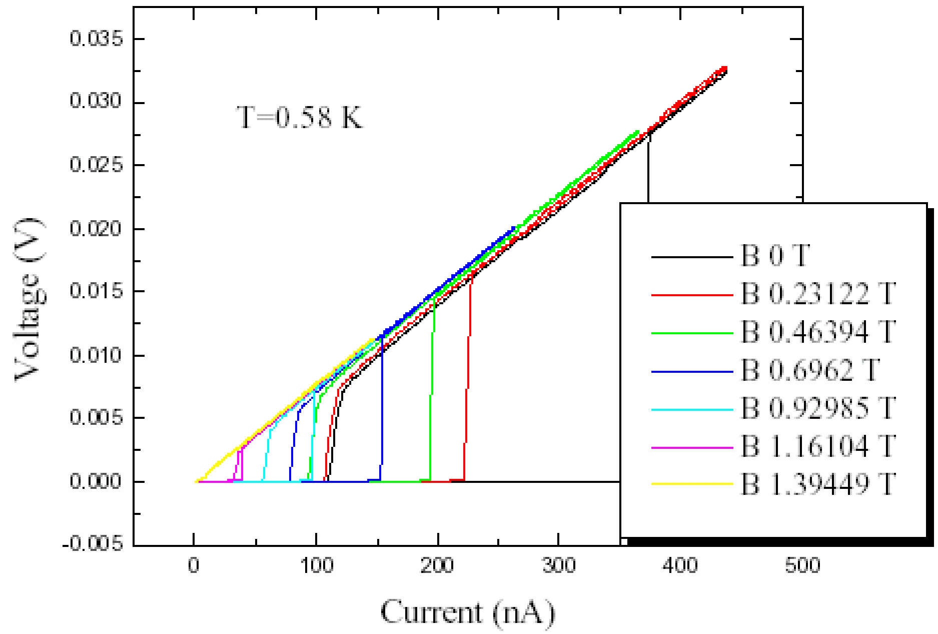
$$\lim_{I \to \infty} V \propto I^{\nu}$$

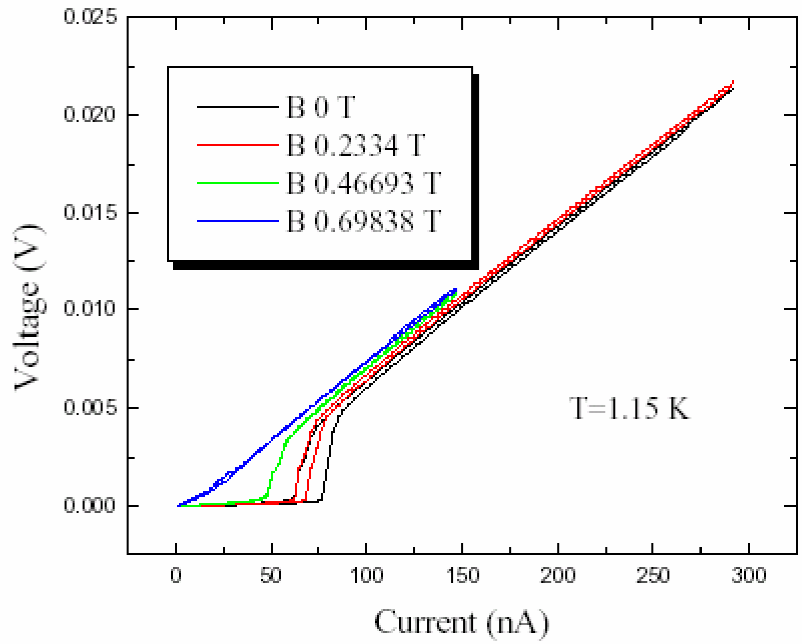
$$V \propto \frac{1}{(\phi_0 \eta)^{2\nu}} \sinh(I\phi_0\eta)$$

$$\eta \propto \frac{1}{T\pi} arctg\left(\frac{(2\nu+1)\pi T}{\phi_0 I}\right)$$

S. Khlebnikov unpublished; S. Khlebnikov and L. P. Pryadko, Phys. Rev. Lett. **95**, 107007 (2005)




T=0.58 K
Voltage (V)
Current (nA)
B 0 T
B 0.23122 T
B 0.46394 T
B 0.6962 T
B 0.92985 T
B 1.16104 T
B 1.39449 T


Voltage (V)
Current (nA)
T=1.15 K
B 0 T
B 0.2334 T
B 0.46693 T
B 0.69838 T