HW#6, Due 9:00am, Nov.4 (Wed).

No late HW will be accepted. So turn in whatever you have done.

Reading assignment: Sakurai 1.6-1.7.

1. The classical Poisson brackets are defined as

$$\{A(q,p), B(q,p)\}_P \equiv \sum_i \left(\frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i} \right)$$

where A, B are functions of general coordinates $q = \{q_1, q_2, ...\}$ and canonical momentum $p = \{p_1, p_2, ...\}$. Prove the Jacobi identity:

$$\{A, \{B, C\}_P\}_P + \{B, \{C, A\}_P\}_P + \{C, \{A, B\}_P\}_P = 0$$

2. With the orbital angular momentum $\vec{L} = \vec{q} \times \vec{p}$ for a point particle, calculate the Poisson brackets among them:

$$\{L_i, L_j\}_P$$

Does the result amaze you?

3. Construct a quantity of pure mass dimensionality from three fundamental constants: $\hbar$, the speed of light c, and the Newton gravitational constant G. This mass is called the Planck mass. How large is it? (Specify what unit you adopt.)

4. A particle moving in one dimension has a wave function in position representation:

$$\psi(x) = A \exp(-x^2/4\triangle^2)$$

where $\triangle$ is a constant.
(a) Determine the normalization factor A
(b) The probability that the particle has linear momentum in the range p to $p + dp$ is $Prob(p) \cdot dp$. What is $Prob(p)$?
(c) Show that the product of the uncertainties in position and momentum has the minimum value allowed by the uncertainty principle.

5. Let $|\psi\rangle$ be the state of a spinless particle in three dimensions, and let $\phi(\vec{p}) = \langle \vec{p}|\psi\rangle$ be its momentum space wave function. Find the momentum space wave function of the state $T(\vec{a})|\psi\rangle$, namely, find the action of the translation operator $T(\vec{a})$ in the momentum representation.