HW#8, Due 9:00am, Dec.9 (Wed).

No late HW will be accepted. So turn in whatever you have done.

1. Apply the Hamilton-Jacobi method to the S.H.O. problem in classical mechanics. The Hamiltonian for the 1D S.H.O. is

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2.$$

Since the Hamiltonian does not explicitly involve the time t, we can separate the action into

$$S(q,t) = W(q) - Et$$

where W is called the Hamilton's characteristic function.
(1) What is the corresponding Hamilton-Jacobi equation in terms of W?
(2) From (1), obtain the analytic solution for W
(3)The change from S to W can be viewed as a Legendre transformation. Therefore, the inverse Legendre transform, $t = \partial W/\partial E$, can be used to find the solution of time t. Work out the expression for t.
(4) From (3), obtain the solutions for q and p.

2. A particle lives in an 1D infinite well, V(x)=0 for $0 < x < L$ and $V(x) = \infty$ elsewhere. Use the exact eigenfunctions to verify that the nodes interleaving does happen in this case.

3. Consider a particle moving in a potential well that has a rigid wall at $x = x_1$, or $V(x) = \infty$ for $x < x_1$, and by a certain monotonically increasing function $V(x)$ for $x > x_1$. The classically allowed region is specified by $x_1 < x < x_2$; x_1 and x_2 are the turning points.
Show that the WKB energy quantization rule is now given by

$$\int_{x_1}^{x_2} dx\sqrt{2m(E - V(x))} = \left(n + \frac{3}{4}\right)\pi\hbar,\ n = 0, 1, 2, 3\cdots$$

4. A particle of mass m is moving in the 1D potential

$$V(x) = \frac{-V_0}{\cosh^2\left(\frac{x}{a}\right)}$$

where V_0 is a constant energy. Apply the WKB approximation to estimate
(1) the bound state energy?
(2) how many bound states are allowed in this potential?