HW#12, Due 9:00am, Jan.6 (Wed).

No late HW will be accepted. So turn in whatever you have done.

1. (25%) We have shown how to quickly obtain the answer for the spin-1/2 magnetic resonance by going to a rotating frame. Now obtain the Rabi's flopping formula for a spin-1/2 particle by directly solving the Schrodinger equation in the fixed frame.

2. (25%) g-factor in classical EM.
   Consider a uniformly charged solid sphere of radius R and mass M. It carries a total charge Q, and is set spinning with angular velocity ω about the z axis. What are the angular momentum and the magnetic dipole moment of the sphere? What is the gyromagnetic ratio g in this case?

3. (25%) Here is another way to derive the precession of a spin-1/2 in a const magnetic field. Consider the density matrix for a particle of spin 1/2 and magnetic moment μ in a magnetic field, where the Hamiltonian is:

$$H = -\frac{1}{2} g\mu\vec{\sigma}\cdot\vec{B}$$

   where g is a constant. We use α and β to denote the spin up and down states respectively,

$$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

   Show that the corresponding density matrix is

$$M = \frac{1}{2}[1 + \hat{p}\cdot\sigma]$$

   Also, derive the equation of motion for the polarization vector $\hat{p} = \langle\vec{\sigma}\rangle$ and see how the precession happens.

4. (25%) Second spherical harmonic.
   (a)Starting from the equation that

$$L_-|2,-2\rangle = 0$$

   to obtain the expression of spherical harmonic $Y_2^{-2}(\theta,\phi)$. Make sure that its normalization is correct.
   (b) Applying the L_+ operator to raise m and obtain the $Y_2^{-1}(\theta,\phi)$ and $Y_2^0(\theta,\phi)$.