Quantum Mechanics-2 HW#1
Due 9:00am, March 10, 2010 (Wed).

No late HW will be accepted. So turn in whatever you have done.

1. A 3D square well with potential

$$V(r) = \begin{cases} -V_0 = -\frac{a^2}{2R^2} & \text{for } r < R \\ 0 & \text{elsewhere} \end{cases}$$

The parameter a controls the depth of the well. If the potential is just deep enough to accommodate a $p-$wave ($l = 1$) bound state. Verify that the reduced wave function is described by

$$\chi_1(\rho) = \begin{cases} C\left(\frac{\sin\rho}{\rho} - \cos\rho\right), & \text{for } r < R \\ D\rho^{-1} & \text{for } R \le r \end{cases}$$

where $\rho = kr$, C and D will be determined by matching the boundary conditions. Assume that $m = \hbar = 1$,
(a)(15%) Find the value of a.
(b)(15%) For this particular a, what is the ground state energy of the $s-$wave ($l = 0$)?

2. Given $\exp(ikz) = \sum_{l=0}^{\infty}(2l+1)i^l j_l(kr)P_l(\cos\theta)$, and $P_l(x) = (2^l l!)^{-1}(d/dx)^l(x^2-1)^l$, with $\int_{-1}^{1} dx P_l^2(x) = 2/(2l+1)$,
(a)(15%) find the constant C in

$$j_l(\rho) = C\int_{-1}^{1} dy e^{i\rho y} P_l(y).$$

(b)(15%) Use integration by parts to determine the function $f(\rho)$ in the following relation,

$$j_l(\rho) = f(\rho)\int_{-1}^{1} e^{i\rho y}(1-y^2)^l dy.$$

3. Given the Runge-Lenz vector operator

$$\hat{M}_0 = \frac{1}{2}\left(\vec{p}\times\vec{L} - \vec{L}\times\vec{p}\right) - Z\frac{\vec{r}}{r},$$

and $\hat{H} = \vec{p}^2/2 - Z/r$, show that
(a) (15%) $\hat{M}_0\cdot\hat{M}_0 = 2\hat{H}(\vec{L}^2+1) + Z^2$,
(b) (25%) and $[\hat{M}_0, H] = 0$.