Quantum Mechanics-2 HW#3
Due 9:00am, April 14, 2010 (Wed).

No late HW will be accepted. So turn in whatever you have done.

1. (25%) (Variation method) To estimate the ground state energy of the hydrogen atom, we choose the spherically symmetrical trial function

$$u_\lambda(r) = \begin{cases} c(1 - r/\lambda) & \text{for } r \le \lambda \\ 0 & \text{for } r > \lambda \end{cases}$$

where c is a normalization constant and λ is the variational parameter.
(a) Calculate the mean value of the energy.
(b) Find the optimal value λ_0 of λ. Compare λ_0 with the Bohr radius a_0.
(c)Compare the approximate value obtained for the ground state energy with the exact value.

2. (25%) (non-degenerate perturbation) A particle of charge q and mass m, which is moving in a 1-D harmonic potential of frequency ω, is subject to a weak electric field E in the $x-$direction.
(a) Find the exact expression for the energy.
(b) Calculate the energy to first nonzero correction and compare it with the exact result obtained in (a)

3. (25%) (perturbation) Consider a system whose Hamiltonian is given by (with $\lambda \ll 1$)

$$H = E_0 \begin{pmatrix} 3 & 2\lambda & 0 & 0 \\ 2\lambda & -3 & 0 & 0 \\ 0 & 0 & -7 & \sqrt{2}\lambda \\ 0 & 0 & \sqrt{2}\lambda & 7 \end{pmatrix}$$

(a) Calculate the exact eigenvalues of H and expand each of these eigenvalues to the second power of λ.
(b) Calculate the energy eigenvalues to second-order perturbation theory and compare them with the exact results obtained in (a).
(c) Calculate the eigenstates of H up to the first-order correction.

4. (25%) (perturbation) A particle of mass m is moving in a 1-D harmonic oscillator potential, $V(x) = m\omega^2 x^2/2$. Calculate
(a) the ground state energy
(b) the first excited state energy
to the first order perturbation theory when a small perturbation $H' = \lambda\delta(x)$ is added to the potential, with $\lambda \ll 1$.