HW#5, Due May.5, 2010 (Wed) by 9:00.

No late HW will be accepted. So turn in whatever you have done.

1. (30%) Prove the Dirac identity

$$[J^2,[J^2,\vec{V}]] = 2\hbar^2(J^2\vec{V} + \vec{V}J^2) - 4\hbar^2(\vec{V}\cdot\vec{J})\vec{J}$$

2. (30%) A two-component operator Q_x(x=1,2) transforms according to the spin-1/2 representation,

$$[J_a, Q_x] = Q_y \frac{[\sigma_a]_{yx}}{2}$$

where σ_a is the Pauli matrix-a. Given

$$A = \left\langle \frac{3}{2}, -\frac{1}{2}, \alpha|Q_1|1, -1, \beta \right\rangle$$

find

$$\left\langle \frac{3}{2}, -\frac{3}{2}, \alpha|Q_2|1, -1, \beta \right\rangle.$$

3. The quadrupole moment operators can be arranged as: $Q^{(\pm 2)} = \sqrt{3/8}(x \pm iy)^2$, $Q^{(\pm 1)} = \mp\sqrt{3/2}(x \pm iy)z$, and $Q^{(0)} = \frac{1}{2}(3z^2 - r^2)$. Using the form of the wave function $\psi_{lm} = R(r)Y_l^m(\theta, \phi)$,
(a) (10%) Evaluate $< \psi_{33}|Q^{(0)}|\psi_{33} >$
(b) (10%) Predict all other $< \psi_{3,m'}|Q^{(k)}|\psi_{3m} >$ by using the Wigner-Eckart theorem in terms of Clebsch-Gordan coefficients.
(c) (20%) Verify them with explicit calculations for $< \psi_{3,1}|Q^{(1)}|\psi_{3,0} >$, $< \psi_{3,-1}|Q^{(-2)}|\psi_{3,1} >$, and $< \psi_{3,-2}|Q^{(0)}|\psi_{3,-3} >$.
You can just leave the $\int_0^\infty r^2 dr R(r)^2 r^2$ as an overall constant and drop it.