QM2 Midterm, April 19, 2010.

You must provide the procedure and reasoning of how you obtain the result.

1. (25%) Using perturbation to estimate the eigenvectors and eigenvalues of the matrix

$$\begin{pmatrix} 4.1 & 0.2 & 0.2 \\ 0.2 & 8.3 & 0.1 \\ 0.2 & 0.1 & 11.9 \end{pmatrix}$$

   Use the leading digit structure as the unperturbed matrix and give the final answer with two-digit accuracy.

2. (25%) A particle of mass m lives in an infinitely deep square well of width a,

$$V(x) = \begin{cases} \infty, & x < 0 \\ 0, & 0 < x < a, \\ \infty, & x > a \end{cases}$$

   (1) Write down all the normalized energy eigenfuctions and their energy eigenvalues. Then the bottom of the potential is deformed by a small perturbation,

$$\triangle V = v \cos \frac{\pi x}{a}$$

   (2) The action of $\triangle V$ on the ground state $\psi_1(x)$ produces a new state which is proportional to one of $\psi_n(x)$. Work out the details.
   (3) What is the change in the ground state energy in lowest non-zero order perturbation theory?

3. (25%) Find the $l = 0$ energy and wave function, $\psi(r, \theta, \phi)$, of a particle of mass m which is subject to a central potential

$$V(r) = \begin{cases} 0, & a < r < b \\ \infty, & \text{elsewhere} \end{cases}$$

4. (25%) Given the hyperfine splitting in the hydrogen atom produces 21 cm radiation, find the corresponding wavelength for the deuterium atom with the nucleus spin-1, $g_p = 2(2.79)$, and $g_n = 2(-1.91)$.

5. (25%)Consider a spinless 'electron' moving in the EM field of two spinless 'protons', which are fixed at location $z = \pm a$ on the z-axis. The Hamiltonian is

$$H = \frac{p^2}{2} - \frac{1}{\sqrt{x^2 + y^2 + (z-a)^2}} - \frac{1}{\sqrt{x^2 + y^2 + (z+a)^2}}, \quad (\hbar = m = 1)$$

and we ignore the fine structure effect and so on.
(1) List all the obvious symmetries of this Hamiltonian, i.e., list all the operators (either symmetry operators or generators of symmetry operators) which commute with H.
(2) Discuss the orders of the degeneracy due to these symmetries.