# Quantum Physics Final Exam, Jan. 10, 2012.

## 1 Single Choice (4% each)

1. A student looks at a dim light of intensity about $10^{-10} W/m^2$. Determine approximately how many photons per second enter the pupil of his eye. Assume that the wavelength is 580nm and the pupil diameter is 4.0mm. (a) 4 (b) 37 (c) 370 (d) 3700 (e) 37000

2. A Helium ion has a hydrogen-like spectrum, except the nucleus charge has to be taken as 2 rather than 1 as is the case for hydrogen. It makes a transition from the $n = 3$ state to $n = 2$ state. What is the wavelength of the radiation emitted in the transition? (a) 164 nm (b) 291 nm (c) 549 nm (d) 660 nm (e) 690 nm

3. Of the following sets of quantum numbers for an electron in a hydrogen atom, which is possible? (a) $n = 5, l = 3, m_l = -3$ (b) $n = 3, l = 3, m_l = -2$ (c) $n = 5, l = -3, m_l = 2$ (d) $n = 3, l = 2, m_l = -3$ (e) $n = 4, l = 5, m_l = -2$

4. A magnetic dipole $\vec{\mu}$ is placed in a strong uniform magnetic field $\vec{B}$. The associated force exerted on the dipole is: (a) along $\vec{\mu}$ (b) along $-\vec{\mu}$ (c) along $\vec{B}$ (d) along $\vec{\mu} \times \vec{B}$ (e) zero

5. Assume the interaction between proton and antiproton is purely electromagnetic. What is the ground state energy of the proton-antiproton bond state? (a) $-136$ eV (b) $-272$ eV (c) $-13.6$ keV (d) $-27.2$ keV (e) $-6.8$ keV

6. A particle is confined to a 1-D trap by infinite potential energy walls. Of the following states, designed by the quantum number $n$, which one has the greatest probability density near the center of the well? (a) $n = 2$ (b) $n = 3$ (c) $n = 4$ (d) $n = 5$ (e) $n = 6$

7. Which one of the following operator pairs has nonzero chance to be measured simultaneously? (a) $(\hat{x}, \hat{p}_x)$ (b) $(\hat{L}_x, \hat{L}^2)$ (c) $(\hat{L}_x, \hat{L}_y)$ (d) $(\hat{L}^2, \hat{x} + i\hat{p}_y)$ (e) $(\hat{x}^2, \hat{x} - i\hat{p}_y)$

8. In a 2-dimensional Hilbert space, which of the following is a possible matrix representation for a projection operator? (a)$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ (b)$\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$ (c)$\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ (d)$\frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ (e)$\frac{1}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

9. A particle with mass $m$ lives in a box of dimension $L \times 2L \times 2L$. What is the needed photon energy to excite the first excited state to the second excited state (in the units of $\hbar^2\pi^2/2mL^2$)? (a) 1/4 (b) 1/2 (c) 3/4 (d) 5/4 (e)3/2

## 2 Short Questions (5% each)

**Explain your answer,** *in Chinese please*, **with one or two sentences.**

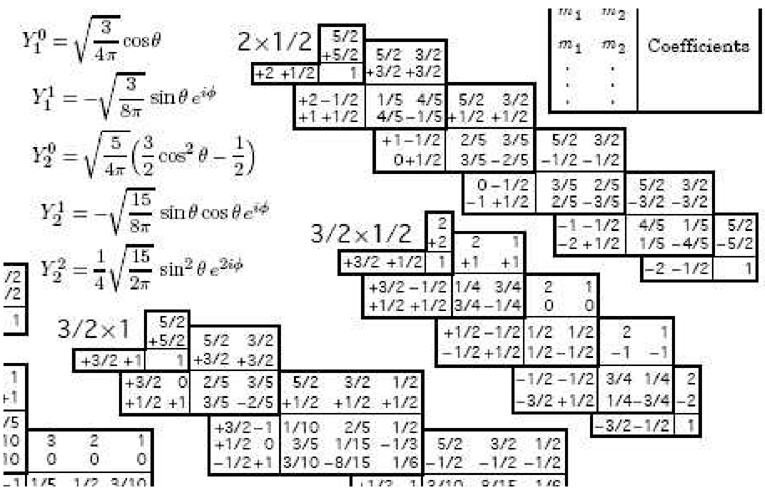
1.

$$-\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]Y_7^6(\theta,\phi)=?$$

2. Define $r=\sqrt{x^2+y^2+z^2}$,

$$\left[(y-ix)\frac{\partial}{\partial z}-z\frac{\partial}{\partial y}+iz\frac{\partial}{\partial x}\right]\left[1-3r^2+r^4\right]\exp[-r^5+r^3/2]=?$$

3. A system is consist of two subsystems with angular momenta 2 and 1/2. Use the following table to write down the decomposition of total angular momentum state $|3/2,-1/2\rangle$.




4. An operator $\hat{Q}\equiv a\hat{x}+i\,b\,\hat{p}$, where $a,b$ are real numbers. Is $\hat{Q}$ a physical observable? Why?

5. What are Fermion and Boson? Name one example for each type.

# 3 Long Questions

**Justify your answers with necessary details or no credit point will be given .**

1. (20%) Consider a system of two non-identical fermions, each with spin 1/2. One is in a state with $S_{1x} = \hbar/2$ while the other is in a state with $S_{2y} = -\hbar/2$. What is the probability of finding the system in a state with total spin quantum numbers $s = 1, m_s = 0$, where $m_s$ refers to the $z$-component of the total spin?

2. (15%) A neutron with mass $m$ is placed on top of a perfect mirror lying on the $xy$ plane at $z = 0$. The system is in the gravitational potential $V(z) = gmz$. Use the uncertainty principle to estimate the minimal distance between the neutron and the mirror.

3. (15%) Consider a particle of mass $m$ interacting with an attractive central potential

$$V(r) = -\frac{\lambda}{r^{2(1-\eta)}}$$

where $0 < \eta < 1$. With the help of dimensional analysis determine the dependence of the discrete energy eigenvalues on the parameters of the system. What is the characteristic size of the discrete eigenfunctions?

4. (20%) Find the $l = 0$ energy and wave function of a particle of mass $m$ that is subject to the following central potential

$$V(r) = \begin{cases} 0, & a < r < b \\ \infty, & \text{elsewhere}\,. \end{cases}$$

Recall that in spherical coordinate the Laplacian is

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \phi^2}$$