Quantum Physics Midterm, Oct. 21, 2011.

You must provide the details or reasonings to justify your answers.

1. (10+10+10%) Consider the following hypothetic wave function for a particle confined in the region $-4 \le x \le 4$:

$$\psi(x) = \begin{cases} A(4+x), & -4 \le x \le 2, \\ B(x^2 - C), & 2 \le x \le 4 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine A, B, C, and sketch the wave function. (b) Calculate σ_x, (c) and σ_p.

2. (10+10%) In 1-D simple harmonic oscillator, ψ_n denotes the eigenfunction with energy $(n + 1/2)\hbar\omega$. Use the raising ($a^\dagger$) and lowering operator (a) to show that only when $m = n \pm 1$,

$$\int_{-\infty}^{\infty} dx\ \psi_m^*\ \hat{x}\ \psi_n \neq 0$$

What's the condition to obtain a nonzero integral

$$\int_{-\infty}^{\infty} dx\ \psi_m^*\ \hat{p}^2\ \psi_n ?$$

3. (10+10+10%) For a given initial wave function

$$\psi(x, t = 0) = e^{ik_1 x} + c\, e^{-ik_2 x},$$

where k_1, k_2, c are positive real numbers, calculate the corresponding (a) **probability density**, and (b) **current** as functions of x and t, (c)Do your best discussing the physical meaning(consider especially when $k_1 \sim k_2$, you can set $k_2 = k_1 + \epsilon$).

4. (20%) A particle of mass m is in the potential

$$V(x) = \begin{cases} \infty & (x < 0), \\ -32\hbar^2/mL^2 & (0 \le x \le L), \\ 0 & (x > L). \end{cases}$$

How many bound states are there?

5. (10+10+10%) Consider the potential step ($V_0 > 0$)

$$V(x) = \begin{cases} V_0 & (-L \le x \le L), \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Is there any bound state, why?
(b) Calculate the transmission coefficient for a free particle with energy $E > V_0$.
(c) Recycle the derivation in previous problem (you only need to change a few things), and obtain the transmission coefficient for $E < V_0$.