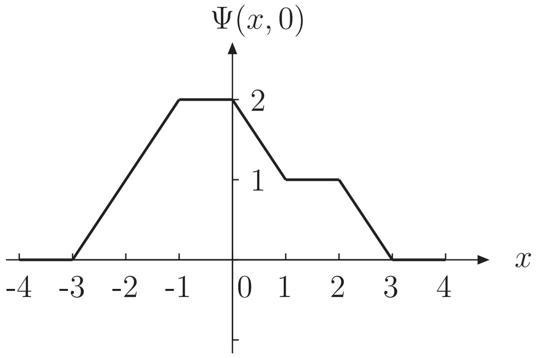
Quantum Physics Midterm, Dec. 2, 2011.

The details or reasonings must be provided to justify your answers.

1. (10+10%) (a) For a 1-dimensional harmonic oscillator, give the matrix representations of $a^\dagger$ and a in the basis in which the Hamiltonian is diagonal.
   (b) Using the matrices found in (a), calculate the commutator $[a, a^\dagger]$.

2. (10+10+10%) At $t = 0$, the wave function $\Psi(x, 0)$ for a free particle with mass m is prepared as shown.



   If one performs the $\hat{x}$ measurement at $t = 0$ and it is found the particle is in the range of $-1 \leq x \leq 0$.
   (a) Sketch the wave function, $\psi(x, 0)$, right after the measurement.
   (b) Calculate the σ_x right after the measurement.
   (c) By Fourier decomposition, obtain the analytical expression for $\psi(x, t)$. (Do your best to simplify the result, but leave it in the integral form.)

3. (10+10+10+10%) The Hamiltonian for a certain three-level system is represented by the matrix

$$\hat{H} = E_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

   Two other observables, $\hat{A}$ and $\hat{B}$, are represented by the matrices

$$\hat{A} = a \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}, \quad \hat{B} = b \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

   (a) Find the eigenvalues and the normalized eigenvectors of $\hat{H}$, $\hat{A}$ and $\hat{B}$.
   (b) Suppose the system is described by the state at $t = 0$

$$|\psi(0)\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

with $|c_1|^2 + |c_2|^2 + |c_3|^2 = 1$. Find (at $t = 0$) $\langle \hat{H} \rangle$,$\langle \hat{A} \rangle$, and $\langle \hat{B} \rangle$.
(c) What is $|\psi(t)\rangle$? What is the probability that you measure $\hat{H} = 2E_0$ first and then $\hat{A} = -a$?
(d) What are $d\langle \hat{A} \rangle/dt$ and $d\langle \hat{B} \rangle/dt$?.

4. (10+10%) A particle of mass m in the gravitational potential near the surface of Earth can be described by the Hamiltonian $\hat{H} = \frac{\hat{P_Z}^2}{2m} - mg\hat{Z}$ where $g = 9.8m/sec^2$.
(a) Calculate $d\langle \hat{Z} \rangle/dt$, $d\langle \hat{P}_Z \rangle/dt$, and $d\langle \hat{H} \rangle/dt$
(b) Obtain $d^2\langle \hat{Z} \rangle/dt^2$ and solve the equation to get $\langle \hat{Z} \rangle(t)$ such that $\langle \hat{Z} \rangle(0) = h$ and $\langle \hat{P}_Z \rangle(0) = 0$.

5. (10+10%) A 2-state system posses the normalized energy eigenstates $|1\rangle$ and $|2\rangle$ such that $\hat{H}|1\rangle = E_1|1\rangle$ and $\hat{H}|2\rangle = E_2|2\rangle$. There is another observable $\hat{Q}$ with the known "*experimental*" results that: (1) $\langle 1|\hat{Q}|1\rangle = 1/2$, (2) $\langle 1|\hat{Q}^2|1\rangle = 1/4$, and (3) $\langle 2|\hat{Q}^3|2\rangle = 1$.
(a) Find the eigenvalues and the eigenvectors of $\hat{Q}$.
(b) What kind of state $|\psi\rangle$ can make $\sigma_H\sigma_Q = 0$?