Quantum Physics Homework 4(partial) due Nov. 29, 2011.

1. Consider the following hypothetic wave function for a particle confined in the region $-2 \leq x \leq 1$:
$$\psi(x) = \begin{cases} A(2+x), & -2 \leq x \leq 0, \\ B(x^2 - C), & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$
(a) Determine A, B, C, and sketch the wave function. (b) Calculate σ_x, (c) and σ_p.

2. Consider a charged oscillator, of positive charge q and mass m, which is subject to an oscillating electric field $E_0 \cos \omega t$; the particle's Hamiltonian is $\hat{H} = \hat{p}^2/2m + k\hat{x}^2/2 + qE_0\hat{x}\cos\omega t$.
(a) Calculate $d\langle\hat{x}\rangle/dt$, $d\langle\hat{p}\rangle/dt$, and $d\langle\hat{H}\rangle/dt$.
(b) Solve the equation for $d\langle\hat{x}\rangle/dt$ and obtain $\langle\hat{x}\rangle(t)$ such that $\langle\hat{x}\rangle(t=0) = 0$.

3. A particle of mass m, which moves freely inside an infinite potential well of length L, is initially in the state $\psi(x,0) = \sqrt{3/5L}\sin(3\pi x/L) + (1/\sqrt{5L})\sin(5\pi x/L)$.
(a) Find $\psi(x,t)$ at any later time t.
(b) Calculate the probability density $\rho(x,t)$ and the current density $\vec{J}(x,t)$.
(c) Verify that the probability is conserved, namely, $\partial\rho(x,t)/\partial t + \vec{\nabla} \cdot \vec{J}(x,t) = 0$.