what we have learnt last class: (SHO as an example)

① WF is associated with a linear vector space.

$$1 = \int \underbrace{\psi^*}_{\langle\psi|} \underbrace{\psi}_{|\psi\rangle} \, dx \rightsquigarrow \sum_i c_i^* c_i = 1 \qquad |\psi\rangle \sim \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \end{pmatrix}$$

Dirac bracket

inner product map to complex #

$\langle\psi| \sim (c_0^*, c_1^*, c_2^*, \dots)$

$\langle a|b\rangle = (\langle b|a\rangle)^*$

②. Observables are hermitian.

$$\hat{x} \rightsquigarrow \begin{pmatrix} & \end{pmatrix} \qquad M_x = (M_x)^\dagger$$

$$\hat{p} \rightsquigarrow \begin{pmatrix} & \end{pmatrix} \qquad M_p = (M_p)^\dagger$$

$$\hat{x}\hat{p} \rightsquigarrow M_{\hat{x}\hat{p}} = M_x \cdot M_p \qquad (M_{xp})^\dagger = M_p^\dagger M_x^\dagger$$

$$\hat{p}\hat{x} \rightsquigarrow M_{\hat{p}\hat{x}} = M_p \cdot M_x$$

Einstein convention

$$A \Rightarrow A_{ij}$$

$$A^\dagger = (A^T)^* \quad (A^\dagger)_{ij} = A_{ji}^*$$

$$(M_{xp})_{ij} = (M_x)_{ik} (M_p)_{kj}$$

$$(M_{xp}^\dagger)_{ij} = ((M_{xp})_{ji})^* = M_{x\,jk}^* M_{p\,ki}^* = (M_p^\dagger)_{ik} (M_x^\dagger)_{kj}$$

$$\Rightarrow M_{xp}^\dagger = M_p^\dagger \cdot M_x^\dagger = M_p \cdot M_x = M_{px} \neq M_{xp}$$

· In W.F. fun

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$$\int \psi_1^* \hat{x}\hat{p}\, \psi_2\, dx = \int \psi_1^* x \left(-i\hbar \frac{d}{dx}\right) \psi_2\, dx$$

$$= \int -i\hbar \left[\frac{d}{dx}(\psi_1^* x \psi_2) - \frac{d}{dx}(\psi_1^* x)\psi_2\right] dx$$

$$= +i\hbar \left[\int \left(\frac{d}{dx}\psi_1^*\right) x \psi_2 + \int \psi_1^* \psi_2\right] dx$$

$$= \int \left(\hat{x}\left(\frac{-i\hbar\, d}{dx}\right)\psi_1\right)^* \psi_2 + i\hbar \int \psi_1^* \psi_2\, dx$$

$$\left(x\left(-i\hbar \frac{d}{dx}\right)\right)^\dagger = \left(-i\hbar \frac{d}{dx}\right) x \neq x\left(-i\hbar \frac{d}{dx}\right)$$

is not Hermitian

$$\int \psi_1^* x \psi_2 = \int (x\psi_1)^* \psi_2$$

$$\int \psi_1^* \left(-i\hbar \frac{d}{dx}\right)\psi_2 = \int -i\hbar \frac{d}{dx}(\psi_1^* \psi_2) + \left(i\hbar \frac{d}{dx}\psi_1^*\right)\psi_2$$

$$= \int \left(-i\hbar \frac{d}{dx}\psi_1\right)^* \psi_2$$

$$\langle a|b\rangle = \sum_i a_i^* b_i \qquad Q \to Q_{ij}$$

$$\langle a|Q|b\rangle = \sum_i a_i^* Q_{ik} b_k \qquad (Q|b\rangle)_i = Q_{ik} b_k \qquad \Rightarrow (Q_{ki}^* a_i)^* a_k$$

2a

A physical measurement $\int \psi^* \hat{Q} \psi$ should be real.

$\Rightarrow \langle \psi | \hat{Q} | \psi \rangle$

is real

$$c_i^* Q_{ij} c_j = c_i Q_{ij}^* c_j^*$$

$$= c_j^* (Q^{T*})_{ji} c_i$$

$$= c_j^* (Q^+)_{ji} c_i$$

$$= c_i^* (Q^+)_{ij} c_j$$

$$\Rightarrow \boxed{Q = Q^+}$$

Secondly,

$$\langle a | Q | b \rangle = \langle a | Q b \rangle$$

$$= \langle Q^+ a | b \rangle$$

$$a_i^* Q_{ij} b_j$$

$$= a_i^* (Q_{ij}^*)^* b_j$$

$$= a_i^* (Q^+)_{ji} b_j = (Q^+ a)_j^* b_j$$

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APCTP 2009, 2009

① Linear vector space.
② Linear-independence & dimensionality
③ Unitary space, scalar product, Hilbert space.
④ Orthonormal basis, Schmidt process
⑤. Schwartz's inequality
⑥. Linear operator. $A \Rightarrow A^{+}$
⑦. dual & Adjoint operator.
basis
$|u\rangle \to |k\rangle$ Hermitian conjugate
⑧ Hermitian operator $A = A^{+}$
⑨. The eigenvalue problem, spectrum of A.
if $A = A^{+}$, → real eigenvalues.
⑩. projection operator & completeness.
⑪. unitary operator. diagonalization of Hermitian operator.

~ 2010.03.16
APCTP 2010
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