

Quantum Physics Midterm 1, Oct. 26, 2012.

You must provide the details or reasonings to justify your answers. It is your responsibility to clearly state the logic of your answers. I will not make any attempt to “guess” your results. If I cannot follow what you write, I cannot give you the credit.

1. (10%) Is it always true that $\hat{H}\psi = E\psi$?
2. (10+10+10+10%) Consider the following hypothetic wave function for a particle of mass m confined in the region $-a \leq x \leq a$:

$$\psi(x) = \begin{cases} N(a-x), & 0 < x < a, \\ 0, & a \leq x \\ \psi(-x), & x \leq 0. \end{cases}$$

- (a) By dimensional analysis, determine the relation between N and a .
 - (b) By dimensional analysis, use a , m , and \hbar to construct a quantity which has the dimension of momentum.
 - (c) Determine N , and check with what you got in (a).
 - (d) Calculate σ_p , and compare it with your answer to problem(b). (You might need to be careful about the delta functions at several places.)
3. (10+10+10%) In an 1-D simple harmonic oscillator, ψ_n denotes the eigenfunction with energy $(n + 1/2)\hbar\omega$. The raising operator (a^\dagger) and lowering operator (a) are defined as

$$a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}} \left(-\hbar \frac{d}{dx} + m\omega x \right), \quad a = \frac{1}{\sqrt{2\hbar m\omega}} \left(\hbar \frac{d}{dx} + m\omega x \right)$$

Moreover, $[a, a] = [a^\dagger, a^\dagger] = 0$, and $[a, a^\dagger] = 1$.

- (a) Express \hat{x}, \hat{p} in terms of a and a^\dagger to calculate $\hat{x}\psi_n$, and $\hat{p}\psi_n$. Express your result in terms of stationary states.
 - (b) Define an operator $\hat{Q} \equiv \hat{x}\hat{p} - \hat{p}\hat{x}$. Use what you got in problem(a) to obtain $\hat{Q}\psi_n = ?$
 - (c) What if Q acts on a general wave function? Are you surprised by the result? Explain why.
4. (10+10+10+10%) In a potential free 1D system, an initial wave function for a particle of mass m is given by

$$\psi(x, t = 0) = e^{ikx} + c e^{-ikx},$$

where k is positive real, and c is a complex number.

- (a) Write down the wave function for $t \neq 0$.
- (b) Calculate the corresponding **probability density**, and (c) **current** as functions of x and t .
- (d) Do your best discussing the physical meaning of what you got in (b) and (c).