Quantum Physics Midterm 2, Nov. 30, 2012.

Single Choice (7 % each)

Just collect and summarize your answers(only) inside the boxes 1-10 on the cover page

1. For a given state $|\psi\rangle = \frac{1}{2}|1\rangle + \frac{i}{3}|2\rangle - \frac{1}{\sqrt{3}}|3\rangle$, where $|1\rangle, |2\rangle, |3\rangle$ are three orthonormal basis. What's the probability of finding $|\psi\rangle$ in $|2\rangle$? (a)4/25, (b) 2/5, (c)1/3, (d) -1/9, (e)1/9.

2. Continue from previous question, if there is another state $|\phi\rangle = \frac{1}{4}|1\rangle - \frac{2}{\sqrt{3}}|2\rangle - \frac{i}{2}|3\rangle$, which of the below is incorrect? (a) $\langle\phi|\phi\rangle = 79/48$, (b) $\langle\psi|\phi\rangle = 1/8 + 7i/6\sqrt{3}$, (c) $\langle\psi|\psi\rangle = 25/36$, (d) $\langle 2|\psi\rangle\langle\phi|2\rangle = 2i/3\sqrt{3}$, (e) $\langle 2|\phi\rangle\langle\phi|3\rangle = -i/\sqrt{3}$.

3. In a 3-state system, there are three operators in some basis:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Which of the following is correct (a) A, B can be measured simultaneously. (b) There is one common eigenstate of B, C and it has the same eigenvalue under B and C. (c) There are two simultaneous eigenstates of B, C and both of them have the same eigenvalue under B and C. (d)A, B, C can share the same basis. (e) A, C commute .

4. In a 3-dim Hilbert space, the matrix representation for measurement $\hat{Q} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & i \\ 0 & -i & 0 \end{pmatrix}$ in the orthonormal basis $\{|1\rangle, |2\rangle, |3\rangle\}$. Which one of the following corresponds to the projection operator for eigenvalue = 0? (a)$\frac{1}{2}\begin{pmatrix} 1 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 1 \end{pmatrix}$ (b)$\begin{pmatrix} 1 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 1 \end{pmatrix}$ (c)$\frac{1}{2}\begin{pmatrix} 1 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & -1 \end{pmatrix}$ (d)$\begin{pmatrix} 1 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & -1 \end{pmatrix}$ (e)$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

5. In previous question, if at $t = 0$, we perform the Q measurement on a prepared state $|\psi\rangle = \frac{-i}{2}|1\rangle + |2\rangle - |3\rangle$. What is the normalized wave function right after we obtain 0 as the experimental result? (a) $\begin{pmatrix} i \\ 0 \\ 1 \end{pmatrix}$ (b) $\frac{1}{4}\begin{pmatrix} i \\ 0 \\ -1 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix} 0 \\ i \\ -1 \end{pmatrix}$ (e) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

6. A wave function $\psi = c_1\psi_1 + c_2\psi_2$ is a linear combination of two energy eigenstates ψ_1, ψ_2 with two complex coefficients c_1, c_2. Which of the following is the incorrect statement: (a) ψ is an energy determine state for $c_1 = 0, c_2 = 3$. (b) $\sigma_E = 0$ for $c_1 = 1, c_2 = 0$. (c) ψ is not a stationary state for $c_1 = c_2 = 1/\sqrt{2}$. (d) $\sigma_E \neq 0$ for $c_1 = -c_2 = 1/\sqrt{2}$. (e) To normalize ψ, $c_1^2 + c_2^2 = 1$.

7. $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is the second Pauli matrix. What is $e^{i\sigma_2\theta}$ =? (a) $\frac{1}{\sqrt{2}}\begin{pmatrix} \cos\frac{\theta}{2} & 0 \\ 0 & \sin\frac{\theta}{2} \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (c)$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos\theta & -i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$ (e) $\begin{pmatrix} \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$

8. Here, we exercise the general uncertainty principle on 1D simple harmonic oscillator. What is the lower bound of $\sigma_{x^2}\sigma_p$ for the n-th energy state $|n\rangle$ with energy $(n+1/2)\hbar\omega$? (a)$2\hbar$, (b) $n\hbar$, (c) 0, (d) $n\sqrt{\hbar^3/m\omega^2}$, (e) $n\sqrt{\hbar^3/m\omega^5}$.

9. Which of the following is a possible physical observable? (a) $\begin{pmatrix} 1 & i/3 \\ i/3 & 2 \end{pmatrix}$ for a 2- state system, (b) $-i\hat{x}\hat{p}$, (c) $\hat{x}\hat{p}$, (d) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ for a 2- state system, (e) $(\hat{x}\hat{p}+\hat{p}\hat{x})$.

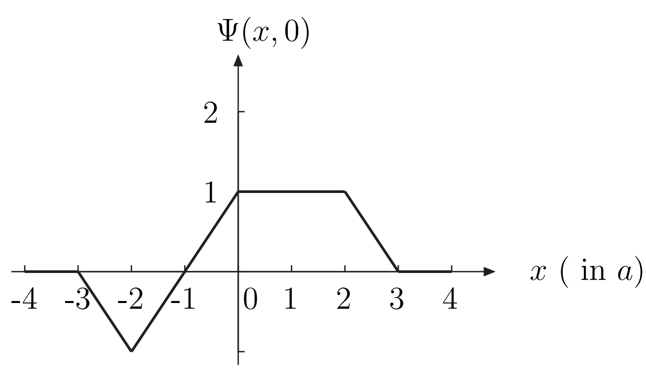
10. In the momentum space, $\phi_p(x)$ denotes the corresponding wave function of a point particle in 1D. Which is not true: (a) $\hat{p}\phi_p(x) = p\phi_p(x)$, (b) $[\hat{x},\hat{p}] = -i\hbar$, (c) $\hat{x} = i\hbar\frac{\partial}{\partial p}$, (d) $\hat{H} = p^2/2m + V(\frac{\hbar}{-i}\frac{\partial}{\partial p})$, (e) $\phi_p(x)$ is the Fourier transform of the position space wave function.

Long Questions

You must provide the details or reasonings to justify your answers. It is your responsibility to clearly state the logic of your answers. I will not make any attempt to "guess" your results. If I cannot follow what you write, I cannot give you the credit.

11. (10%+10%) A 2-state system has orthonormal basis $|1\rangle$ and $|2\rangle$, such that $\langle 1|\hat{H}|1\rangle = \epsilon_0$, $\langle 1|\hat{H}|2\rangle = i\epsilon_0/2$, and $\langle 2|\hat{H}|2\rangle = 5\epsilon_0$, where ϵ_0 is some energy unit. An observable $\hat{Q}$ admits the known "experimental" results that $\langle 2|\hat{Q}|2\rangle = 1/3$, $\langle 2|\hat{Q}^2|2\rangle = 1/9$, and $\langle 1|\hat{Q}^3|1\rangle = -1/8$.
(a) What is the matrix representation of $\hat{Q}$ in the basis of $\{|1\rangle, |2\rangle\}$?
(b) At $t=0$, a state is given as $|\psi\rangle = \frac{1}{\sqrt{5}}|1\rangle + \frac{2}{\sqrt{5}}|2\rangle$. What is $\frac{d}{dt}\langle\psi|\hat{Q}|\psi\rangle$?

12. (10+10+10%) At $t=0$, the unnormalized wave function $\Psi(x,0)$ for an 1D free particle with mass m is prepared as shown, where x is in some length unit a.



If one performs the $\hat{x}$ measurement at $t=0$ and the particle is found to be in the range of $0 \le x \le 2a$.
(a)Sketch the normalized wave function, $\psi(x,0)$, right after the measurement and state clearly what is your unit for the y-axis.
(b) Calculate the σ_x at $t=0^+$ right after the measurement.
(c) Give a rough estimation (with explanation) at what time the new wave function will double its width.