Theory

General phenomenological approach to second order phase transitions

The order parameter field and spontaneous symmetry breaking

A second order phase transition is generally well described phenomenologically if one identifies:

a. The order parameter field $\phi_i(x)$

b. Symmetry group G and its spontaneous breaking pattern.

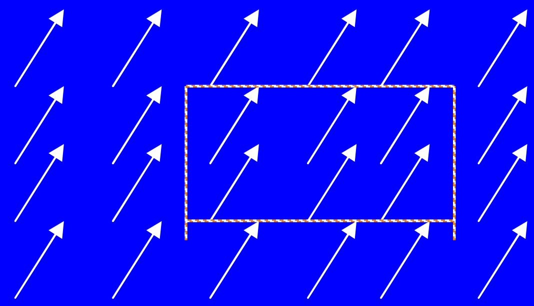
L.D. Landau (1937)

Rest of degrees of freedom are “irrelevant” sufficiently close to the critical temperature Tc. Later, the “relevant” part, namely the symmetry breaking pattern and dimensionality was termed “the universality class”.

An example: XY- (anti) ferromagnet

T=0

well ordered

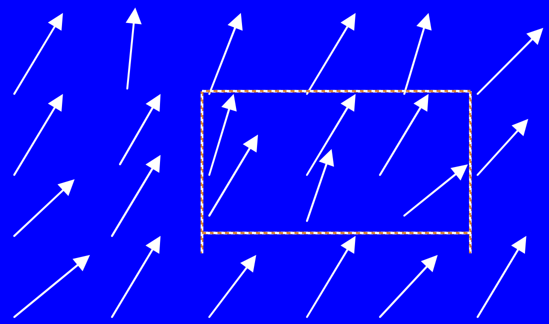


$$\left|\vec{M}\right| \equiv \left|\left\langle \vec{S} \right\rangle\right| > 0$$

large

0<T<Tc

ordered

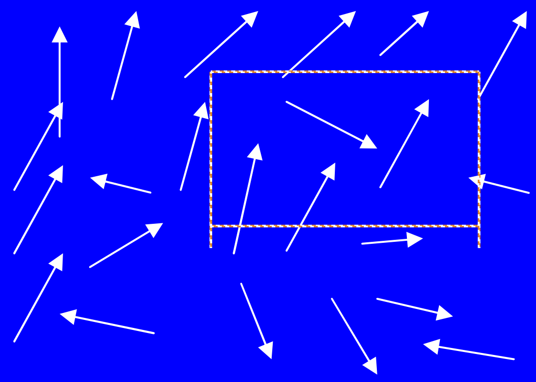


$$\left|\vec{M}\right| > 0$$

small

T>Tc

disordered



$$\left|\vec{M}\right| = \left|\left\langle \vec{S} \right\rangle\right| = 0$$

a. Order parameter : magnetization.

$$\vec{M} \equiv (M_x, M_y)$$

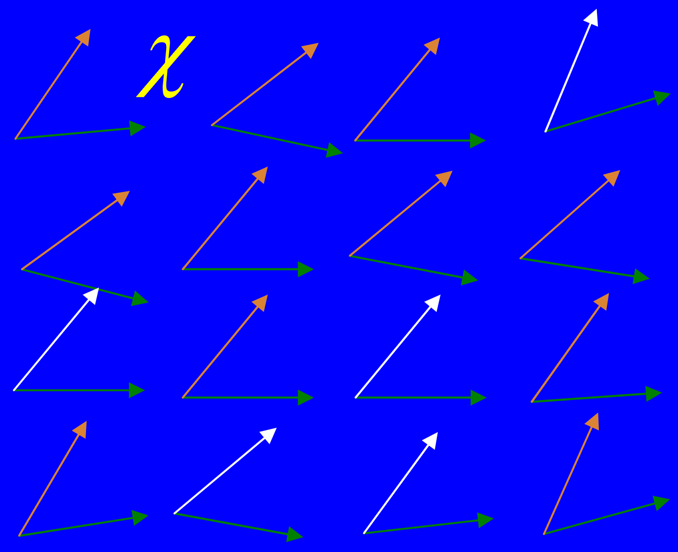
or, using complex numbers,

$$\phi = M_x + iM_y$$

b . Symmetry : 2D rotations

$$M_x \to \cos\chi\, M_x + \sin\chi\, M_y \equiv M_x'$$

$$M_y \to -\sin\chi\, M_x + \cos\chi\, M_y \equiv M_y'$$



Using complex numbers the symmetry transformation becomes U(1):

$$\phi \rightarrow e^{i\chi}\phi$$

Symmetry means that the energy of the rotated state is the same as that of the original (not rotated) one.

$$F(\vec{M}) = F(\vec{M}\,')$$

Effective free energy near the phase transition

Most general functional symmetric under $\phi \rightarrow e^{i\chi}\phi$ and space rotations, with lowest possible powers of ϕ and lowest number of gradients is

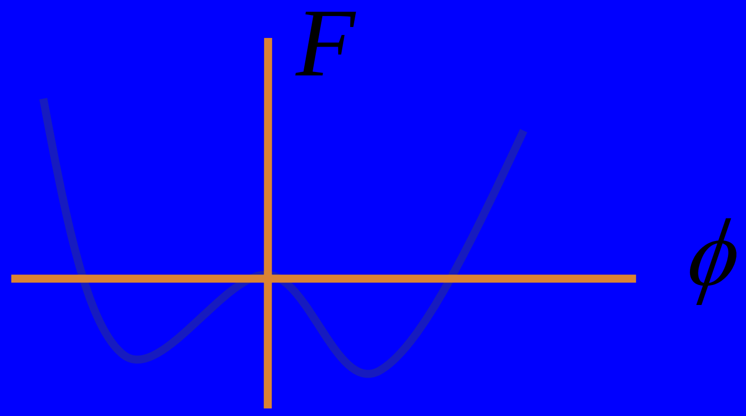
$$F = \int d^D x \left[\vec{\nabla}\phi^* \vec{\nabla}\phi + a(T)\phi^*\phi + \frac{b(T)}{2}(\phi^*\phi)^2 \right]$$

Higher order terms $\left(\nabla\phi^*\nabla\phi\right)^2, (\phi^*\phi)^3, ..$

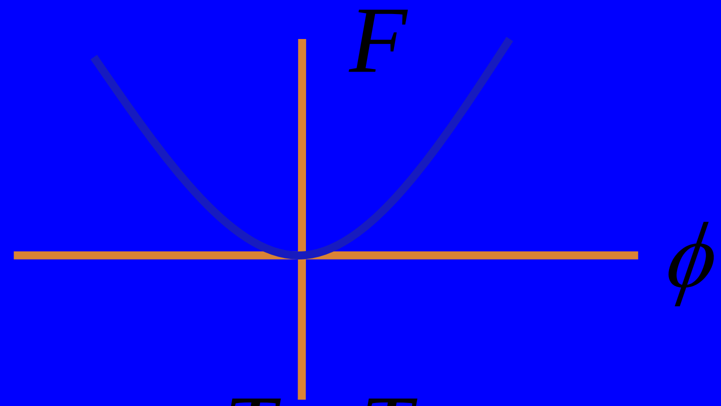
are expected to be smaller close enough to Tc.

The remaining coefficients can be expanded around Tc:

$$a(T) = \alpha(T - T_c) + ...,$$
$$b(T) = \beta + ...$$



$T < T_c$



$T > T_c$

Now we apply this general considerations to the superconductor – normal metal phase transition.

a. Order parameter

The complex order parameter is "amplitude of the Cooper pair center of mass"

$$\Psi(x) \propto \Delta(x), \quad \Delta(k) \propto \langle c_{\uparrow}(k) c_{\downarrow}(-k) \rangle$$

which is "the gap function" of BCS or any other (no matter how "unconventional") microscopic theory .

b. Symmetry

The broken symmetry is charge U(1) mathematically the same symmetry as that of the XY magnet.

Without external magnetic field the free energy near transition therefore is:

$$F[\Psi] = \int d^3x \left[\frac{\hbar^2}{2m^*} \vec{\nabla}\Psi^* \vec{\nabla}\Psi + \alpha(T - T_c)\,\Psi^*\Psi + \frac{\beta}{2}(\Psi^*\Psi)^2 \right]$$

$$m^* = 2\, m_e$$

Ginzburg and Landau (1950) postulated a reasonable way to generalize this to the case of arbitrary magnetic field $\vec{B}(x)$:

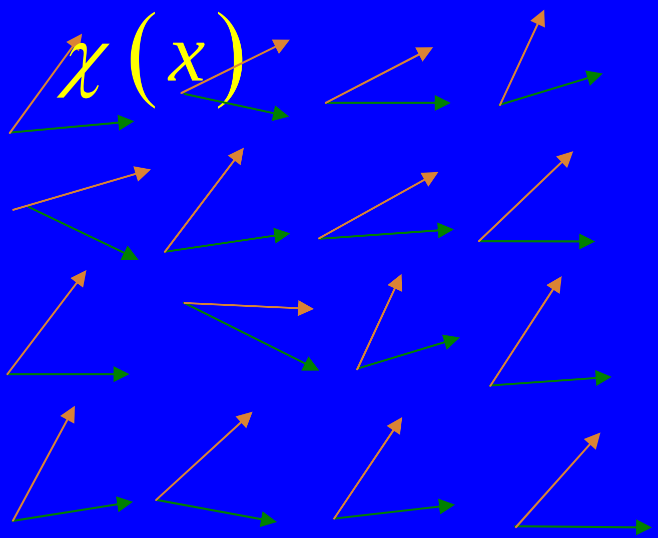
One is using the "principle" of local gauge invariance of electrodynamics.

Electrodynamics is invariant under local gauge transformations:

$$\begin{cases} \Psi(x) \to e^{i\chi(x)}\Psi(x) \\ \vec{A}(x) \to \vec{A}(x) + \dfrac{\hbar c}{e^*}\vec{\nabla}\chi(x) \end{cases}$$

$$e^* = 2e$$

$$\chi(x)$$



Leading to "minimal substitution":

$$\vec{\nabla} \to \vec{D} = \vec{\nabla} - i\frac{e^*}{\hbar c}\vec{A}$$

Scales

GL equations possess two scales. Coherence length ξ characterizes variations of order parameter, while the penetration depth λ characterizes variations of magnetic field

Properties of solutions crucially depend on the GL parameter. When

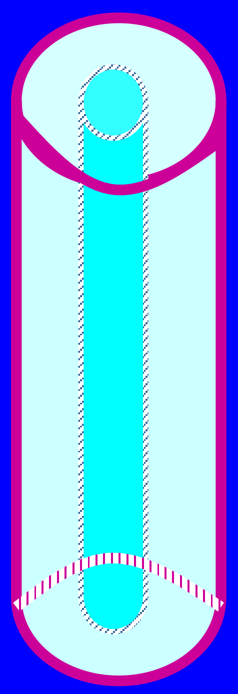
$$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$$

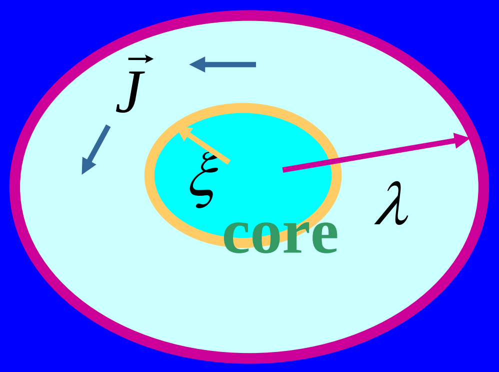
(the type II superconductivity) there exist "topologically nontrivial" solutions – the Abrikosov vortices.

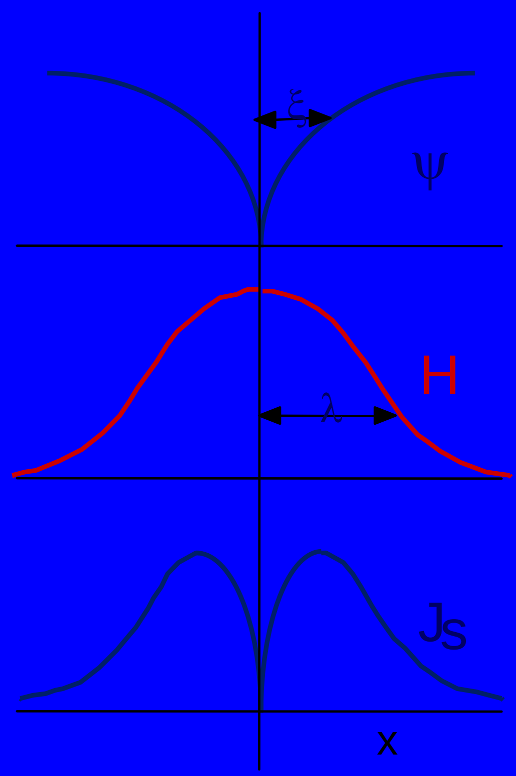
Abrikosov (1957)

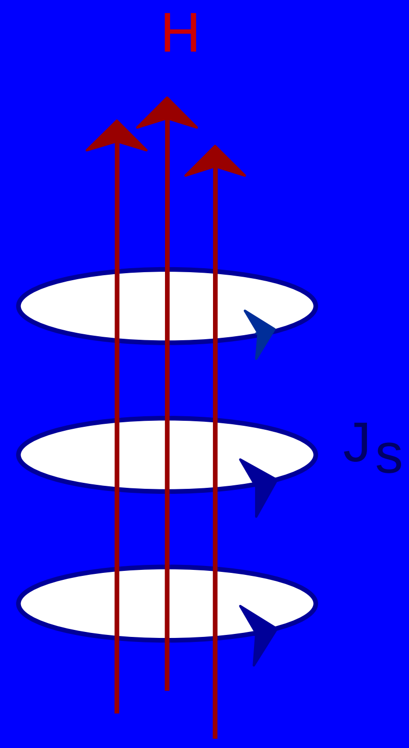
Abrikosov vortices

linear topological defect.





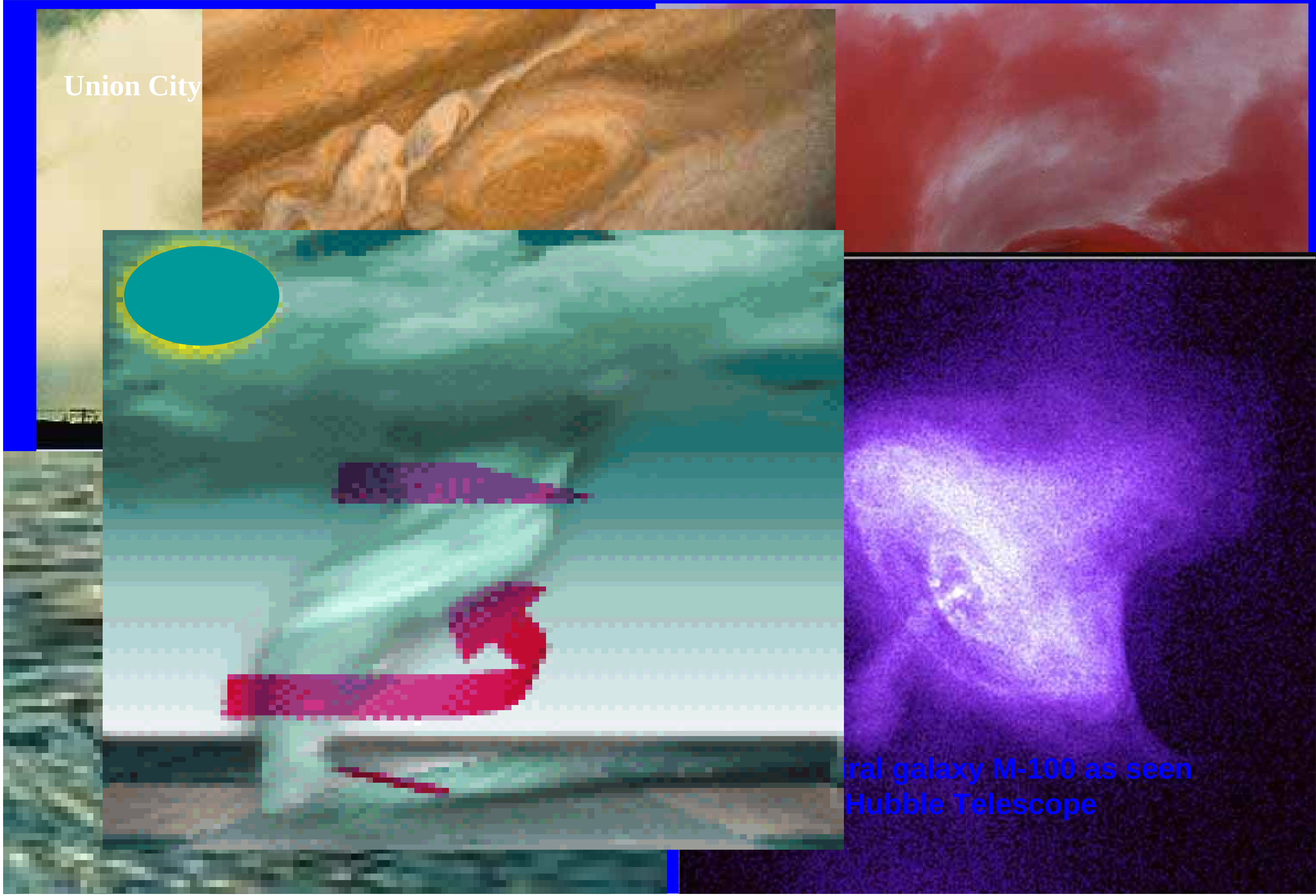


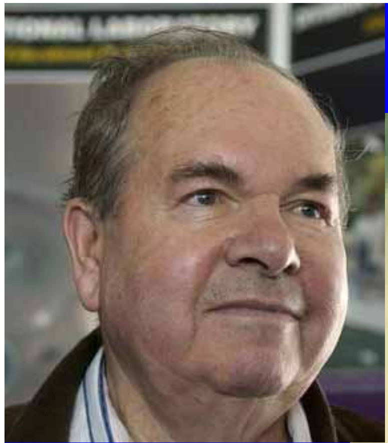


quantum of flux
hc/2e

$\lambda \gg \xi$

… vortices…





Abrikosov vortices can be viewed as magnetic tornados: the core surrounded by encircling electrons

Time dependent GL

The friction dominated dynamics is described by the TDGL

$$\frac{\hbar^2\gamma}{2m^*}\frac{\partial}{\partial t}\psi\left(x,t\right) = -\frac{\delta}{\delta\psi^*\left(x,t\right)}F\left[\psi,\psi^*\right]$$

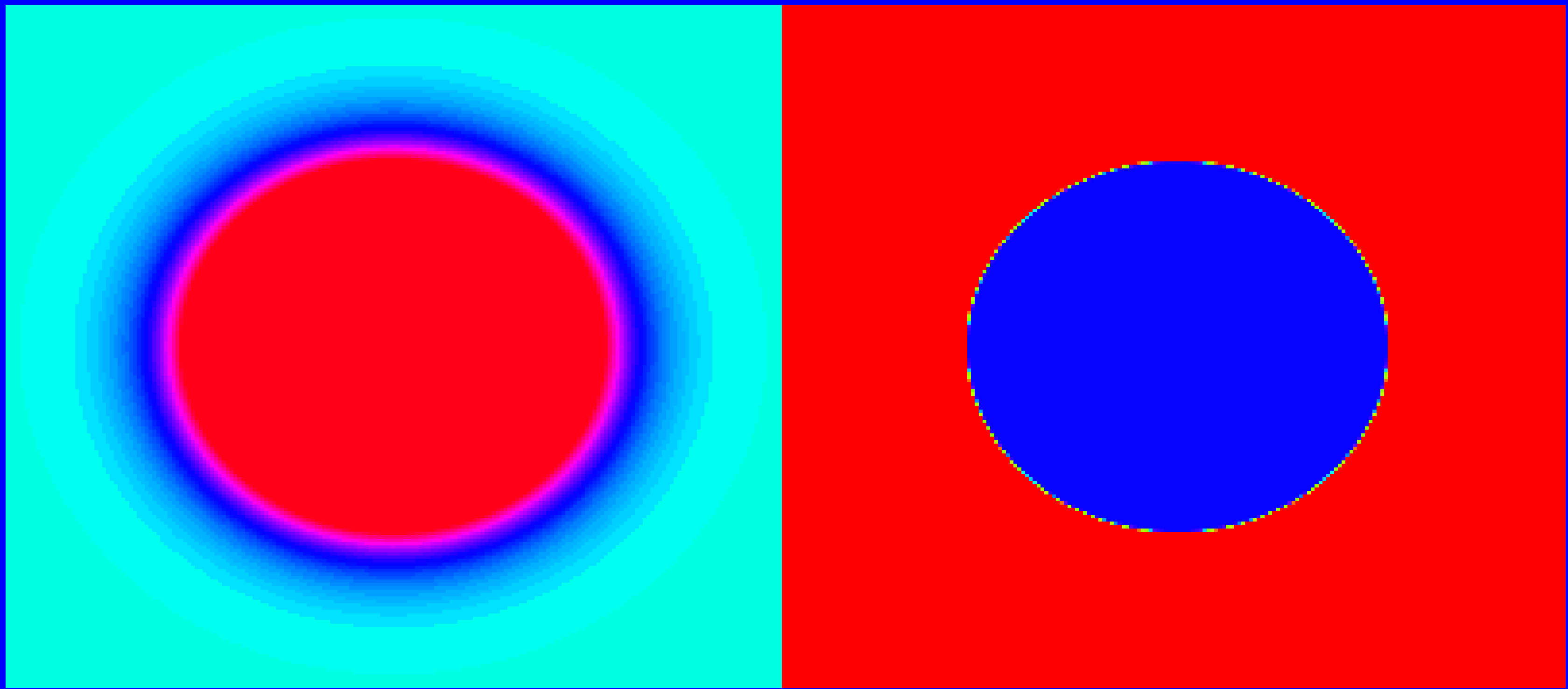
where γ **is the normal state inverse diffusion constant**

The driving force is introduced via (homogeneous far from Hc1) electric field

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \frac{ie^*}{\hbar}\Phi\left(x,t\right)$$

The vortex dynamics then can be simulated

Creating of vortices and anti-vortices at the hot spot



Magnetic field　　　　　　　　Order parameter

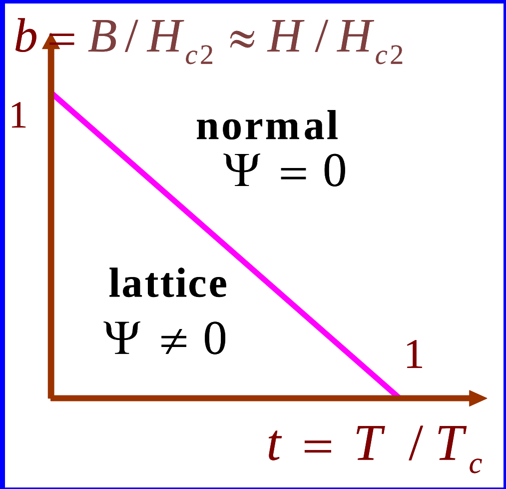
Disintegration of the magnetic flux at the normal line into vortices at type II SC

Same in type I SC

Ginzburg – Landau energy and the Abrikosov lattice solution

$$G = \int d^3x \frac{\hbar^2}{2m_{ab}} \left| \left(\nabla_{ab} - \frac{2ie}{\hbar c}\mathbf{A} \right) \psi \right|^2 + \frac{\hbar^2}{2m_c} \left| \nabla_c \psi \right|^2 + \alpha (T - T_c) \left| \psi \right|^2$$

$$+ \frac{\beta}{2} \left| \psi \right|^4 + \frac{(B-H)^2}{8\pi}$$



Near H_{c2}, neglecting fluctuations, Abrikosov found a hexagonal lattice solution

$$\varphi(x,y) \propto \sum_{e=-\infty}^{\infty} \exp\left\{ 2\pi i \left[\frac{(l-1)l}{4} + l\frac{x}{a} \right] \right\}$$

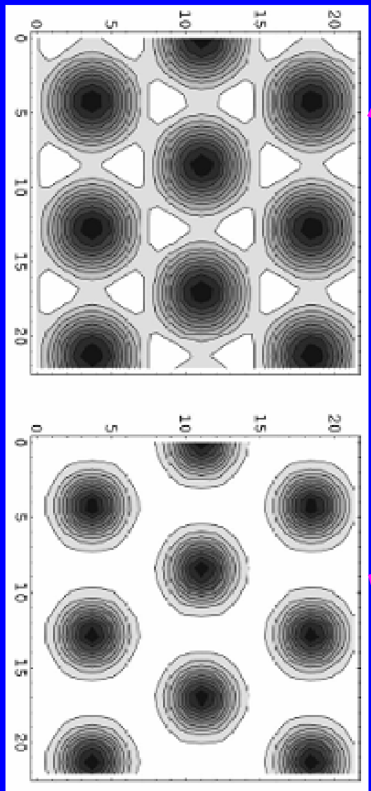
$$\exp\left[-\frac{1}{2} \left(y - \frac{2\pi l}{a} \right)^2 \right]$$

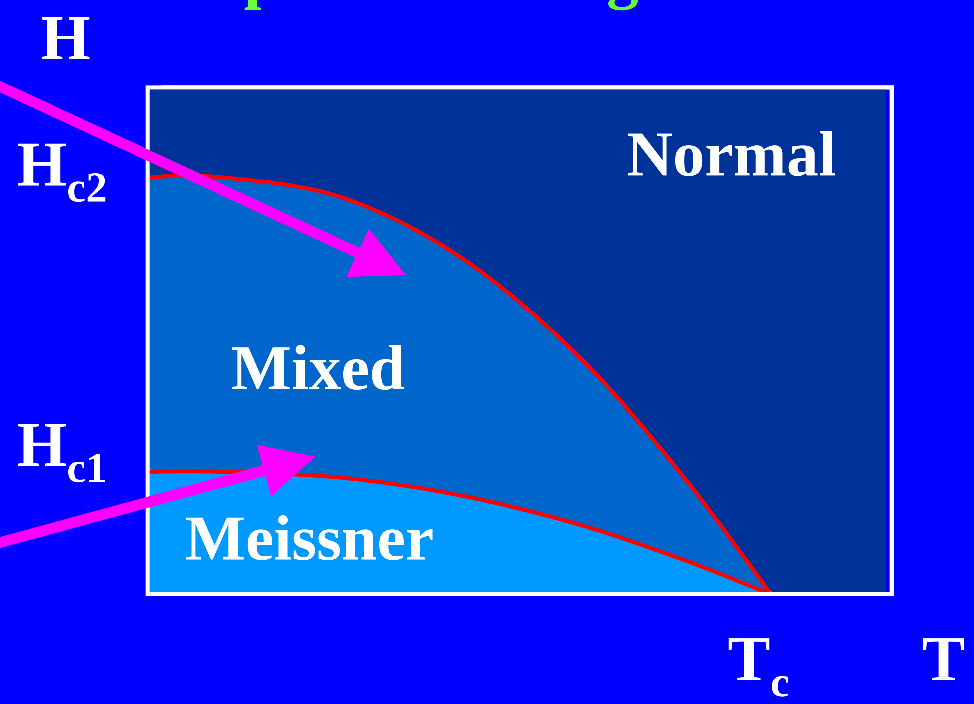
Second order transition at

$$b + t = 1 \quad b = \frac{B}{H_{c2}}; \quad t = \frac{T}{T_c}$$

Two complementary theoretical approaches to the mixed state

For $H \gg H_{c1}$ vortex cores almost overlap. Instead of lines one just sees array of superconducting “islands”



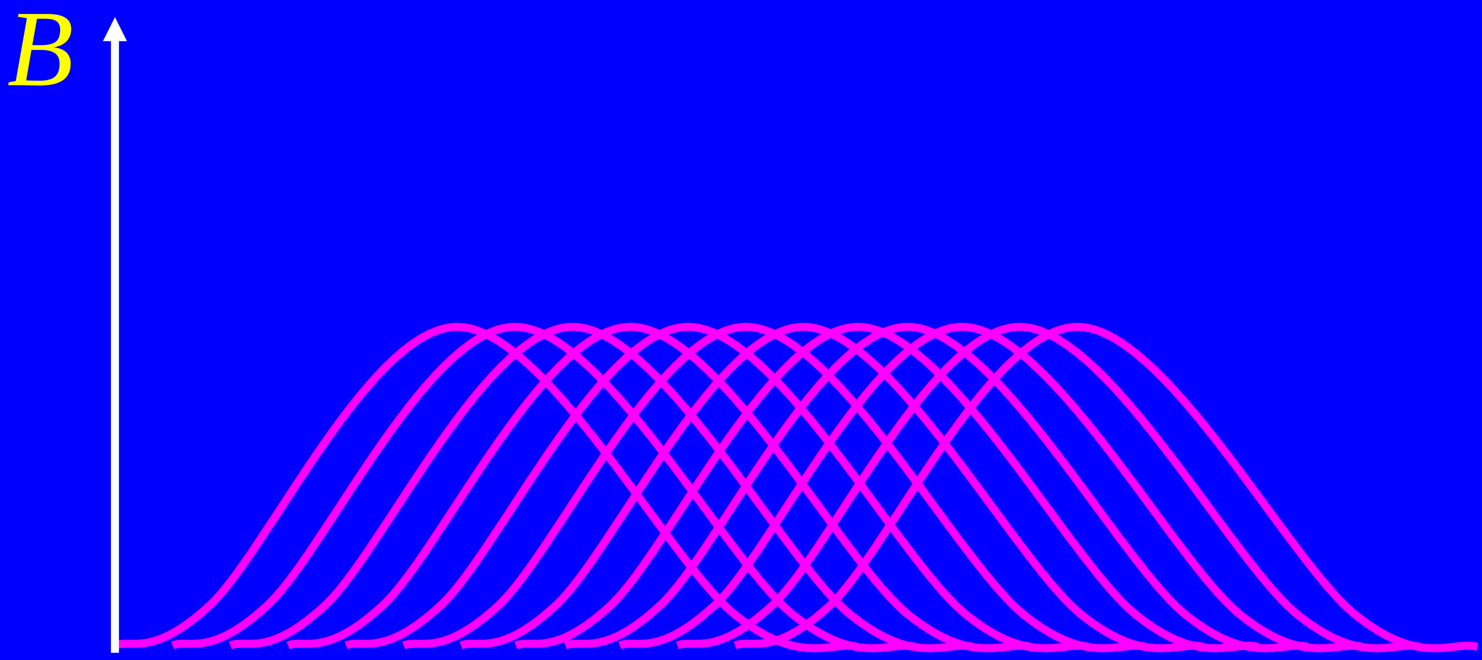


Lowest Landau level appr. for constant magnetic induction **B**

London appr. for infinitely thin lines constant order parameter

For $H \ll H_{c2}$ vortices are well separated and have very thin cores

Homogeneity of magnetic field



Homoheneity of magnetic induction B is a result of overlap of magnetic fields of roughly $\frac{B}{H_{c2}}\kappa^2$ **magnetic fields of individual vortices**

Magnetization (although inhomogeneous) is small ($\kappa^{-2}H_{c2}$**) and one replaces B(r)=H**

Ginzburg – Landau energy and a systematic expansion for the Abrikosov lattice solution

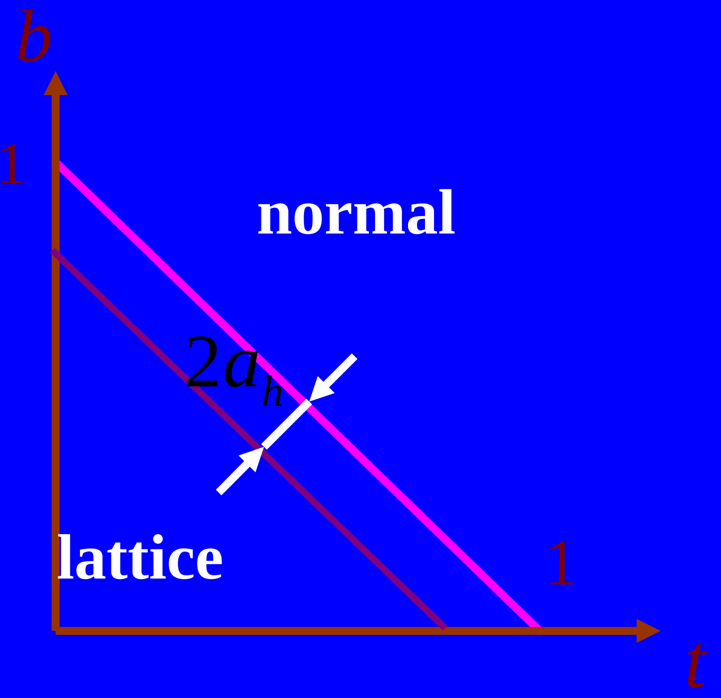
$$G=\int d^3x\,\frac{\hbar^2}{2m^*}\left|\left(\nabla_{xy}-\frac{ie^*}{\hbar c}\mathbf{A}\right)\Psi\right|^2+\frac{\hbar^2}{2m_c}\left|\nabla_z\Psi\right|^2+\alpha(T-T_c)\left|\Psi\right|^2+\frac{\beta}{2}\left|\Psi\right|^4$$

GL equations (using ξ as unit of length, $\psi^2=\Psi^2/\left(2\Psi_0^2\right)$)

$$\frac{\delta G}{\delta\psi}=\hat{H}\psi-a_h\psi+\psi\left|\psi\right|^2=0$$

$$\hat{H}=-\frac{1}{2}D^2-\frac{b}{2}$$

$$t=\frac{T}{T_c},\quad b=\frac{B}{H_{c2}},\quad a_h=\frac{1-t-b}{2}$$



Variational solution

The operator H has a discrete spectrum: $E_N = Nb$

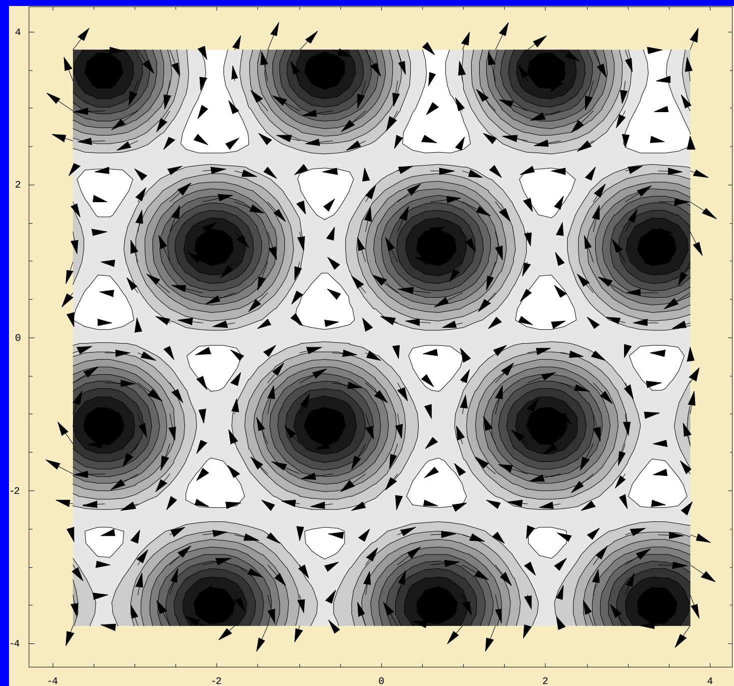
$$\phi_{Nk}(r) \propto e^{ikx} H_N\left[b^{1/2}(y-k/b)\right]\exp\left[-\frac{b}{2}(y-k/b)^2\right]$$

One can look for a state of minimal energy on the LLL subspace

$$\psi = \int_k c_k \phi_{N=0,k}$$

Symmetry leads to the hexagonal lattice superposition

$$\varphi_\Delta(r) = C_0 \sum_{e=-\infty}^{\infty} \exp\left[i\frac{\pi}{2}l^2\right]\phi_{\frac{2\pi}{a}l}(r)$$



Normalization is fixed by the minimization of GL energy:

$$C_0 = \frac{a_h^{1/2}}{\beta_A^{1/2}}; \quad \beta_A = \left\langle |\varphi|^4 \right\rangle$$

It is not clear how to extend this variational method to dynamics with dissipation, described by TDGL,

$$\frac{\hbar^2}{2m^*}(\gamma + i\gamma')\frac{\partial}{\partial t}\psi_{xt} = -\frac{\delta}{\delta\psi_{xt}^{\ *}}G\left[\psi,\psi^*\right]$$

since there generally is no obvious functional to be minimized. One also would like to estimate how good is the approximation.

The generalization can be achieved, if the variational result is presented as the leading approximant in a perturbation expansion.

Lascher. PR A140, 523 (65)

Perturbation theory in a_h

$$\psi = a_h^{1/2}\left(\psi_0 + a_h\psi_1 + a_h^2\psi_2 + ..\right)$$

The leading ($a_h^{1/2}$) order equation gives the lowest LLL restriction, already motivated in the variational approach:

$$H\psi_0 = 0 \Rightarrow \psi_0 = C_0\varphi_\Delta$$

with normalization undetermined. The next to leading order, $a_h^{3/2}$, equation is:

$$H\psi_1 - C_0\varphi + C_0^2 C_0^*\varphi\left|\varphi\right|^2 = 0$$

Multiplying it with φ^* and integrating one obtains

$$\int_{cell}\varphi^*\left[H\psi_1 - C_0\varphi + C_0^2 C_0^*\varphi\left|\varphi\right|^2\right] = 0$$

$$-1 + \left|C_0\right|^2\int\left|\varphi\right|^4 = 0 \Rightarrow \quad C_0 = \beta_A^{-1/2}$$

Corrections

Higher order correction will include the higher Landau level contributions

$$\psi_1 = \sum_{N=0}^{\infty} C_1^N \varphi_N$$

Multiplying the GL equation by $\varphi^*_N, \quad N > 0$ **,and integrating over the unit cell, then using orthonormality relations one obtains:**

$$\int \varphi_N^* \hat{H} \psi_1 = N\, b\, C_1^N = \sqrt{C_0} \int \varphi_N^* \varphi \left(1 + C_0 |\varphi|^2\right)$$

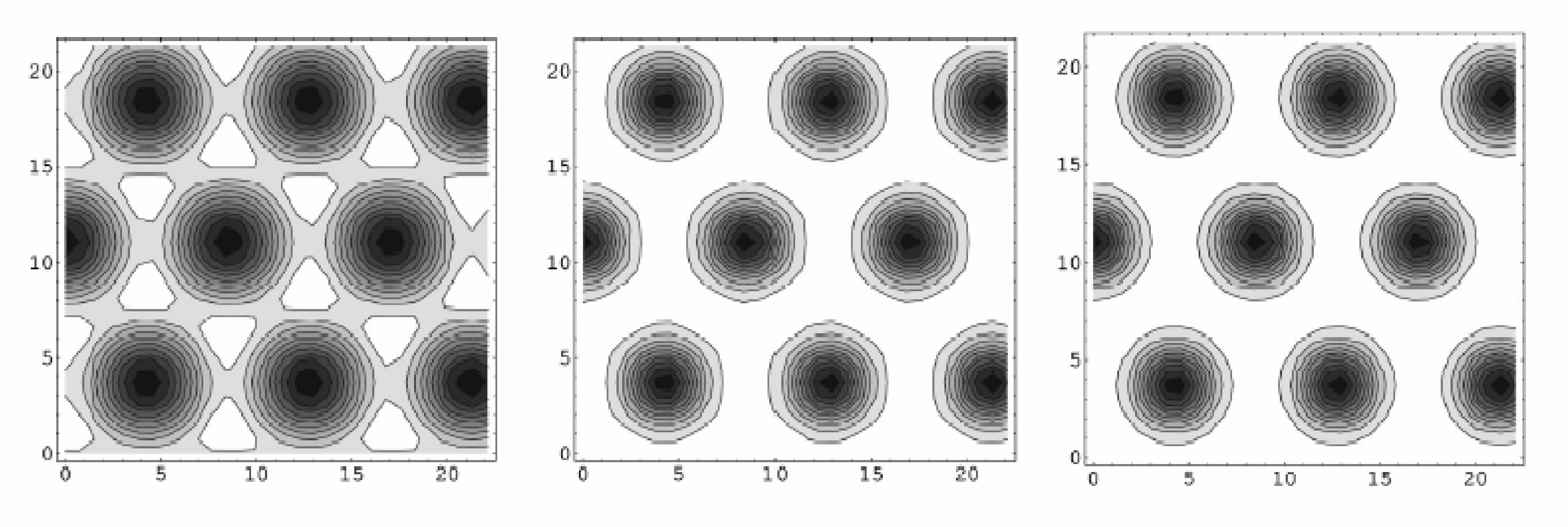
$$C_1^N = \frac{C_0^{3/2}}{N\, b} \left\langle \varphi_N{}^* \varphi |\varphi|^2 \right\rangle_{period}$$

The LLL component is found from the order $a_h{}^{5/2}$ **etc.**

All the corrections are very small numerically partially due to factors of 1/N with multiples of 6 contributing due to the hexagonal symmetry

$$C_1^{(6)} = -\frac{0.279}{6b}; C_1^{(12)} = \frac{0.025}{12b}$$

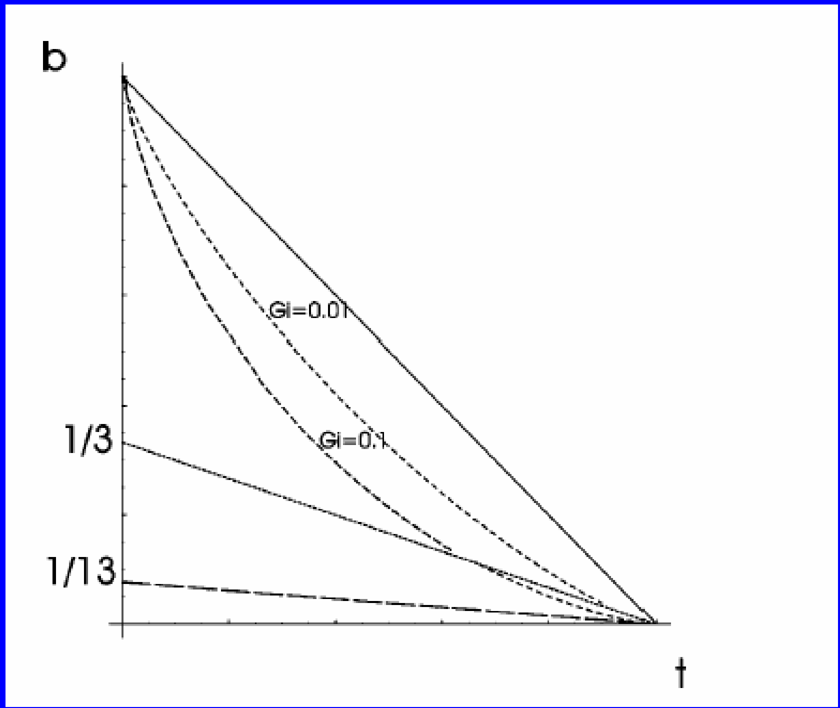
$T/T_c = 0.5 \quad B/H_{c2} = 0.1 \quad a_h = 0.2$



leading **next to leading**

0+6th LL **0+6+12 LL**

Therefore the perturbation theory in a_h is useful up to surprisingly low fields and temperatures, roughly above the line



$$b=\frac{1}{13}(1-t)$$

Li, B.R. PRB60, 9704 (99)

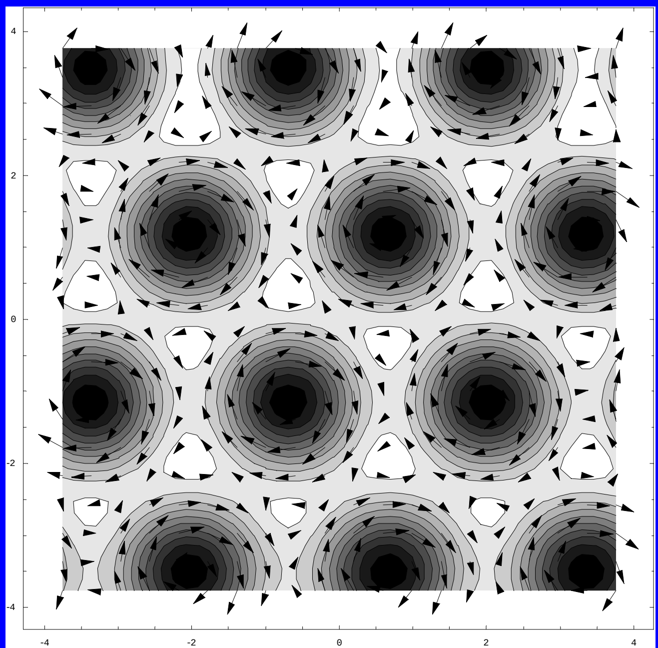
$$f=-\frac{a_h^2}{2\beta_A}-\frac{0.044}{6^2}\frac{a_h^3}{b}+\frac{0.056}{6^3}\frac{a_h^4}{b^2}$$

LLL is by far the leading contribution above this line.

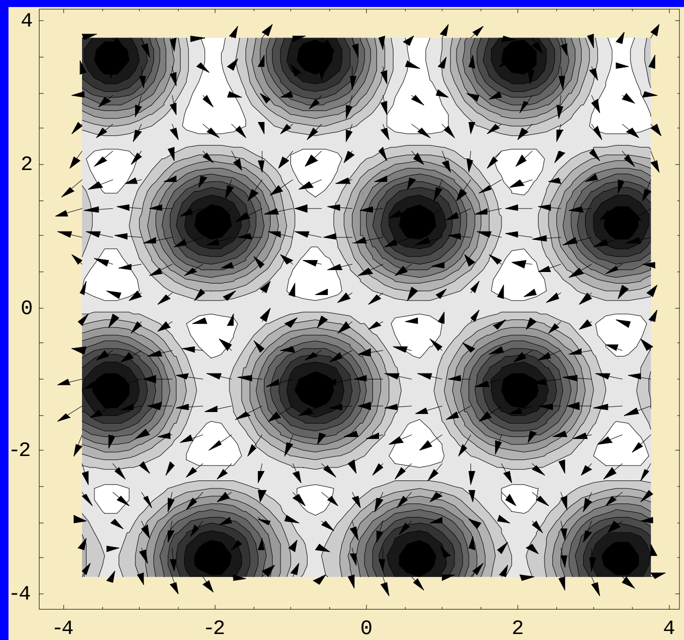
Persistent current in magnetic field

When all the vortices are pinned there might be current without dissipation. The order parameter configuration cannot belong to LLL, since for general LLL configuration

$$J_i = \frac{\hbar e^*}{2m^*}\varepsilon_{ij}\partial_j\left(|\Psi|^2\right)$$



whole current



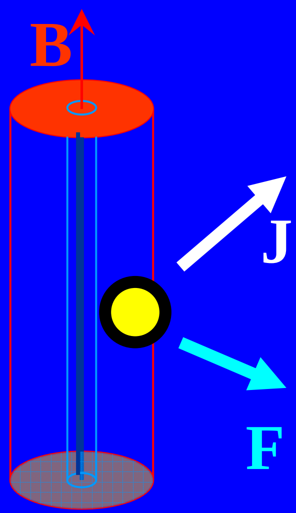
LLL subtracted

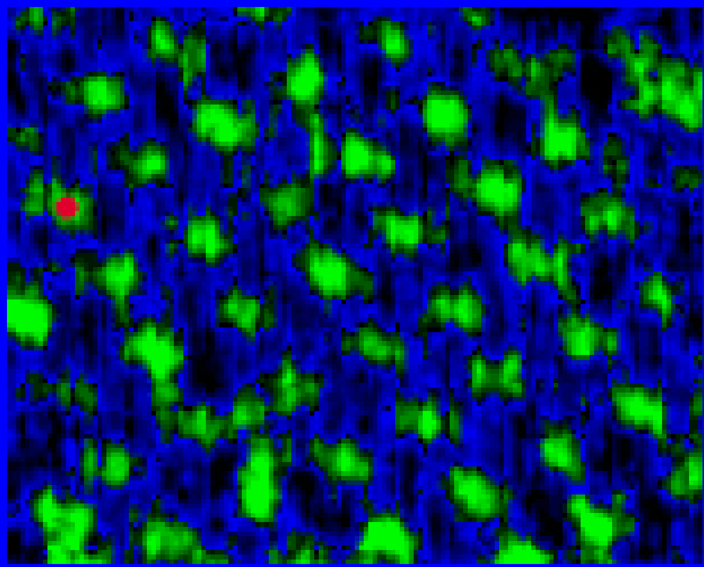
Affleck, Brezin. NPB257, 451 (85)

$$\psi = \varphi_0 + 0.02\varphi_1$$

Flux flow

Fluxons are light and can move. The motion is generally friction dominated with energy dissipated in vortex cores. The current "induces" flux flow, causing voltage via "phase slips".





Electric field is present and, due to superposition between vortices is also homogeneous in sufficiently dense vortex matter

Hu, Thompson, PRL27, 1352 (75)

Field driven flux motion probed by STM on NbSe2

Troyanovsky et al (04)

Time dependent GL in the presence of electric field

The friction dominated dynamics is described by the TDGL

$$\frac{\hbar^2\gamma}{2m^*}D_t\psi_{xt} = -\frac{\delta}{\delta\psi^*_{xt}}F\left[\psi,\psi^*\right]$$

where $\gamma/2$ is the normal state inverse diffusion constant

The driving force is introduced via cov. derivative (unit of time $t_{GL} = \gamma\xi^2$)

$$D_t = \frac{\partial}{\partial t} - \frac{ie^*}{\hbar}\Phi(x)$$

$$\hat{L}\psi - a_h\psi + \psi\,|\psi|^2 = 0$$

$$a_h = -\frac{1}{2}\left(1 - t - b - \mathrm{E}^2/b^2\right)$$

$$\hat{L} = D_t + H + \frac{\mathrm{E}^2}{2b^2}$$

$$E_{GL} = \frac{4\hbar}{e^* t_{GL}\xi} \qquad \mathrm{E} = \frac{E}{E_{GL}}$$

is not Hermitean.

Bifurcation point perturbation theory

Electric field therefore is an additional pair breaker. The critical line beyond which just a trivial normal solution exists is

$$1-t-b-\mathrm{E}^2/b^2=0 \Rightarrow \; H_{c2}\left(T,E\right)=H_{c2}\left(1-t-\mathrm{E}^2/b^2\right)$$

Hu, Thompson, PRL27, 1352 (75)

The perturbation in a_h can be applied again. One first looks for eigenfunctions of the linear part of the equation

$$\hat{L}\phi_{Np\omega}=\Theta_{Np\omega}\phi_{Np\omega} \qquad \Theta_{Np\omega}=Nb+i\left(\omega-vk\right) \qquad v=\mathrm{E}b^{-3/2}$$

The right eigenfunctions are:

$$\phi_{Np\omega}=\mathrm{e}^{i(kx-\omega t)}\,H_N\left[b^{1/2}\left(y-k/b+iv\right)\right]\exp\left[-\frac{b}{2}\left(y-k/b+iv\right)^2\right]$$

Note the "wave" exponential despite absence of Galileo invariance (due to microscopic disorder tied to the rest frame)

Within the bifurcation method one uses scalar products. In the present case these should be formed with the left eigenfunctions:

$$\bar{\phi}_{Nk\omega} = \mathrm{e}^{-i(kx-\omega t)} H_N \left[b^{1/2}(y-k/b+iv) \right] \exp\left[-\frac{b}{2}\left(y-k/b+iv\right)^2 \right] \neq \phi^*_{Nk\omega}$$

The orthonormality relations take a form:

$$\int_{x,y,t} \bar{\phi}_{Nk\omega}(x,y,t)\phi_{N'k'\omega'}(x,y,t) = \sqrt{\pi}\,\delta_{NN'}\delta(k-k')\delta(\omega-\omega')$$

Assuming, as in statics, an expansion $\psi = a_h^{1/2}\left(\psi_0 + a_h\psi_1 + ..\right)$ **,**

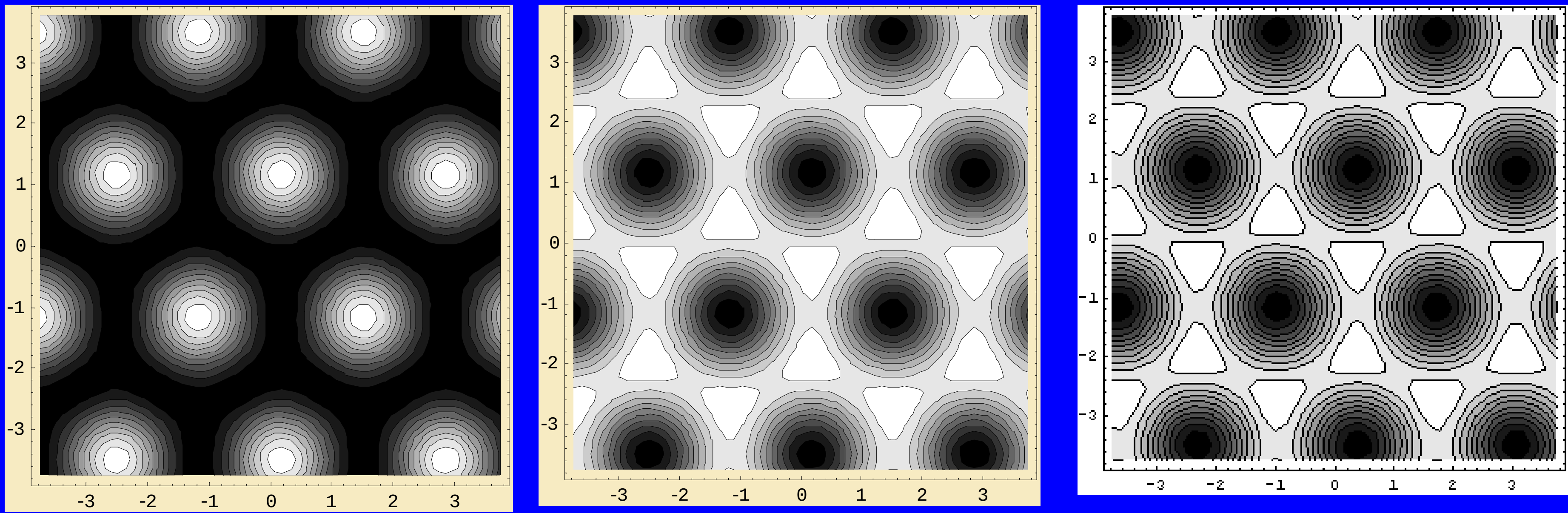
the leading, $a_h^{1/2}$**, order equation is the LLL constraint**

$$\hat{L}\,\psi_0 = 0$$

implying $$N = 0, \quad \omega = vk$$

For each moving lattice symmetry one gets normalization from the $a_h^{3/2}$ order:

$$C_0 = \beta_A^{-1/2}(v); \quad \beta_A(v) = \left\langle \overline{\varphi}\varphi^*\varphi^2 \right\rangle$$



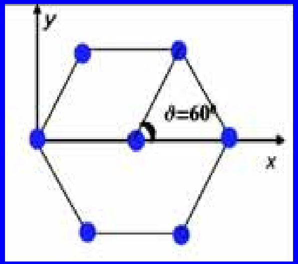
dissipation **superfluid density** $p = \left\langle E \cdot J \right\rangle = \dfrac{\hbar^2\gamma}{2m^*}\left\langle \left| D_t\psi \right|^2 \right\rangle$

LLL supercurrent density: $$J_i = \frac{\hbar e^*}{m^*}\left[\frac{1}{2}\partial_j \left(\left|\Psi\right|^2 \right) + v_i \left|\Psi\right|^2 \right]$$

Shape of the moving lattice, I-V

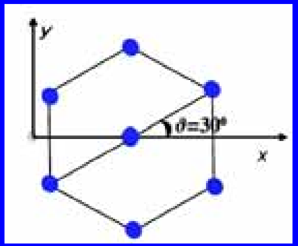
As in statics, one should maximally use the symmetries available, but now there are less due to electric field.

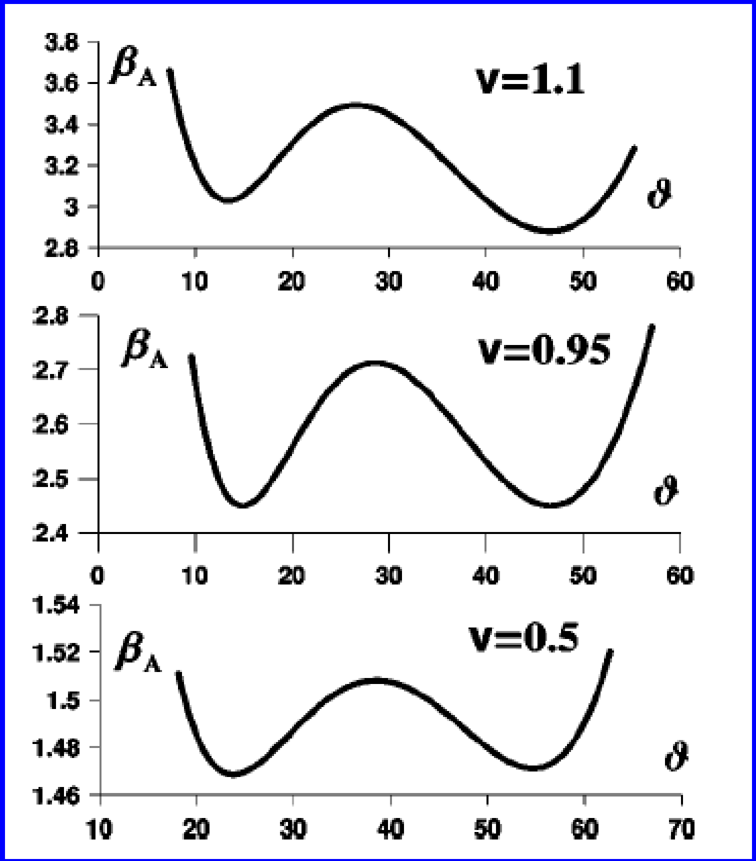
Generally there are the following options:



Flux flow

Li, Malkin, B.R.
PRB70, 214528
(04)





Conductivity also has small non – Ohmic corrections due to HLL

Thermal fluctuations on the mesoscopic level in the flux lattice

Thermal fluctuations are taken into account via statistical sum

$$Z = \int D\psi^*(x) D\psi(x) e^{-G[\psi]/T}$$

For not very small fields one can consider effective lowest Landau level (LLL) only in which case the energy simplifies

$$\frac{G}{T} = \int d^3x \left[\frac{1}{2} \left|\partial_z \psi\right|^2 - a_T \left|\psi\right|^2 + \frac{1}{2} \left|\psi\right|^4 \right]$$

With only one dimensionless parameter: the LLL scaled temperature

$$a_T \equiv -\frac{1-t-b}{\left(t\, b \sqrt{Gi} \right)^{2/3}}$$

Supersoft phonons in vortex solid near Hc2

There are two types of the fluctuation modes in expansion around the Abrikosov solution:

$$\psi(x,y,z)=\varphi(x,y)+\int_{k\in B.Z,k_z}\varphi_k(x,y)e^{ik_z z}(O_k+iA_k)$$

Substituting this into energy and diagonalizing quadratic part of free energy one obtains:

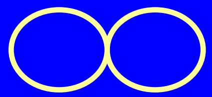
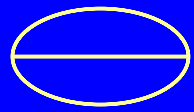
$$F=F_{mf}+\int_k e_O(k)O_k^* O_k+e_A(k)A_k^* A_k+AAA+AAAA$$

$$e_A(k)=a_T\left(1-2\frac{\beta_k}{\beta_A}+\frac{\gamma_k}{\beta_A}\right)+k_z^2\approx .12\,|a_T|\,k^4+k_z^2$$

Eilenberger, PR 164, 622 (1967); Maki&Takayama, PTP 46, 1651 (1971)

IR divergences

Naively higher order contributions to energy are hopelessly divergent:

 $\rightarrow \log^2 L$  $\rightarrow L^4$

$$\int_{k\in B.Z, k_z} \frac{1}{\left(k_x^2+k_y^2\right)^2+k_z^2} = \int_{k\in B.Z} \frac{1}{k_x^2+k_y^2} = \log L$$

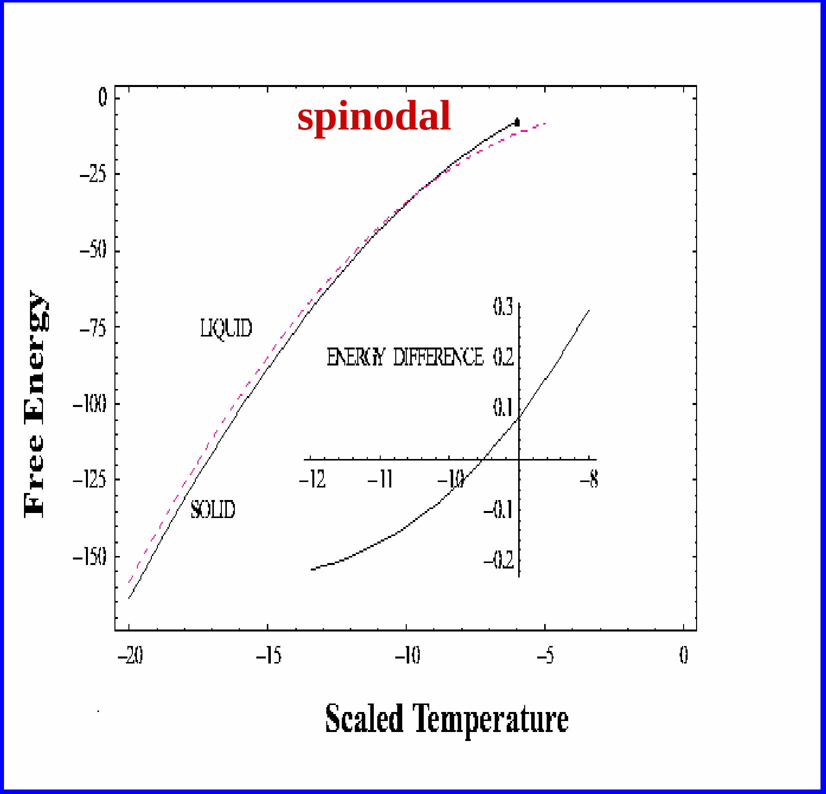
Since experimentally the corrections are small one can speculate that it is not analytic. The perturbation theory was abandoned.
Is this correct?

A question of principle

Is there a thermodynamic solid state for T>0 or experimentally observed vortex lattice is just a finite size effect?

Answers

1. All the IR divergences exactly cancel like "spurious IR divergences" in the theory of critical phenomena. There exist a stable crystalline state
2. For the free energy, which is invariant under translation, one gets to the two loop order:

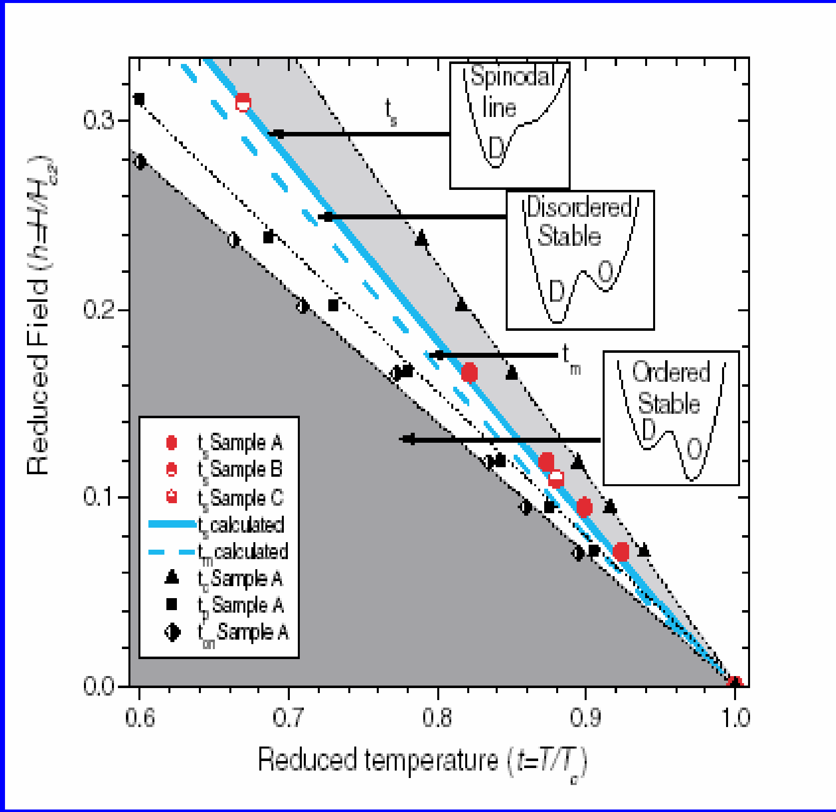


$$f_{sol} = -\frac{a_T^2}{2\beta_A} + 2.848|a_T|^{1/2} + \frac{2.4}{a_T}$$

B.R. PRB 60,4268 (1999)

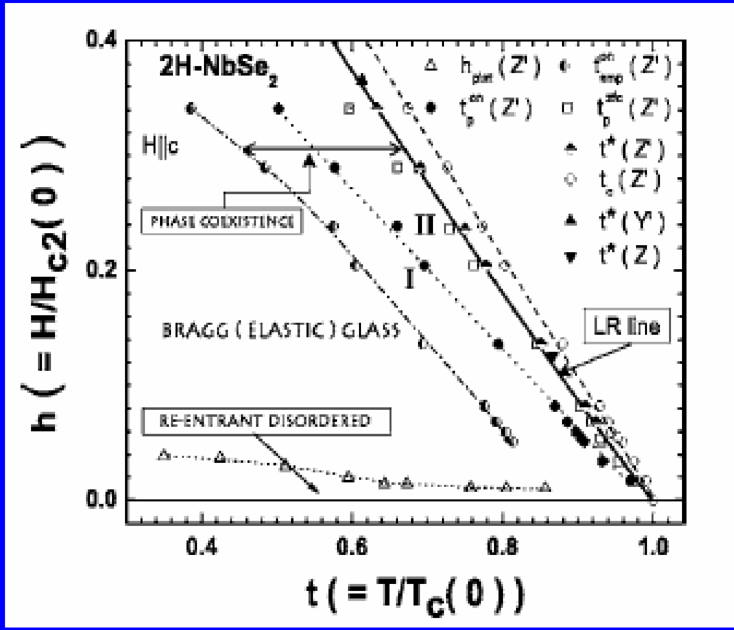
Even at melt ($a_T^m = -9.5$) the precision is 0.1%. From this one calculates magnetization, specific heat …

Metastable solid and spinodal point



Metastable solid state becomes unstable at a spinodal point :

$$a_T^{spinodal} = -5.5$$



Thakur et al PRB72, 134524 (05)

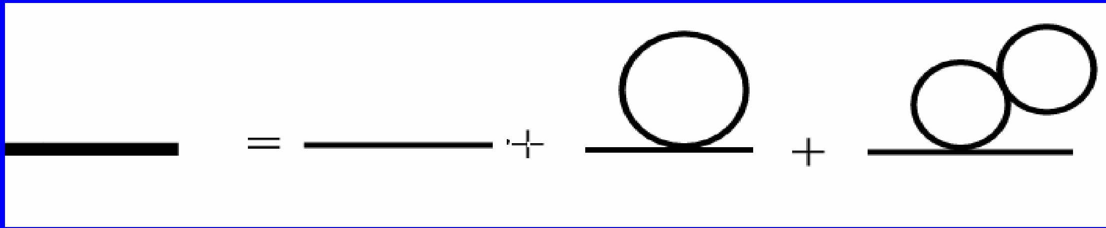
Xiao et al PRL92, 227004 (04)

Appears not far from the melting line in weakly fluctuating superconductors like NbSe2

Vortex liquid

Perturbation theory at high temperatures and the Gaussian resummation (mean field)

Standard high temperature perturbation theory is defined only only for $a_T > 0$ with the excitation energy a_T and is therefore useless for the experimentally interesting region around melting.



To get to negative a_T one can perform "bubble resummation" also called Hartree – Fock, gaussian, renormalized…

The gap equation for the excitation energy

$$e^{3/2} - a_T e^{1/2} - 4 = 0$$

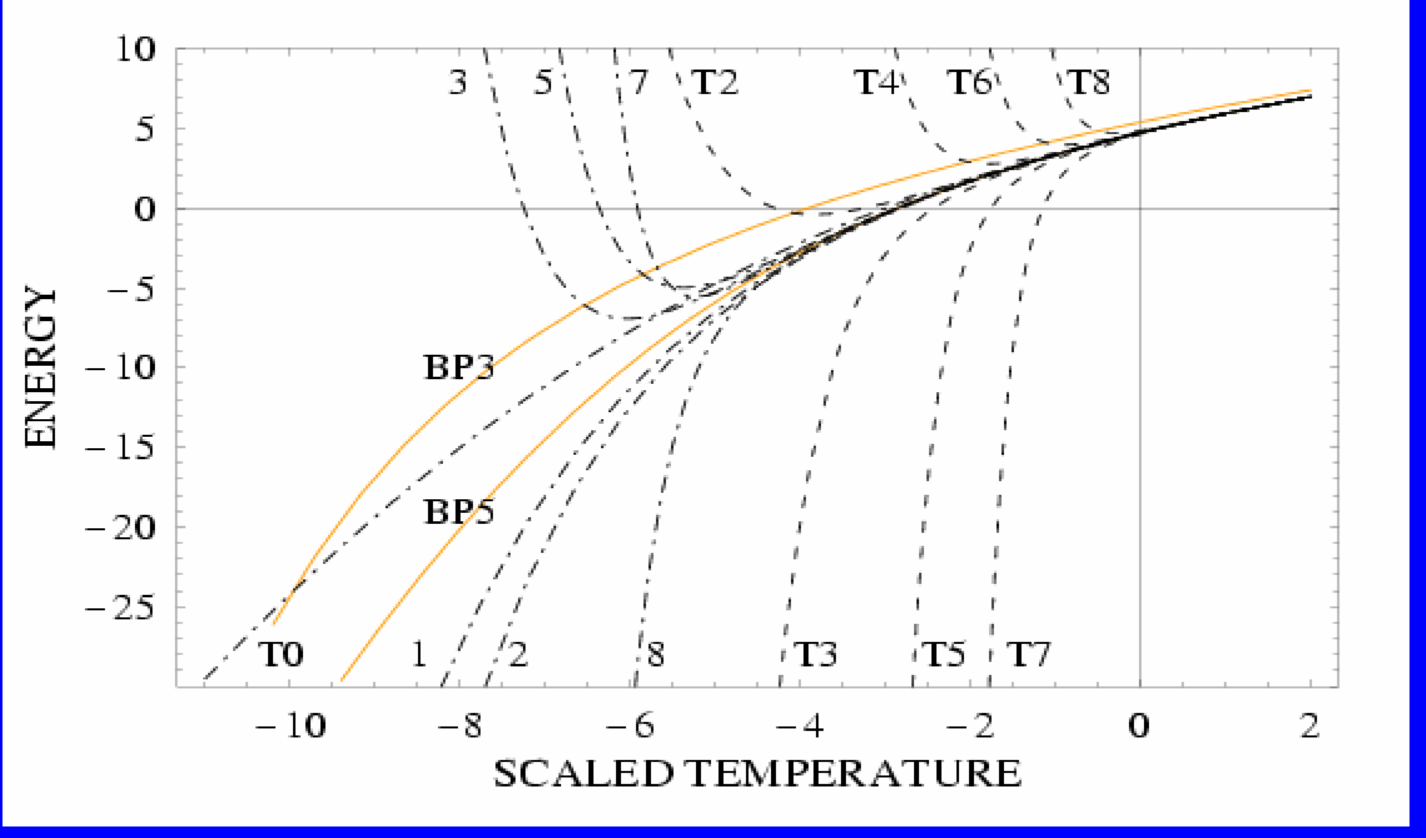
Thouless, PRL 34, 946 (1975)

has a solution for any temperature.

It shows that overcooled liquid is metastable all the way down to T=0 with excitation energy vanishing as $e \approx -4 / a_T$

Perturbations around Gaussian state were pushed to the 9th order. Unfortunately the series are asymptotic and can be used only for $a_T > -2$

Brezin et al, PRL65,1949 (1990)

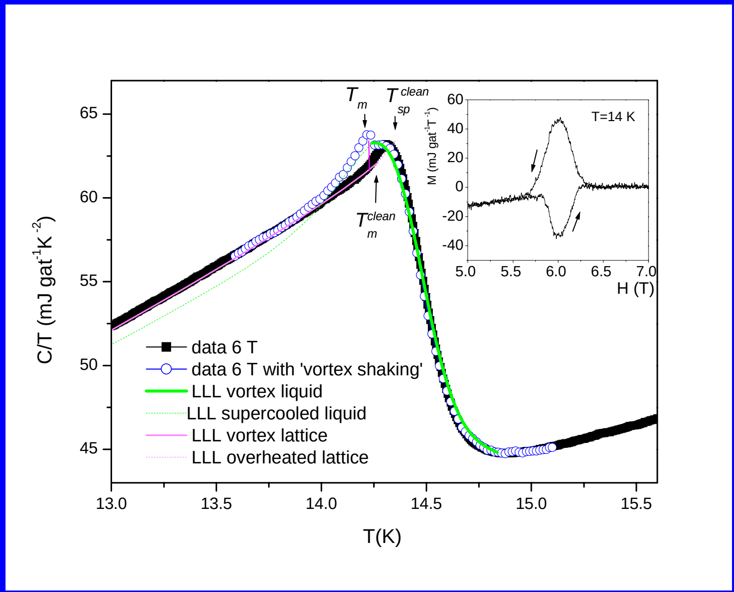


Beyond the renormalized perturbation theory

We constructed the optimized gaussian series, which are convergent rather than asymptotic DP Li, B.R. PRL86, 3618 (01)

Radius of convergence was found to be $a_T = -4.5$ still a bit short of melting.

However it allowed us to verify unambiguously the validity of the Borel-Pade method which provided a convergent scheme everywhere down to T=0. It uses the fact that for repelling objects there exists a pseudo-critical fixed point at T=0.



Lortz et al PRB74, 227004 (06)

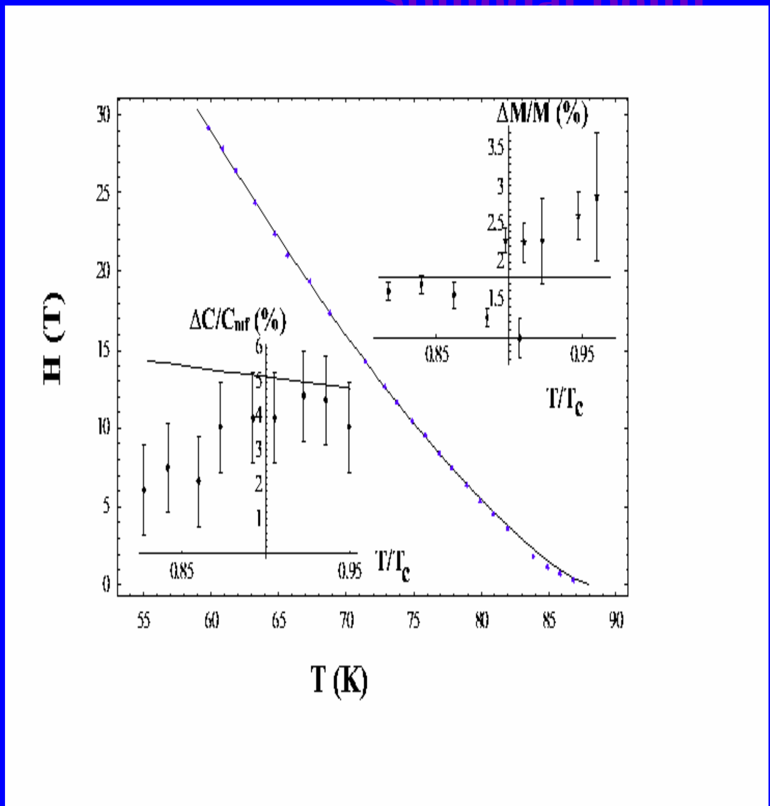
Precision of the BP is finally good enough (0.1%) to study melting quantitatively

Melting line, discontinuities at melting

The melting point: $a_T^m = -9.5$

Nishizaki et al, Physica C341,957 (00)

fully oxidized YBCO



The magnetization jump:

$$\frac{\Delta M}{M_{sol}} = 1.8\ \%$$

Welp et al, PRL76,4809 (1996)

Specific heat jump:

$$\frac{\Delta C}{C_{mf}} = 0.75\left(\frac{2-2b+t}{t}\right)^2\ \%$$

Willemin et al, PRL81,4236 (1998)

Effects of the quenched disorder

In the framework of GL a point like disorder leads to a local random modification of coefficients in the free energy. For example the " ΔT_c " disorder is described by a random field

$$\alpha \to \alpha\left[1+V\left(x\right)\right]$$

with certain phenomenologically determined variance

$$\overline{V(x)\,V(y)} = n\xi_{ab}^{2}\xi_{c}\delta(x-y)$$

To find the influence of the disorder on the melting line in some cases perturbation theory in n is sufficient, but to find irreversibility line, study dynamics nonperturbative methods like the replica method or the dynamical MSR are needed.

Disorder and thermal fluctuations in dynamics

The thermal fluctuations are introduced into the time dependent GL by the Langeven thermal noise for each degree of freedom

$$\frac{\hbar^2\gamma}{2m^*}\frac{\partial}{\partial t}\psi(x,t) = -\frac{\delta}{\delta\psi^*(x,t)}F\left[\psi,\psi^*,V\right]+\zeta(x,t)$$

$$\left\langle \zeta(x,t)\zeta(x',t')\right\rangle = 2T\gamma\delta(x-x')\delta(t-t')$$

Direct simulation becomes impractical and refined field theoretical methods of disorder averaging are necessary. The generalization of the Martin – Siggia – Rose method to GL is straightforward, but complicated.

The LLL approximation can also be developed in this case. Gaussian approximation allows calculation of the correlator and the response function in both ergodic and non-ergodic phases and subsequently the calculation of current.

Results

The irreversibility line and disorder effect on the crystalline – homogeneous transition

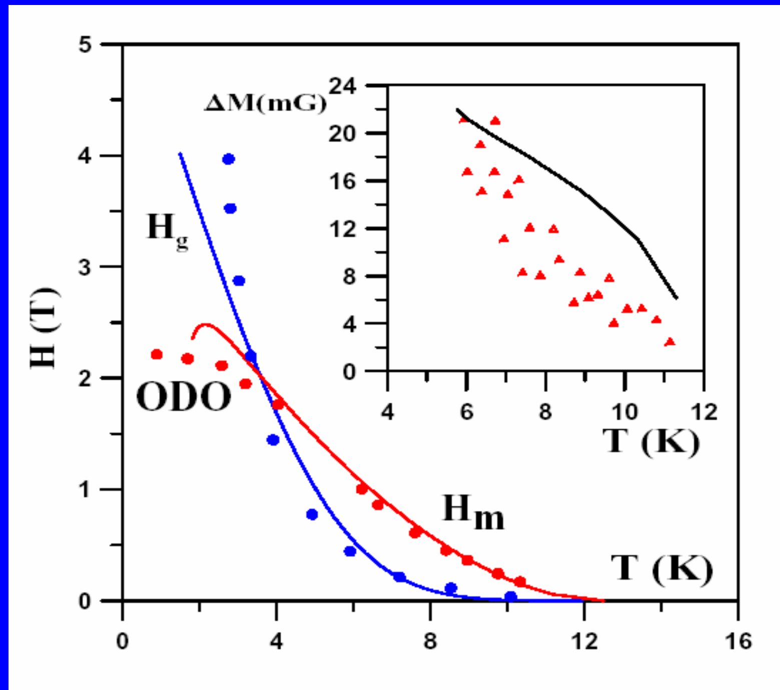
Replica method applied to the GL model with disorder show that below the following line there is continuous RSB.

$$a_T^g(t,b) = (2r)^{2/3}\left(3 - \frac{2}{r}\right); \qquad r = \frac{n}{4\pi\sqrt{2Git}}(1-t-b)^2$$

Liquid gains more than solid from pinning. When disorder (and thermal fluctuations) is strong it creates a Kauzmann point in which the entropy jump vanishes.

Shibauchi et al, PRB57, R5622 (1998)

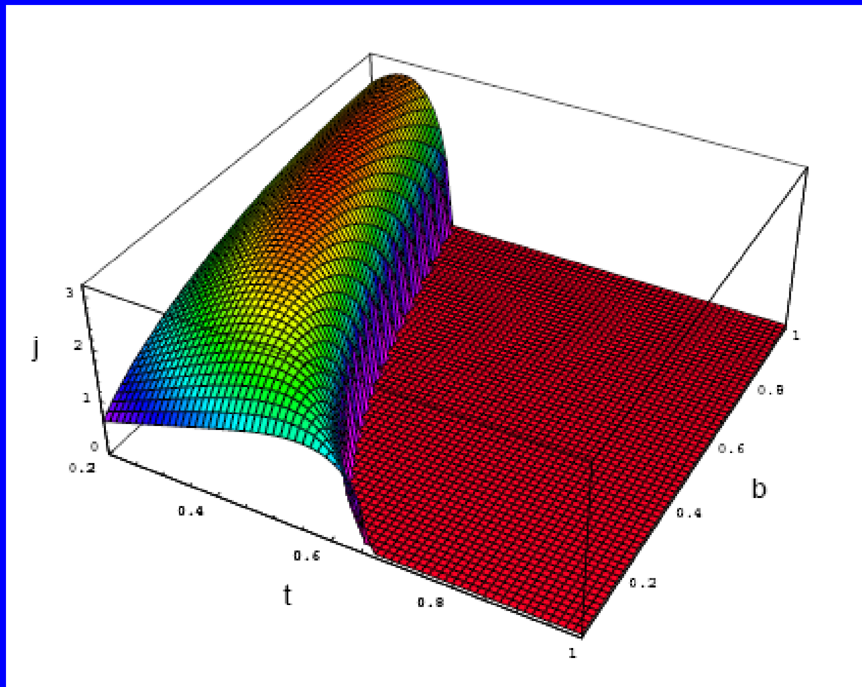
$kappa\ \text{-}(BEDT\text{-}TTF)_2Cu[N(CN)_2]Br$



Critical current

Dynamical (bulk) irreversibility surface

$J_c(T,B)$ **Critical current in the homogeneous pinned state**



$$j = J / J_0$$

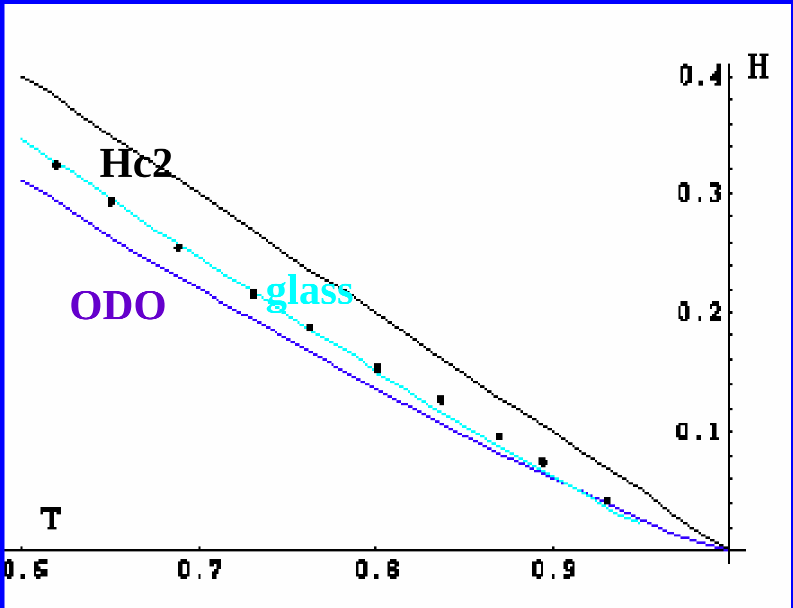
$$J_0 = \frac{16e^* T_c}{\hbar (2Gi)^{1/2} \xi_{ab} \xi_c}$$

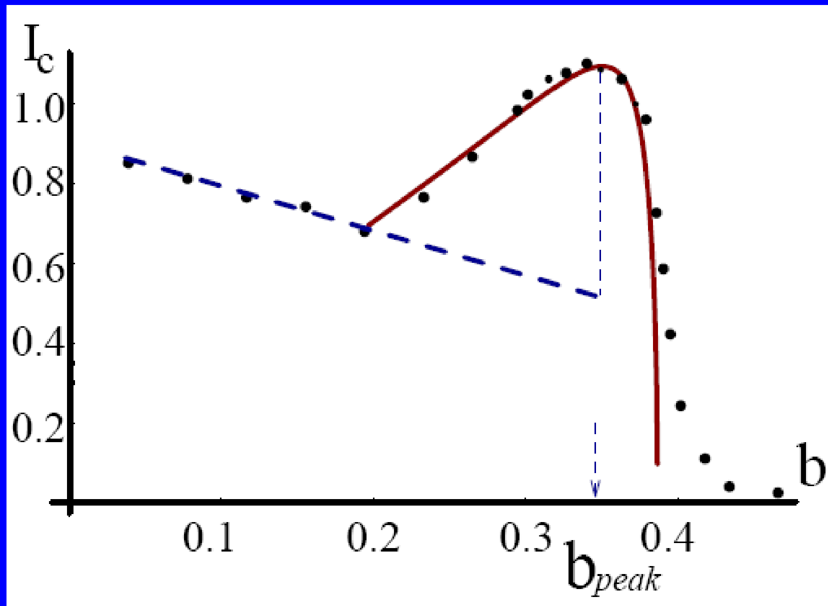
$$j_c = \frac{Gi^{2/3}}{32\pi^{4/3}} \frac{(bt)^{4/3}}{r^{1/3}} \left[a_T(t,b) - a_T^g(t,b) \right]$$

consistent with the replica results for static irreversibility line

Peak effect

Dependence of the critical current in the homogeneous state on magnetic field, is different from the monotonic dependence in the crystalline state. As a result the stable state current jumps upon crossing the ODO line.





Banerjee et al
Physica C355,39 (01)

NbSe2

Kokubo et al
PRL95,167004
(05)

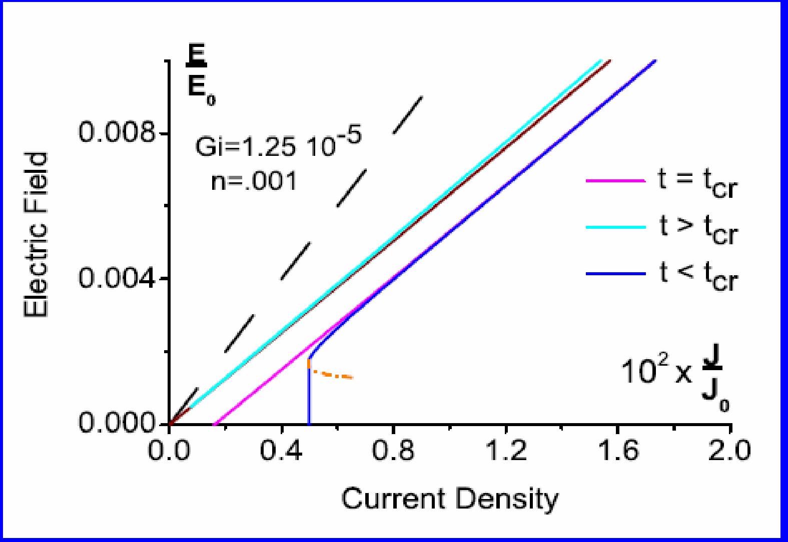
I – V curves

The LLL part is dissipative, but does not have the persistent current part which appears as a correction due to first LL

$$E_0 = \frac{16T_c}{e^*\xi} \qquad \mathrm{E} = \frac{E}{E_0} \qquad j = j_{LLL} + j_d + j_n$$

$$j_{LLL} = \frac{R_0(\mathrm{E})}{32\pi}\left(\frac{t^2 Gi}{b}\right)^{1/3} \mathrm{E}$$

$$j_d = \left(\frac{\sqrt{2}-1}{3\pi}\right)^{1/2} (Gi)^{5/6}\left(b^2 t^5\right)^{1/3} r^{1/6}$$



Where the response function is solution of

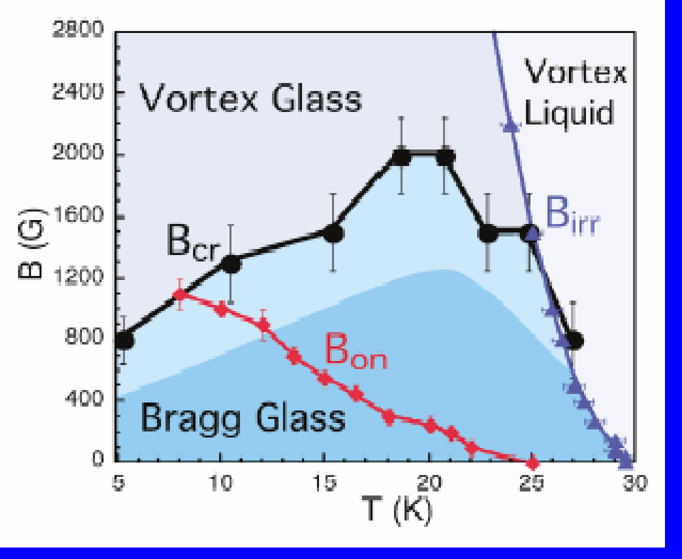
$$-4(1-r)R_0^3 - a_T(\mathrm{E})R_0^2 + 1 = 0$$

$$a_T(\mathrm{E}) = -\left(t\, b\sqrt{G\, i}\right)^{-2/3}\left(1 - b - t - \mathrm{E}^2/b^2\right)$$

Vortex matter phase diagram of a HTSC

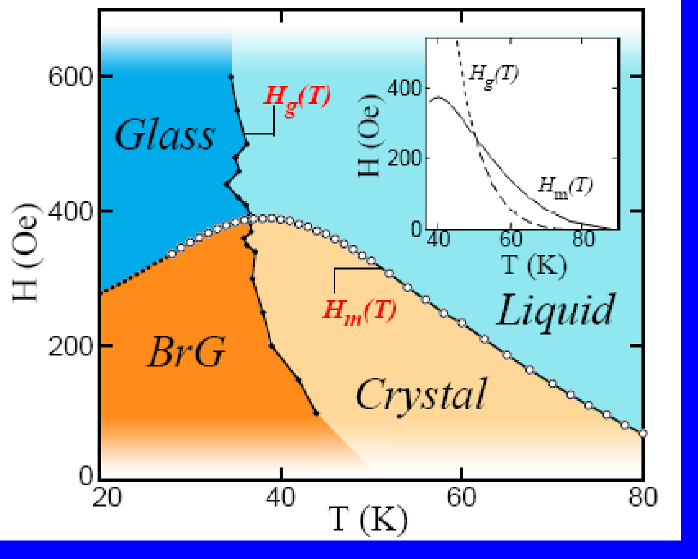
LaSCO

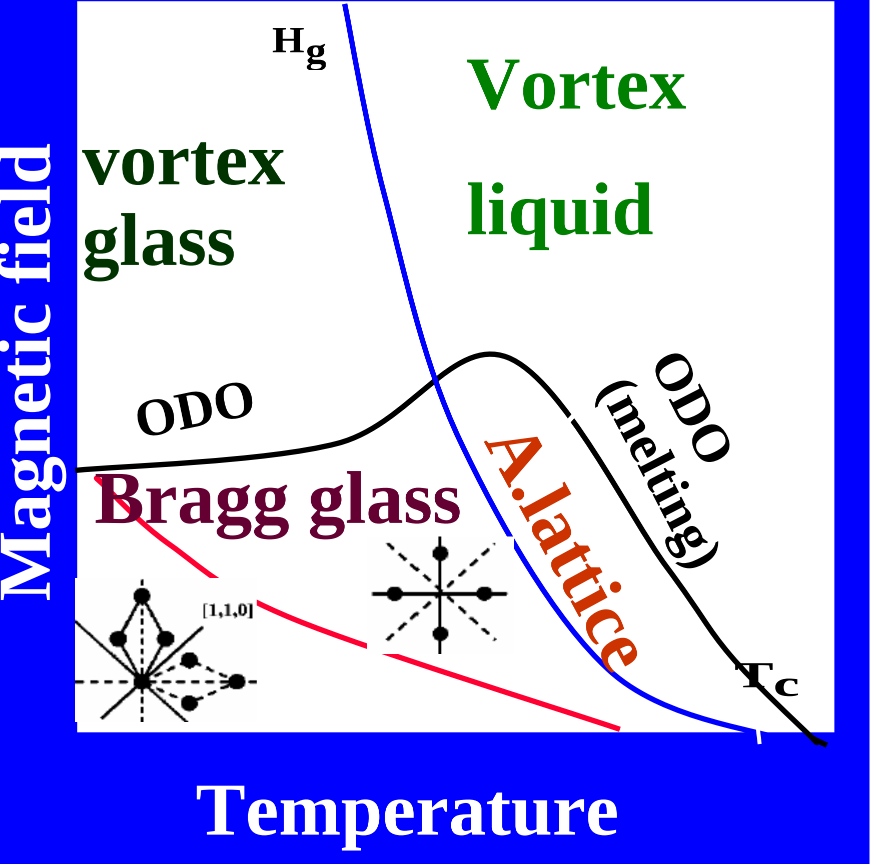
Divakar et al, PRL92,237004 (04)



BSCCO

Beidenkopf et al, PRL95,257006 (05)





Li, B.R, V.Vinokur JS&M90,167 (07)