# Quantum Chromodynamics and B Physics



### Presented at NTHU 03/02/2007

# **Titles of lectures**

- Strong Interaction
- Factorization Theorem and LHC Physics
- Quantum Mechanics in B Physics

# **Strong Interaction**

# Outlines

- Introduction
- History
- Asymptotic Freedom
- Confinement
- Summary

### Introduction

### Understood composition of our universe



### Force carriers

- Graviton propagates gravity
- Photon propagates EM force
- Gluon propagates strong interaction



#### 4 fundamental interactions



### 簡短描述

- 重力:決定星系,棒球的運動,無所不在
- 電磁力:控制電器,化學反應,摩擦力的來源,
  無所不在
- •弱作用力:宇宙存在的要素,感受不到,僅可 感受其效應---陽光
- 強作用力:產生物體90%的質量,

只存在於原子核內,感受不到

# History

# Electric Charges & Photons

- Our understanding of EM interaction at beginning of 20<sup>th</sup> century…
- EM interaction between electric charges through exchanging photons.
- Electron carries an electric charge, but a photon does not.
- Described by U(1): electron charge does not change as emitting a photon.
- No EM interaction between photons.
- All electric charges add into electric neutrality.

# Why nucleus exists?

- Nucleus full of protons, which repel each other.
- New interaction must exist, which differs from gravity and EM force.
- New interaction → new force carrier, like EM force → photon
- Yukawa called this new carrier as meson.
- Nucleus size 10E(-13) cm, where meson propagates.
- Uncertainty principle > meson mass 100 MeV
- Strong interaction appeared. Yukawa was awarded in 1949.

# **Gauge Field Theories**

- Describe interactions in the viewpoint of group theory (Yang, Mills 1954)
- Symmetry dictates interaction!
- EM (QED) weak strong (QCD)

3X3

matrix

SU(3), color charge

red

blue

green

- U(1) SU(2)
- It was a math model at that time.
- Pauli: long-range forces, but not observed

# Difficulties of gauge theory

- It is difficult to make sense out of gauge theory…
- Gauge theory is local, ultraviolet divergence (infinite energy) associated with point particle
- Systematic way to remove this divergence in order to extract physical prediction
  - $\Rightarrow$  Renormalization!
- A physical quantum field theory must be renormalizable

# **Quark Model**

- Chemical "elements" are classified according to proton numbers, leading to the periodic table.
- Many "elementary" particles were found in 60's. There should exist more elementary particles.
- Gell-Mann proposed that "elementary" particles are composed of quarks (1964).
- "Elementary" particles were classified, and new particles were predicted.

# **Eightfold Ways**

- Quark was only a math identity, not real.
- DIS at SLAC indicated existence of "quarks" inside a proton at the end of 60's.



# Deep inelastic scattering

- DIS at SLAC showed something like quarks inside a proton at end of 60's.
- Gell-Mann was awarded in 1969.
- No interaction among quarks, like free particles.
- Incredible, because interaction should get stronger, when particles are closer.
- If free particles, why can't get quarks out of a proton
- DIS was awarded in 1990.

People were puzzled, when pieces of knowledge could not be connected together

### Asymptotic Freedom

Gross, Politzer, Wilczek Nobel prize 2004

### 2004 Nobel Prize awarded to







#### David J. Gross

Kavli Institute for Theoretical Physics, University of California, Santa Barbara, USA, **H. David Politzer** 

> California Institute of Technology (Caltech), Pasadena, USA, and Frank Wilczek

Massachusetts Institute of Technology (MIT), Cambridge, USA



Violate energy conservation  $\Delta E \neq 0$ Impossible in classical mechanics h = 0Allowed in quantum mechanics As long as electrons exist in A sufficiently short time.  $\Delta t \rightarrow 0$ 

## Vacuum Polarization

Electron and positron pop out from the vacuum



#### Distance vs. energy



#### EM interaction $\implies$ screening effect



Precision measurement of hydrogen atom's energy levels has confirmed this effect, Lamb shift (Nobel prize, 1955)

# Color Charges & Gluons

- Strong interaction between color charges through exchanging gluons.
- Both quark and gluon carry color charges.
- Strong interaction between gluons.
- Electric charges: positive and negative; color charges: red, blue, green
- All color charges add into color neutrality. <sup>g</sup>

# Nonlinearity of QCD

- Gluon-gluon interaction corresponds to nonlinearity of QCD.
- Field strength between two quarks:



### Anti-screening of QCD



Anti-screening > screening

# Beta function

- T'Hooft, Veltman renormalized gauge theory in 1971.
  - Gauge theory was then applicable to meaningful calculation.
- T'Hooft, Veltman were awarded in 1999.
- Sign of beta function determines relation of interaction to energy.
- Beta function of EM (strong) force has a plus (minus) sign.
  - Gross, Wilczek (Apl. 27, 1973), Politzer (May 3, 1973) published beta function of SU(3) gauge theory.

#### This minus sign worth of million dollars





# Suddenly...

- Gell-Mann's quarks are not math identity.
- Interaction among quarks is described by gauge theory.
- Quarks behave like free particles at high energy (long distance).
  - Gauge theory is no math model, but real physics.
  - 3 in SU(3) means 3 colors  $\implies$  QCD.
- Quarks can not be separated. They form bound states at long distance.
- Yukawa's meson model describes interaction among bound states.

### Confinement

#### Next Nobel prize in QCD

# Ionization of an Atom

- Atom is a bound state of a nucleus and electrons.
- They are bounded by Coulomb's potential energy, electromagnetic (EM) interaction,
  V(r) = ((+Z) e) (-e) /r
- Can see a free electron via ionization  $A(g) \rightarrow A+(g)+e-(g)$
- Ionization energy is finite.

## **Confinement of Quarks**

- Hadron is a bound state of quarks, such as pion, proton, neutron,...
- They are bounded by strong interaction, color potential energy.
- Never see free quarks, no matter how much energy is supplied.
- Why is the confinement?



# Flux Tube

- To produce a strong potential, consider 1dim QED
- E=constant,  $V \propto r$
- Field lines are parallel
- To separate the two plates, infinite energy is needed
  - $\Rightarrow$  confinement
- Conjecture: field lines between a pair of quarks are deformed into a tube.





# String Model

- Regge trajectory
- J=E<sup>2</sup>/2πσ spin, mass





- Multiplicity
- As dosed energy exceeds the mass of quark pair, string breaks, and new pair appears








# Multiplicity dn (No. particles)/dy (rapidity)







#### Our understanding of confinement mainly comes from numerics.

No one solves QCD after decades

# Factorization theorem and LHC physics

# Outlines

- Introduction
- Factorization theorem
- Application
- LHC physics
- Summary

#### Introduction

- QCD Lagrangian  $\mathcal{L} = \overline{\psi}(iD^{\mu}\gamma_{\mu}-m)\psi F^{\mu\nu}F_{\mu\nu}/4$
- $\psi$ : quark field, F: gluon field strength
- Confinement at low energy, hadronic bound states: pion, proton, B meson,...
- Asymptotic freedom at high energy: a small coupling constant ⇒ perturbation
- Test QCD at hgih-energy scattering!
- Nontrivial due to initial hadrons
- A sophisticated prescription is necessary ---factorization theorem

#### Factorization theorem

#### Deep inelastic scattering

- Electron-proton DIS I(k)+N(p) $\rightarrow$  I(k')+X
- Large momentum transfer -q<sup>2</sup>=(k-k')<sup>2</sup>=Q<sup>2</sup>
- Calculation of cross section  $\sigma$  requires the nonperturbative quark distribution in the proton
- Is it possible to factor the perturbative part, and the nonpert part is left for other methods?



#### Feynman diagrams

- Lowest order
- Cross section= 'q |amplitude|<sup>2</sup>
  q /
- Next-to-leading order, infrared div, except for UV div



# $\begin{array}{c} \text{IR divergence is physical!} \\ \text{q} & \overbrace{t=-\infty}^{\text{Hard}} & \underset{\text{dynamics}}{\text{Soft}} & \underset{\text{scattering occurs}}{\text{Ie}} \end{array}$

- It's a long-distance phenomenon, related to confinement.
- All physical hadronic high-energy processes involve both soft and hard dynamics. How to test QCD?

#### Parton model

- The proton travels huge space-time, before hit by the virtual photon
- As  $Q^2 \rightarrow \infty$ , hard scattering occurs at point space-time
- The quark hit by the virtual photon behaves like a free particle
- It decouples from the rest of the proton
- Cross section is the incoherent sum of the scattered quark of different momentum
- Just need to know the probability of the quark carrying a momentum fraction  $\boldsymbol{\xi}$

#### Factorization formula

- Define Bjorken variable x=Q<sup>2</sup>/(2p· q)
- Following the parton model, the factorization formula for DIS
- $F(x) = \sum_{f} \int_{x}^{1} (d\xi/\xi) H_{f}(x/\xi) \phi_{f/N}(\xi)$
- F: structure function for cross section
- H<sub>f</sub>: hard kernel, cross section of the quark f, calculable in perturbation theory
- $\phi_{f/N}$  : parton distribution function (PDF) for quark f in N,  $\int_0^1 \phi_{f/N}(\xi) d\xi = 1$

#### **Factorization picture**

• Lowest-order  $H_f^{(0)}$ , all-order  $H_f$ 



### Parton distribution function

- PDF is defined by a matrix element of a nonlocal operator
- $\phi_{f/N}(\xi) = \int dy^{-}/(2\pi) \exp(i\xi p^{+}y^{-})$  $\langle N(p)|\overline{f}(y^{-})\gamma^{+}W(y^{-},0)f(0)|N(p)\rangle$
- W(y<sup>-</sup>,0): Wilson link for gauge invariance
- PDF can be computed by nonperturbative methods, like lattice QCD, or extracted from experiment data
- PDF is universal (process-independent)

#### Application

#### Hard kernel

- PDF is infrared divergent, if evaluated in perturbation confinement
- Quark diagram is also IR divergent.
- Difference between the quark diagram and PDF gives the hard kernel H<sup>DIS</sup>



#### Extraction of PDF

• Fit the factorization formula  $F=H^{DIS} \otimes \phi_{f/N}$  to DIS data. Extract  $\phi_{f/N}$  for f=u, d, s,...,g(luon)





#### **Drell-Yan process**

• Derive factorization theorem for Drell-Yan process  $N(p_1)+N(p_2) \rightarrow \mu^+\mu^-(q)+X$ 



#### Hard kernel for DY

- Compute the hard kernel H<sup>DY</sup>
- IR divergences in quark diagram and in PDF must cancel. Otherwise, factorization theorem fails



#### Prediction for DY

 Use σ<sup>DY</sup>=φ<sub>f1/N</sub> ⊗ H<sup>DY</sup> ⊗φ<sub>f2/N</sub> to make predictions for DY process



#### Factorization scheme

- Definition of an IR regulator is arbitrary, like an UV regulator:  $\phi^{(1)} \propto 1/\epsilon_{IR}$ +finite part
- Different finite parts correspond to different factorization schemes.
- Hard kernel depends on schemes
- Extraction of a PDF depends not only on powers and orders, but on schemes.
- Must stick to the same scheme. The dependence of predictions on factorization schemes would be minimized.

# LHC physics

#### LHC will answer

why masses are what they are? why neutrino masses? why symmetry breaking? why Universe dominated by matter? CP violation? why gauge interactions? why SU(3)xSU(2)xU(1)? why 3 generations? what about gravity? why 4 dimensions?

Hope to find Higgs, and new physics signals.

# CERN

- world's largest particle physics laboratory
- Proton-proton collision at E=14 TeV







#### Higgs production channels







#### Higgs production rates



#### Higgs decay modes



 $W_{+}$ H H -----.......

#### Higgs decay rates



# Search channels

- $gg \rightarrow H \rightarrow \gamma \gamma$
- Dominant background:
- QCD continuum production of γγ
- QCD  $\gamma$  jet production with jet fragmenting into  $\pi^0$
- Need to calculate these QCD backgrounds precisely



# Summary

- High-energy QCD processes must involve both perturbative and nonperturbative dynamics.
- Factorization theorem is a powerful tool for highenergy QCD processes. It is predictive.
- Factorization theorem has been extended to many processes, the PQCD approach.
- Accurate calculation of QCD background is crucial for verifying new physics at LHC
- More topics to study, such as B meson decays, CP asymmetries,...

#### Quantum Mechanics in B Physics

# Outlines

- Introduction
- Oscillation
- Basics of Particle Physics
- Particle Oscillation
- B factory
- Summary
## Introduction

- Two-level oscillation is a simple quantum mechanic phenomenon.
- Oscillation frequency reveals the energy difference of the two levels, and the interaction making the splitting.
- This idea can be used to determine fundamental parameters in the standard model of particle physics.

#### Oscillation

## Spin-orbital coupling

- Hamiltonian  $H=H_{coul}+H_{int}$ ,  $H_{int}=c \ s \cdot L$
- With only  $H_{coul},$  |I=1,s=1/2 $\rangle$  are degenerate.
- Adding H<sub>int</sub>, splitting of degenerate levels.
- Mixing matrix:  $\langle I_z, s_z | H | I'_z, s'_z \rangle = \langle H_{coul} \rangle + \langle H_{int} \rangle$

**I**=1

J<sub>+</sub>=3/2

- $\langle H_{coul} \rangle$ : diagonal  $\propto \delta(l_z, l'_z) \delta(s_z, s'_z)$
- $\langle H_{int} \rangle$ : non-diagonal
- Diagonalization gives  $|j,j_z\rangle$  as linear combination of  $|l_z,s_z\rangle$ .

## Splitting of energy levels

- Small c, small splitting, oscillation.
- Oscillation frequency is related to the splitting, and to c.
- Consider



• Ask for the probability of finding electron at the original state.

## Oscillation

- Initial condition,  $||_z,s_z,t=0\rangle = a|j_+,t=0\rangle + b|j_-,t=0\rangle.$
- $||_z,s_z,t\rangle = a \exp(-iE_t)|j_+,t=0\rangle$ +b  $\exp(-iE_t)|j_-,t=0\rangle$
- $\langle I_z, s_z, t=0 \rangle | I_z, s_z, t \rangle = a^2 \exp(-iE_t) + b^2 \exp(-iE_t)$ .

2

4

t

10

8

• Probability =  $a^4 + b^4$ + $2a^2b^2\cos(\Delta E t)$  $\Delta E = E_+ - E_-$ 

Electron goes to other  $|I_z, s_z\rangle^{0.3}$ 

## Complex H<sub>int</sub>

- Imagine complex H<sub>int</sub> Hermitian  $\langle \mathsf{H} \rangle = \begin{pmatrix} E & E_{12} + i\varepsilon \\ E_{12} i\varepsilon & E \end{pmatrix} = \begin{pmatrix} E & E_{12} \\ E_{12} & E \end{pmatrix} + i\varepsilon \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
- Diagonalization of the first matrix gives the eigenstates  $|j_{+}\rangle$ ,  $|j_{-}\rangle$ .
- The second matrix gives nontrivial mixing between  $|j_{+}\rangle$ ,  $|j_{-}\rangle$ .
- True eigenstates,  $|\mathbf{j'}_{+}\rangle = |\mathbf{j}_{+}\rangle + \varepsilon |\mathbf{j}_{-}\rangle, |\mathbf{j'}_{-}\rangle = |\mathbf{j}_{-}\rangle - \varepsilon |\mathbf{j}_{+}\rangle$

### **CP** violation



Weak decay

d

- $\beta$  decay:  $n \rightarrow p e^{-}v_e$
- Quark level d  $\rightarrow$  u e- $v_e$
- Fermi's weak theory inspired by EM
- Decay amplitude:  $G[u\gamma_{\mu}d][e\gamma^{\mu}v]$
- G: phenomenological Fermi constant
- The current  $u\gamma_{\mu}d$  conserves parity.
- $\theta$ - $\tau$  puzzle: same mass,  $\theta \rightarrow 2\pi$ ,  $\tau \rightarrow 3\pi$
- $K^+ \rightarrow 2\pi, 3\pi$ , parity violation (Lee, Yang 56)



 $v_{e}$ 

U

e<sup>-</sup>



- Exp evidence (Wu):  ${}^{60}\text{Co} \rightarrow {}^{60}\text{Ni} e_L v_{eR}$ • J=5. 4
- Weak theory:  $G[u\gamma_{\mu}(1-\gamma_{5})d][e\gamma^{\mu}(1-\gamma_{5})v_{\beta}]$
- God chose it! No right-handed current.
- SU(2) doublet  $(u,d)_L$ ,  $(v_e,e_{-})_L$ ,  $(v_{\mu},\mu_{-})_L$

## Impact from kaon decay

- Naturally, postulate doublet (c,s)<sub>L</sub>
- Amplitude:  $G[c\gamma_{\mu}(1-\gamma_{5})s][l \gamma^{\mu}(1-\gamma_{5})v_{l}]$
- G: universal Fermi constant



- u couples to s?
- Introduce a new coupling constant?

## Cabbibo angle

- Instead of new coupling, Cabbibo proposed "quark mixing" (63).
- Weak eigenstate vs. mass eigenstate

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

 $\theta_C\approx 13^o$  Cabbibo angle

- Doublets (u,d')<sub>L</sub>, (c,s')<sub>L</sub>
- Beautiful phenomenology!





#### **Particle Oscillation**

# K-K oscillation

- C: charge conjugate.  $CP|K^0\rangle = |K^0\rangle$
- Oscillation between K<sup>0</sup> and K<sup>0</sup> through a box diagram



## CKM matrix

- CP violation implies nontrivial admixture,  $|K'_{S}\rangle = |K_{S}\rangle + \epsilon |K_{L}\rangle, |K'_{L}\rangle = |K_{L}\rangle - \epsilon |K_{S}\rangle$
- The mixing matrix must be complex.
- To get it, Kobayashi and Maskawa (73) proposed the third generation of quarks.
- Doublets (u,d')<sub>L</sub>, (c,s')<sub>L</sub>, (t,b')<sub>L</sub>
- Top and bottom were not discovered then.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



#### Weak phase

- In 2-generation model, no physical weak phase in mixing matrix, since all phases can be absorbed into quark fields.
- The minimal number of generations for complex mixing matrix is 3.
- 18-9(unitarity)-5(unphysical)=3(rotation)+1
- Again, beautiful phenomenology!

$$\begin{array}{c|cccc} & \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ \end{pmatrix} & \overline{\mathsf{K}^{0}} & \underbrace{\mathsf{u},\mathsf{c},\mathsf{t}}_{\mathsf{d}} & \underbrace{\mathsf{u},\mathsf{c},\mathsf{t}}_{\mathsf{d}} & \underbrace{\mathsf{u},\mathsf{c},\mathsf{t}}_{\mathsf{d}} & \underbrace{\mathsf{u},\mathsf{c},\mathsf{t}}_{\mathsf{d}} & \mathsf{K}^{\mathsf{0}} \\ \hline \mathsf{d} & \mathsf{V}_{\mathsf{td}} & \mathsf{s} \\ \end{array}$$

# Measure $V_{CKM}$

- Re  $V_{CKM}$ : splitting or oscillation frequency
- Im  $V_{CKM}$ : CP violation
- Measure K- $\overline{K}$  mixing, determine V<sub>CKM</sub>
- But Im  $[V_{td}V_{ts}^*]$  is too small!
- Unitarity: V<sup>†</sup> V=I (magnitude of a vector is invariant under rotation).
- $V_{ud}V_{us}^*+V_{cd}V_{cs}^*+V_{td}V_{ts}^*=0.$
- A flat CKM triangle.  $V_{ud}V_{us}^*$

$$V_{cd}V_{cs}^{*}$$

 $V_{td}V_{te}^{*}$ 



- An ideal but small CKM triangle (Bigi, Sanda).
- $V_{ud}V_{ub}^*+V_{cd}V_{cb}^*+V_{td}V_{tb}^*=0.$
- Relatively large Im[V<sub>td</sub>V\*<sub>tb</sub>]



## How?

• In kaon case, just produce K<sub>0</sub>



- In B meson case,  $\tau(B_S) \approx \tau(B_L)$ . The above strategy does not work.
- To produce abundant b quarks, use electron collider with E at the threshold. B and B are produced at the same time.
- Need a different strategy!

# Bigi, Sanda's clever idea

Nuclear Physics B193 (1981) 85-108 © North-Holland Publishing Company



#### Need millions of B, challenge for experimentalists **NOTES ON THE OBSERVABILITY OF** *CP* **VIOLATION IN B DECAYS**

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We describe a general method of exposing CP violations in on-shell transitions of B mesons. Such CP asymmetries can reach values of the order of up to 10% within the Kobayashi-Maskawa model for plausible values of the model parameters. Our discussion focuses on those (mainly non-leptonic) decay modes which carry the promise of exhibiting clean and relatively large CPasymmetries at the expense of a reduction in counting rates. Accordingly we address the complexities encountered when performing CP tests with a high statistics B meson factory like the  $Z^0$  (and a toponium) resonance.



#### **B** Factories

#### Time-dependent CP Violation

• Still measure CP violation, but different

$$A_{CP}(\Delta t) = \frac{\Gamma(\overline{B}^{0}(\Delta t) \to f_{CP}) - \Gamma(\overline{B}^{0}(\Delta t) \to f_{CP})}{\Gamma(\overline{B}^{0}(\Delta t) \to f_{CP}) + \Gamma(\overline{B}^{0}(\Delta t) \to f_{CP})}$$

- Recall  $\underset{K_0}{K_0}=(1+\epsilon) \underset{S}{K'_{S}} + (1-\epsilon) \underset{L}{K'_{L}}, K'_{CP} \neq 0.$
- To produce abundant B mesons, E(e<sup>+</sup>e<sup>-</sup>)=Mass(BB).
- Then the two B meson sit at rest. How to distinguish B or B meson decay?
- Very clever idea: asymmetric collider!!

#### Time-dependent CP asymmetry measurement



#### **Babar at SLAC**



#### KEKB & Belle Layout





Amplitude (CP violation) related to Im  $V_{CKM}$ weak phase sin(2 $\phi_1$ ) or sin(2 $\beta$ )

Frequency related to mass (energy) difference, Re V<sub>CKM</sub>

## Summary

- Simple Quantum Mechanics is useful in determining fundamental parameters in the standard model.
- Beauty of phenomenology is appreciated in exploring property of fundamental interaction.
- Cabbibo: quark mixing, KM: CP model. All are insightful.
- Measurement of  $sin(2\phi_1)$  from oscillation requires clever theoretical and experimental ideas.