Ph.D. Qualify Exam: **Classical Electrodynamics**

September 2011

1. (10%, 10%) Green function for the wave equation

(a) Find the Green's function $G(\vec{r},\vec{r}')$ which is the solution of $(\nabla^2+k^2)G(\vec{r},\vec{r}')=-\delta(\vec{r}-\vec{r}')$. [Hint: Assume no boundary surface and depend only on $R=\left|\vec{r}-\vec{r}'\right|$.]

(b) Use the results of (a) to obtain the retarded potential

$V(\vec{r},t)=\dfrac{1}{4\pi\varepsilon_0}\displaystyle\int\frac{\rho(\vec{r},t-\left|\vec{r}-\vec{r}'\right|/c)}{\left|\vec{r}-\vec{r}'\right|}d^3r'$ from the wave equation $V(\vec{r},t)$ under Lorentz gauge:

$\nabla^2V(\vec{r},t)-\dfrac{1}{c^2}\dfrac{\partial^2V(\vec{r},t)}{\partial t^2}=-\dfrac{\rho(\vec{r},t)}{\varepsilon_0}$. [Hint: Express $V(\vec{r},t)$ and $\rho(\vec{r},t)$ as Fourier integrals.

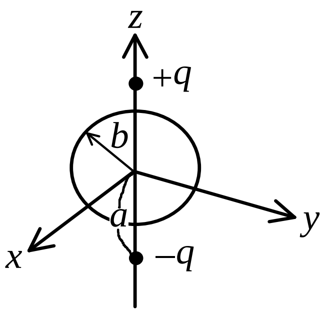
[Note: All quantities in this question are expressed in SI units]

2. (10%, 10%) Electrostatics

Two point charges $+q$ and $-q$ are located on the z axis at $z=+a$ and $z=-a$, respectively.

(a) Find the electrostatic potential as an expansion in spherical harmonics and powers of r for $r>a$ in the absence of the grounded sphere.

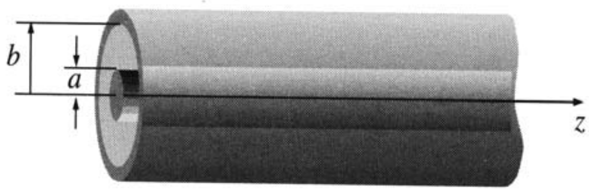
(b) If there is a grounded conducting sphere of radius b ($b<a$) centered at the origin, use the linear superposition to satisfy the boundary conditions and find the potential everywhere for $r\gg a$.



3. (10%, 10%) For a dielectric-filled $(\varepsilon=4\varepsilon_0,\ \mu=\mu_0)$ coaxial waveguide with inner radius a and outer radius b as shown below,

(a) find the electric and magnetic fields,

(b) and find the time-averaged energy density u and the energy flow **S** (energy per unit area per unit time) along the line.



4. (10%, 10%) Magnetostatics

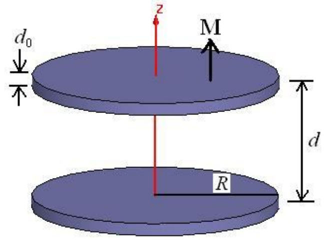
Consider two thin ferrite disks of radius R and thickness d_0, separated by a distance d ($>>d_0$). Assume the ferrite disks carry a uniform magnetization, $\mathbf{M} = M_r \hat{z}$.

(a) Find the bound surface currents $\mathbf{K}_b$ on the outer surfaces of the ferrite disk and the bound volume current density $\mathbf{J}_b$.

(b) Find the magnetic field at the midpoint of the central axis when $d=R$.

[Hint: The arrangement is similar to a Helmholtz coil.]

[Hint: $\mathbf{A}_M(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{\nabla' \times \mathbf{M}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x' + \frac{\mu_0}{4\pi}\oint_S \frac{\mathbf{M}(\mathbf{x}') \times \hat{\mathbf{n}}'}{|\mathbf{x}-\mathbf{x}'|} da'$].



5. (10%, 10%) Wave: skin depth

(a)Starting from the Maxwell equations, derive the dispersion relation (i.e. the relation between the wave frequency ω and the propagation constant k) for a plane electromagnetic wave in an infinite and uniform medium of conductivity σ, electrical permittivity ε, and magnetic permeability μ.

(b)Assume that the medium is a good conductor, derive an expression for its skin depth δ.

[vector formula: $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$]