

Qualification Exam for Statistical Mechanics (September 2011)

You should derive your results step by step clearly.

1. (10%) Consider a model of gas where all particles move with the same speed v in random directions. Calculate the average value of the relative speed.
2. (15%) An ideal classical gas composed of N particles, each of mass m , is enclosed in a vertical cylinder of height L , placed in a uniform gravitational field of acceleration g and is in thermal equilibrium. Evaluate the partition function of the gas.
3. (10 %) Estimate how long (in seconds) it takes a molecule of air to move to a position 5 meters away. Assume that the mean free path is 5×10^{-6} m and the average speed is 500 m/s.
4. (15 %) The atmosphere is often in a state of constant entropy ($PV^\gamma = \text{constant}$). In a uniform gravitational field, find (dT/dz) where T is the temperature at altitude z .
5. (10 %) Calculate the density of (quantum mechanical) states of a particle with mass m confined in a cubic box with volume V , constant k is known to have a total energy E , but its starting time is not known. Find the probability density function, $P(x)$, where $P(x)dx$ is the probability that the mass would be found in the interval dx at x .
6. (15%) A classical harmonic oscillator of mass m and spring constant k is known to have a total energy E , but its starting time is not known. Find the probability density function, $P(x)$, where $P(x)dx$ is the probability that the mass would be found in the interval dx at x .
7. (10 %) A crystal consists of N identical atoms. At very low temperature, the interaction between atoms can be neglected. If the ground state of each atom is doubly degenerate, what is the entropy of the crystal at zero absolute temperature?

8. (15%) Consider the Ising model on a one dimensional open chain. The Hamiltonian is

$$H = -J \sum_{i=1}^{N-1} \sigma_i \sigma_{i+1}$$

where $\sigma_i = 1$ or -1 . Calculate the partition function

$$Q_N = \sum_{\sigma_1} \cdots \sum_{\sigma_N} \exp\left(-\frac{H}{kT}\right).$$